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Method of resolving functions for the pursuit game with a pure time-lag

Consider a controlled object whose dynamics is described by the linear pure time-lag differential system in an Euclidean space R^n:

$$\dot{z}(t) = Bz(t-\tau) + \phi(u, v), \qquad z \in \mathrm{R}^n, \ u \in U, \ v \in V, \tag{1}$$

where B is a square constant matrix of order n; U and V are nonempty compact sets; the function $\phi(u, v)$, $\phi : U \times V \to \mathrm{R}^n$, is jointly continuous function of its variables, u and v are the control parameters of the pursuer and the evader chosen from the control domains U and V, respectively; z is the state vector, involving geometric coordinates, velocities, accelerations of the pursuer and the evader; $\tau = const > 0$.

The initial condition

$$z(t) = z^0(t), \qquad -\tau \leqslant t \leqslant 0, \tag{2}$$

is absolutely continuous function.

The terminal set M^* is a cylinder:

$$M^* = M_0 + M, \tag{3}$$

where M_0 is a linear subspace of the space R^n, and M is a compact set in the orthogonal complement L to M_0 in R^n.

The players choose their controls in the form of certain functions [1]. In this way the players affect the process (1) pursuing their own goals. The goal of the pursuer is to bring a trajectory of the process to the terminal set M^* (3) in the shortest time. The goal of the evader is to avoid the meeting of the trajectory of the process with this set on a whole semi-infinite interval of time or if it's impossible to maximally postpone the moment of meeting.

We consider the local problem of approach within fixed interval of time [1]. If the controls are chosen in the form of Lebesgue measurable functions $u(s)$ and $v(s)$ [2], which take its values in sets U and V, respectively, then a solution of the system (1) with the initial condition (2) can be presented by the formula:

$$z(t) = exp_\tau\{B,t\}z^0(-\tau) + \int_{-\tau}^{0} exp_\tau\{B, t-\tau-s\}z'^0(s)\,ds + \int_0^t exp_\tau\{B, t-\tau-s\}\phi(u,v)\,ds,$$

where $exp_\tau\{B,t\}$ is the time-lag exponential [3], $t \geqslant 0$.

The game is evolving on the closed time interval $[0;T]$, where T, $T > 0$, is the end point of game, and it is such that $z(T) \in M^*$ or $\pi z(T) \in M$. Here π is the orthogonal projector from R^n onto the subspace L.

Consider the multivalued mappings

$$W(t,v) = \pi exp_\tau\{B,t\}\phi(U,v), \qquad W(t) = \bigcap_{v \in V} W(t,v).$$

Pontryagin's condition. The mapping $W(t) \neq \emptyset$ for all $t \geqslant 0$.

For the game, described above, satisfying Pontryagin's condition, the scheme of the method of resolving functions is developed. The guaranteed time of the game termination is found, and corresponding control law is constructed.

References. **1.** Chikrii, A.A. (2013). Conflict-Controlled Processes. Springer Science & Business Media. **2.** Baranovskaya, G.G., & Baranovskaya, L.V. (1997). Group Pursuit in Quasilinear Differential-Difference Games. Journal of Automation and Information Science, 29(1), 55–62. **3.** Хусаинов Д.Я., Диблик Й., Ружичкова М. Линейные динамические системы с последействием. Представление решений, устойчивость, управление, стабилизация. – Киев: ГП Информ.-аналит. агенство, 2015. – 252 с.