PHYSICS AND MATHEMATICS

MIXED STRATEGY NASH EQUILIBRIUM IN ONE GAME AND RATIONALITY

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Abstract. *We consider some card game as a simple two person game. This game was solved for the mixed strategy Nash equilibrium. It gives us game theoretic predictions. We compared the experimental results, our behavior and the Nash equilibrium prediction.*

Keywords: *game theory, mixed strategy Nash equilibrium, two-person zerosum games.*

People often say that game theory is a science about rationality. So we're going to examine the relationship between people's rationality and the outcome of games. And we're going to consider the whole spectrum of intellectual capacity of the players. Let us consider a simple card game, whish was first invented by Barry O'Neill [1]. There are two players, red and black. And each player has those four cards: King (K), Ace (1), 2, 3. Each player carefully chooses one card and shows it simultaneously to their opponent. If both players choose K, red wins. And if players choose different number the card like 1 and 3 also red wins. In all other cases, black wins. So black player wins if only one player chooses king or players choose cards with the same number such as 1 and 1.

As we can see rules are not symmetric, so maybe one of the players may have an advantage. So we have first question, who has as advantage, red or black? Second question, in particular, what is the winning rate of each player? What's the probability of winning of red and black? The third question, can you say anything about the distribution of cards of each player? We can get some idea about the first question by inspecting the nature of the game. But the second and third question, calculating the winning rate of each player and calculating the distribution of cards of each player, it's a hard question.

Let's formulate this card game as a game. We can write down payoff table, and calculate which strategy Nash equilibrium. So the only equilibrium here is a mixed strategy equilibrium. So let's find a mixed strategy Nash equilibrium where each player mixes those four cards with certain probabilities.

First, we have to write down the payoff table (see Figure 1). The rules say red player wins if both players choose K, or different numbers are chosen. So if both players choose K, red player wins, and his payoff is 1, and black player's payoff is 0. And also, red player wins if they choose different numbers such as 1 and 2 and 1 and 3. And also red player wins and obtains payoff 1, and black player loses, his payoff is 0. In all other cases, black player wins.

Red \ Black	K	3	2	1
K	1;0	0;1	0;1	0;1
3	0;1	0;1	1;0	1;0
2	0;1	1;0	0;1	1;0
1	0;1	1;0	1;0	0;1

Fig. 1.

According to the Nash theorem [2], there exists at least one Nash equilibrium, possibly involving mixed strategies. Let's try to find the mixed strategy equilibrium [3,4]. We have to determine the probability distribution over black player's strategies, K, 1, 2, and 3. So let's suppose black player chooses 1 with probability p, 2 with probability q, 3 with probability r. With the remaining probability, $1 - p - q - r$, the black player chooses K. Our task is to determine those three numbers. But since those three numbers, cards 1, 2, 3 have the very similar roles, let's guess that black player chooses each number card with an equal probability of p. Then, with the remaining probability of $1 - 3q$ black player chooses K. So a profile of mixed strategies is an equilibrium if no player has some strategy that would offer a better payoff than his mixed strategy in reply to the mixed strategies of other players [3]. And our task is to determine this number p. Supposing we are red player, and supposing that our opponent, the black player, is mixing our cards with this probability distribution, what happens to us if we chooses K? So with probability of $1-3p$ our opponent also chooses K, and we win. For all other cases, we lose. So if we chose king, our probability of winning is $1-3p$. Our winning rate, winning probability, is $1-3p$, if we choose K. So we can perform a similar computation. What if we choose 1 as the red player? So we win if our opponent chooses different number cards 2 and 3. In those cases we win, and therefore our probability of winning is $p+p$, that's $2p$. And the situation is very similar to card 2 and card 3. And in those cases, our winning probability is the same, $2p$. Since in the equilibrium probability red player is mixing all those cards, K, 1, 2, 3, those numbers here should be identical. They are equally good. And therefore, red player is mixing between K, 1, 2, and 3. So equilibrium says that those four numbers, or two number, $1-3p$ and $2p$ should be equal. Formally, if red player best-responds with a mixed strategy, black player must make him indifferent between K, 1, 2, 3:

$$U_1(\text{K}, 1\text{-}3p, p, p, p) = U_1(1, 1\text{-}3p, p, p, p) \text{ or } U_1(2, 1\text{-}3p, p, p, p), \text{ or } U_1(3, 1\text{-}3p, p, p, p).$$

Thus we have one equation for one unknown p: $1-3p=2p$, so $p=0.2$ and $2p=0.4$. And by the equilibrium condition, $1-3p=0.4$. So the winning rate of red player is always 0.4. And the probability that black player chooses a number card is 0.2. So we have found those numbers, 0.2 for black players. With the remaining probability, the black player chooses king. So the probability is 0.4 here. This is the equilibrium distribution of cards by a black player. And we have calculated the winning rate of red player, and that was 0.4. So we can perform a similar exercise to determine red player's probability distribution of K, 1, 2, and 3.These results of calculation are shown in Figure 2:

Probability distribution (red)	**Black** / **Red**	***K***	***3***	***2***	***1***
0.4	***K***	1;0	0;1	0;1	0;1
0.2	***3***	0;1	0;1	1;0	1;0
0.2	***2***	0;1	1;0	0;1	1;0
0.2	***1***	0;1	1;0	1;0	0;1
	Probability distribution (black)	↓ **0.6**	↓ **0.6**	↓ **0.6**	↓ **0.6**

Fig. 2.

Let's examine what this result says about the winning rate. Winning rate of red player is 0.4 and, of course, the winning rate of black is the remaining probability, 0.6. As we can see this game favors black player (black player is stronger). And his winning rate should be 60%. It's a very "sharp" prediction.

Then, let's consider the distribution of cards. King is played most often with probability 0.4. And numbers have equal probability of being played. So each number card up here has probability of 0.2.

So let's compare this equilibrium prediction with the actual data. We'd like to explain the results of the card game students of Professor Baranovska L.V. (National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute") played in 2017. A number of people 22 pairs were participated in card game for 30 times. Actually in this game, black player is stronger than red player, and more precisely, the winning rate of black player is 0.5955, and the winning rate of red player is 0.4045.

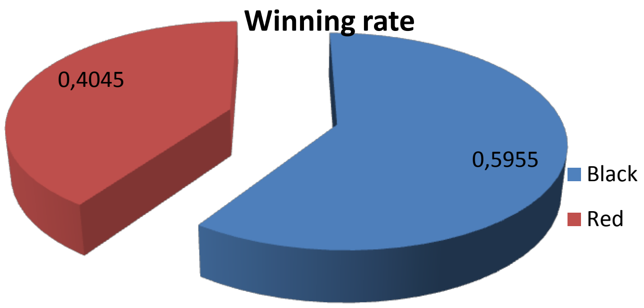

Fig. 3.

As we can see the game theoretic prediction worked amazingly well in terms of the winning rates.

So next we are going to explain the equilibrium prediction of the distribution of the cards. In Nash equilibrium, king should be chosen with a large probability, in particular, 0.4 and the number cards 1, 2, and 3 should be chosen with a smaller probability of 0.2. Those are the equilibrium prediction and let's show the actual data.

Table 1.

№	*RedWins*	*BlackWins*	*rK*	*r1*	*r2*	*r3*	*bK*	*b1*	*b2*	*b3*
1	14	16	12	5	7	6	10	5	8	7
2	6	24	2	6	18	4	12	8	8	2
3	8	22	8	9	6	7	13	7	4	6
4	13	17	13	8	6	3	11	8	7	4
5	15	15	16	5	4	5	14	11	2	3
6	11	19	10	8	5	7	12	8	5	5
7	9	21	8	8	6	8	17	4	6	3
8	11	19	12	8	7	3	10	4	8	8
9	15	15	10	8	7	5	14	9	3	4
10	13	17	12	7	7	4	12	9	2	7
11	12	18	9	7	7	7	12	7	7	4
12	12	18	10	5	7	8	18	7	2	3
13	16	14	17	4	6	3	12	10	4	4
14	12	18	13	7	4	6	11	7	8	4
15	13	17	16	4	6	4	14	9	4	3
16	14	16	8	7	7	8	13	7	5	5
17	11	19	13	5	3	9	5	8	8	9
18	14	16	11	7	6	6	14	6	2	8
19	15	15	15	7	3	5	17	3	4	6
20	8	22	6	9	7	8	11	9	4	6
21	15	15	12	5	6	7	18	6	3	3
22	10	20	7	7	8	8	14	5	6	5
	12,14	*17,86*	*10,91*	*6,64*	*6,50*	*5,95*	*12,91*	*7,14*	*5,00*	*4,95*

The results of our distributions are presented in Figure 4.

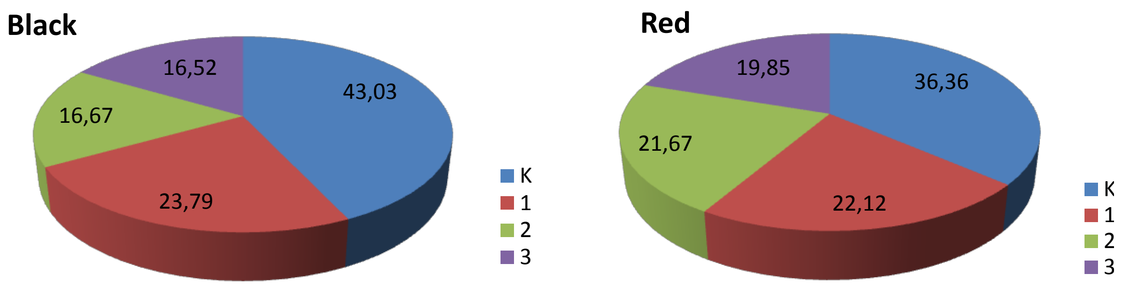

Fig.4.

Game theoretic prediction worked amazing. We've discovered that people's behavior in this card game is amazingly close to Nash equilibrium prediction. Let's compare the results that we obtained with other results. So the first card game was conducted in 2015 with students of Coursera [4] (in 670 pairs 30 times); the second and third ones were conducted in 2014 with students of Professor Michihiro Kandori in his game theory class at the University of Tokyo [4].

	Red	Black
Nash equilibrium	**0.4**	**0.6**
Students of Coursera (2015)	0.415	0.585
Students of University of Tokyo (2014)	0.414	0.586
Students of University of Tokyo (2009)	0.409	0.591
Students of NTUU "KPI" (2017)	0.4045	0.5955

Fig. 5.

The distributions of cards are presented in Figure 6.

	Red				Black			
	K	A	2	3	K	A	2	3
Nash equilibrium	**0.4**	**0.2**	**0.2**	**0.2**	**0.4**	**0.2**	**0.2**	**0.2**
2015	0.36	0.24	0.21	0.2	0.37	0.24	0.19	0.2
2104	0.38	0.21	0.2	0.21	0.44	0.2	0.17	0.18
2009	0.39	0.21	0.2	0.2	0.42	0.2	0.19	0.19
2017	0.36	0.22	0.22	0.2	0.43	0.24	0.17	0.17

Fig. 6.

As we can see in all those experiments, the outcome was amazingly close to

the equilibrium prediction. Surprisingly, people's behavior is closely predicted by Nash equilibrium! So in this card game, given that other players are choosing an equilibrium strategy, it's best for you to follow this equilibrium, so you are naturally attracted to the behavior that is best for you even if you have a free will. And game theory is a very important tool to find out that mechanism operating behind people's behavior. But people may not be so rational and not be completely stupid. So we are going to talk about absolutely zero-intelligent case. What happens if players absolutely have no intelligence? Outcome may be chaotic. And maybe game theory doesn't have anything to say about such a situation. So this is a very important question, which is systemically addressed by economists and game theorists. Let's try to find a dominant strategy simulating experiment simulating game for not rational players.

This is a game generator designed for testing game-theoretic algorithms:

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

intmain(){
    srand(time(NULL));
    inta[4]={0,0,0,0},
                wa[4]={0,0,0,0},
                b[4]={0,0,0,0},
                wb[4]={0,0,0,0};
    double t;
    inti,k1,k2;

    for(i=1;i<=10000;i++){
                k1=rand()%4;
                a[k1]++;
                k2=rand()%4;
                b[k2]++;
                if(k1 == 0 && k2 == 0) wa[k1]++;
                            else if(k1 == k2) wb[k2]++;
                                        else if(k2 == 0 || k1 == 0) wb[k2]++;
                                                    else wa[k1]++;
    }
    printf("\n\nRed:\n");
    printf("K: total=%d win=%d rate=%lg%%\n",a[0],wa[0],100.0*wa[0]/a[0]);
    printf("3: total=%d win=%d rate=%lg%%\n",a[1],wa[1],100.0*wa[1]/a[1]);
    printf("2: total=%d win=%d rate=%lg%%\n",a[2],wa[2],100.0*wa[2]/a[2]);
    printf("A: total=%d win=%d rate=%lg%%\n",a[3],wa[3],100.0*wa[3]/a[3]);
    printf("Total win rate: %lg%%",100.0*(wa[0]+wa[1]+wa[2]+wa[3])/10000);
    printf("\n\nBlack:\n");
    printf("K: total=%d win=%d rate=%lg%%\n",b[0],wb[0],100.0*wb[0]/b[0]);
    printf("3: total=%d win=%d rate=%lg%%\n",b[1],wb[1],100.0*wb[1]/b[1]);
    printf("2: total=%d win=%d rate=%lg%%\n",b[2],wb[2],100.0*wb[2]/b[2]);
    printf("A: total=%d win=%d rate=%lg%%\n",b[3],wb[3],100.0*wb[3]/b[3]);
    printf("Total win rate: %lg%%",100.0*(wb[0]+wb[1]+wb[2]+wb[3])/10000);
    return 0;
}
```

The data obtained in simulation series 10000 parties played by the principle of random principle. So this is our dataset:

Red:
K: total=2483 win=630 rate=25.3725%
3: total=2484 win=1261 rate=50.7649%
2: total=2566 win=1262 rate=49.1816%
1: total=2467 win=1228 rate=49.7771%
Total win rate: 43.81%

Black:
K: total=2489 win=1859 rate=74.6886%
3: total=2524 win=1240 rate=49.1284%
2: total=2570 win=1297 rate=50.4669%
1: total=2417 win=1223 rate=50.5999%
Total win rate: 56.19%

As we can see black player is stronger than red player, and the winning rate of black player is 0.5516, and the winning rate of red player is 0.4381. Still game theory can predict individuals' behavior. As a closing remark, in such game, Nash equilibrium might emerge under very different reasons. So Nash equilibrium may emerge by very careful and sophisticated reasoning. Or Nash equilibrium may emerge out of very low rational trial and error adjustment process. So Nash equilibrium is expected to emerge under a wide range of intellectual capacities of players.

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NUMERICAL SIMULATION OF THE PROBLEM OF THE DAM BREAKS IN THE OPENFOAM PACKAGE

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At present, various methods are used to determine the parameters of breakthrough waves, for example, field studies, physical experiments, numerical modeling in a one-dimensional or two-dimensional formulation of the problem. Within their limits of applicability, they make it possible to predict the appearance time of the leading edge of the wave, the boundaries of the flooding zones, the depth and duration of flooding of the terrain. The actual task is to determine the hydrodynamic effect of the leading edge of a breakthrough wave on various engineering structures, and the existing methods of numerical calculations and models of certain physical phenomena need to be improved.

The mathematical model proposed in this paper is based on the Reynolds-averaged three-dimensional non-stationary Navier-Stokes equations, the closure of which is carried out using the standard k-ε model of turbulence. To determine the position of the free surface changing over time, the liquid volume method, implemented in the multiphase interFoam solver of the OpenFOAM 4.0 open package has been applied [1].