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PHYSICS 1. MECHANICS

Methodical instructions for laboratory works

For specialties

113 APPLIED MATHEMATICS
125 CYBERSECURITY

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Contents

TREATMENT OF EXPERIMENTAL RESULTS

Physics is an experimental science. This means that physical laws are established and verified by the accumulation and comparison of experimental data. However, the results obtained during any physical experiment always contain certain errors, since measurement is practically impossible to do with absolute precision. Possible errors play a significant role in comparing the results of the experiment with the theoretical formulas, so you need to learn how to process the measurement results.

1 Direct and indirect measurements

Measurement is the process of determining the physical value by an experimental way using special technical means. As a result of the measurement we find out how many times the measured value is more (or less) than the corresponding value taken for the unit of measurement. The measurements are direct and indirect.

Direct measurements are called the measurements, during which the required value is found directly from the experimental data.

Indirect measurements are called the measurements, during which the value is found based on the known dependence between this value and the quantities subject to direct measurement. For example, the body density of a cylindrical shape ρ is an indirect measurement and is determined by the formula

$$\rho = \frac{4m}{\pi d^2 h}$$

where the mass, diameter and height of the cylinder (m, d and h) are determined by the results of direct measurements.

2 Errors of direct measurements

The difference between the measured and the true values of the measured magnitude is called the error (measurement error). Errors in measurements of physical quantities are divided into two types: **random** and **systematic**.

2.1 Random (indeterminate) errors

Random errors are related to the measurement process. For example, by measuring the distance of the flight of a body with a roulette, it is impossible to lay it perfectly right; measuring body mass on scales, friction can not be avoided, etc. Therefore, if you perform the same measurement several times, the results will be slightly different.

Assume that, using the same equipment and measurement method, we made N measurements of the quantity x and obtained N values: $x_1, x_2, \ldots, x_N$, where the quantity x_1 is the result of the first measurement, x_2 is the second, x_N- Nth measurement. To process the results, we have to answer two questions: how to find the most probable value of the measured quantity? how to determine a random measurement error? The answers to these questions are given by probability theory and mathematical statistics.

According to the theory of probabilities, the most probable value of the measured quantity (x_{measur}) is equal to average the arithmetic value $\langle x \rangle$ obtained as a result of measurements:

$$x_{\text{measur}} = \langle x \rangle = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N} \tag{E.1}$$

More informations

Only the measurements taken under the same conditions can be averaged, even if it is considered that these conditions do not affect the measurement result. If conditions are different, for example, measurements were made in different laboratories, or the acceleration of free fall was determined on the Atwood machine for different ratios h_1 and h_2, the best estimate of the measured value is a weighted average. Determining the absolute error in this case also has its own characteristics.

Random absolute error (Δx_{rand}) — an error resulting from all measurements, is estimated using the so-called *root mean squared error* or *root-mean-square deviation* $\sigma_{\langle x \rangle}$ and is calculated by the formula:

$$\Delta x_{\text{rand}} = t_{\alpha,N} \sigma_{\langle x \rangle} = \sqrt{\frac{\sum\limits_{i=1}^{N} (x_i - \langle x \rangle)^2}{N \cdot (N-1)}}. \tag{E.2}$$

More informations

In mathematical statistics, it is substantiated that a more reliable estimate of absolute random error (confidence interval) Δx_{rand} is determined by the:

$$\Delta x_{\text{rand}} = t_{\alpha,N} \sigma_{\langle x \rangle},$$

where $t_{\alpha,N}$ — the tabular value of Student's statistical criterion for the selected reliability of the hit measured quantity in the confidence level α and the number of measurements N, $\sigma_{\langle x \rangle}$ — root-mean-square deviation.
For confidence level of $\alpha = 68\%$, sufficient for laboratory work, and the number of measurements $N \leq 10$ Student's criterion $t_{\alpha,N} \approx 1$, therefore, in the following formulas is not given.

2.2 Systematic (determinate) errors

Systematic errors (instrumental) are related to the choice of the device: it is impossible to find a roulette with a perfectly accurate scale, absolutely accurate weights, ideally equal levers. Systematic errors are determined by the quality of the device — its class, so they are often called **instrumental errors**. In Ukraine, by magnitude of error devices are divided into seven classes. The accuracy class is equal to the relative error of the instrument, given in percentages (here 0.1%, 0.2%, 0.5%, respectively). Especially precise (precision) devices used in exact scientific research are devices of classes 0.1; 0.2; 0.5 Such devices work, for example, in the pharmaceutical industry. The technique uses less precise instruments — classes 1; 1.5; 2.5; 4.

For instruments with arrows with a well-known accuracy class r and scale interval A:

$$\Delta x_{\text{syst}} = \frac{rA}{100}. \tag{E.3}$$

Devices with systematic errors used in the laboratory of physics are given in Table T.1.

Table T.1

No	Device	Value of division	Δx_{syst}
1.	Student's ruler	1 mm	±0.5 mm
2.	Ruler for measuring the position of brackets	1 mm	±0.5 mm
3.	Calipers	0,1 mm	±0.01 mm
4.	Micrometer	0,01 mm	±0.005 mm
5.	Electronic timer		±0.001 s
6.	Stopwatch		±1 s
7.	Scales training		±0.01 g

More informations

Sometimes there is no need to measure many times. For example, measuring the length of the same section with a ruler, you are unlikely to get different results. However, this does not mean that there are no random errors, because it is impossible to accurately combine the zero of the ruler scale with the beginning of the segment, in addition, it is quite probable that the end of the segment does not coincide with the division of the scale. In such cases, we assume that the random error is equal to one half of the smallest subdivision given on the measuring device.

2.3 Total error. Absolute and relative errors of direct measurements

In order to correctly evaluaΔx_{syst}) and the random error (Δx_{rand}) due to measuring errors. This total error is called the absolute measurement error (Δx) and are determined by the formula:

$$\Delta x = \sqrt{\Delta x_{\text{syst}}^2 + \Delta x_{\text{rand}}^2} \tag{E.4}$$

In this case, if one of the errors is more than three times smaller than the other one, it can be neglected.

The absolute error itself does not characterize the quality of measurement. In fact, if the distance of 10 m is measured with an error of 0.2 m, this indicates a high quality of measurement. A completely different thing, if the same error has been obtained, measuring a distance of 0.5 m. Therefore, it is better to speak of a relative error.

The relative error ε_x characterizes the measurement quality and is equal to the absolute error (Δx) to the average (measured) value of the measured quantity ($\langle x \rangle$):

$$\varepsilon_x = \frac{\Delta x}{\langle x \rangle} \cdot 100\% \tag{E.5}$$

Relative error is sometimes called *precision*.

3 Errors of indirect measurements. Absolute and relative errors indirect measurements

Many physical quantities can not be measured directly. Their indirect measurement has two steps. First, measure the values x, y, z, ..., which can be obtained by direct measurement, and then, using measured values, calculate the desired value of f. How to determine the absolute and relative errors of measurements in this case? The answer to this question is also given by probability theory.

In a particular case, if in the formula that defines the physical quantity f, only the operations of multiplication and division are present, then the relative error of this value is equal to the sum of *the relative errors of the quantities which are «included» in the formula.* The table shows a number of formulas for calculating relative errors for some functions without derivation.

The absolute error (Δf) can be found using the relative error (ε_f). In fact, by definition $\varepsilon_f = \frac{\Delta f}{f}$ from here:

$$\Delta f = \varepsilon_f \cdot f$$

Formulas for relative errors of some functions are given it Table T.2.

Table T.2

Type of formula (function)	$f = x \pm y$	$f = xy$	$f = x/y$	$f = x^n$
Relative error	$\varepsilon_f = \frac{\Delta x + \Delta y}{x \pm y}$	$\varepsilon_f = \varepsilon_x + \varepsilon_y$		$\varepsilon_f = n \cdot \varepsilon_x$

More informations

According to the theory of errors (J. Taylor's theory), the absolute error Δf of indirect measurements of the value $f(x, y, z, \ldots)$ regardless of the type of function can be calculated by the general formula:

$$Deltaf = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2 + \left(\frac{\partial f}{\partial z}\Delta z\right)^2 + \ldots} \tag{E.6}$$

or approximately:

$$\Delta f = \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y + \left|\frac{\partial f}{\partial z}\right|\Delta z + \ldots \tag{E.7}$$

where $\left|\frac{\partial f}{\partial x}\right|$ — modulus of partial derivative of the function $f(x, y, z, \ldots)$ with respect to x (during differentiation all other variables are considered to be stable), $\left|\frac{\partial f}{\partial y}\right|$, $\left|\frac{\partial f}{\partial z}\right|$ — moduluses of partial derivative of the function with respect to *other variables*, respectively.
By definition, of relative error is $\varepsilon_f = \frac{\Delta f}{f}$, then taking into account (E.6):

$$\varepsilon_f = \frac{\Delta f}{f} = \sqrt{\left(\frac{1}{f}\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{1}{f}\frac{\partial f}{\partial y}\Delta y\right)^2 + \left(\frac{1}{f}\frac{\partial f}{\partial z}\Delta z\right)^2 + \ldots} \tag{E.8}$$

Example

To determine the acceleration of body motion a path $s = 10.000$ m with an error $\Delta s = 0.005$ m at time $t = 20$ s was measured. Error measuring the time $\Delta t = 1$ s. Find the absolute error of acceleration.

We know, that $S = \frac{at^2}{2}$. From here $a = f(S, t) = \frac{2S}{t^2} = \frac{2 \cdot 10}{(20)^2} = 0,050$ m/s^2.
Acceleration a is an indirect measurement, that is, the function of direct measurements s, t.

$$\Delta f = \Delta_a = \sqrt{\left(\frac{\partial a}{\partial s}\Delta s\right)^2 + \left(\frac{\partial a}{\partial t}\Delta t\right)^2} =$$

$$= \sqrt{\left(\frac{2}{t^2}\Delta s\right)^2 + \left(-\frac{4s}{t^3}\Delta t\right)^2} = 0,005 \text{ m/s}^2$$

Result: $a = 0.050 \pm 0.005$ m/s^2.

4 How to write the measurement results correctly

The absolute error of an experiment determines the accuracy with which it makes sense to calculate the measured value.

The absolute error is always rounded off to overstatement of one significant digit, and the result of the measurement – to the same order of magnitude (located in the same decimal position) as absolute error. The final result for the value of x is written as:

$$x_{\text{meas}} = \langle x \rangle \pm \Delta x,$$

where x_{meas} — measured value.

Significant digits are all digits of the number, starting with the first digit to the left, *different from zero*, to the last digit, for the correctness of which one can «guarantee». For example, in the number 320.0 four significant digits $(3; 2; 0; 0)$, in the number 0.32 – two $(3; 2)$, in the number 0.3 – one (3).

The last formula means that the true value of the measured value lies in the interval between $x_{\text{meas}} = \langle x \rangle - \Delta x$ and $x_{\text{meas}} = \langle x \rangle + \Delta x$. The absolute error Δx is assumed to be a positive value, so $x_{\text{meas}} = \langle x \rangle + \Delta x$ is always *the most* probable value of the measured quantity, and $x_{\text{meas}} = \langle x \rangle - \Delta x$ is its *least probable* value.

Example

Let me measure acceleration g of free fall. As a result of the processing of the experimental data obtained, an average value was found: $g = 9.736\ \text{m/s}^2$. For an absolute error, $\Delta g = 0.123\ \text{m/s}^2$ was obtained. The absolute error must be rounded up to one significant digit with an overstatement: $\Delta g = 0.2\ \text{m/s}^2$. Then the result of the measurement is rounded up to the same order of magnitude as the order of error, that is, to the tenth: $g = 9.7\ \text{m/s}^2$.
The answer according to the experiment should be presented as

$$g = (9.7 \pm 0.2)\ \text{m/s}^2.$$

5 Graphical method of processing results

Sometimes it's much easier to process experiment results if to submit them as a graph. Assume that it is necessary to determine the stiffness of the spring. It was decided to use the formula $k = \frac{F_{\text{elast}}}{x}$.

To obtain the most accurate result, spring elongation at different values of elastic strength was measured. A series of measurements at one point (i.e. for one elongation x) was performed to determine the absolute error of the ΔF elastic force. As an error, Δx the instrumental error of the ruler was taken. Considered the errors ΔF and Δx for all points are the same (on the plot of the length of the segments, of which the «crosses» are the same for all points).

The results of the measurements and the absolute errors are given in Table T.3.

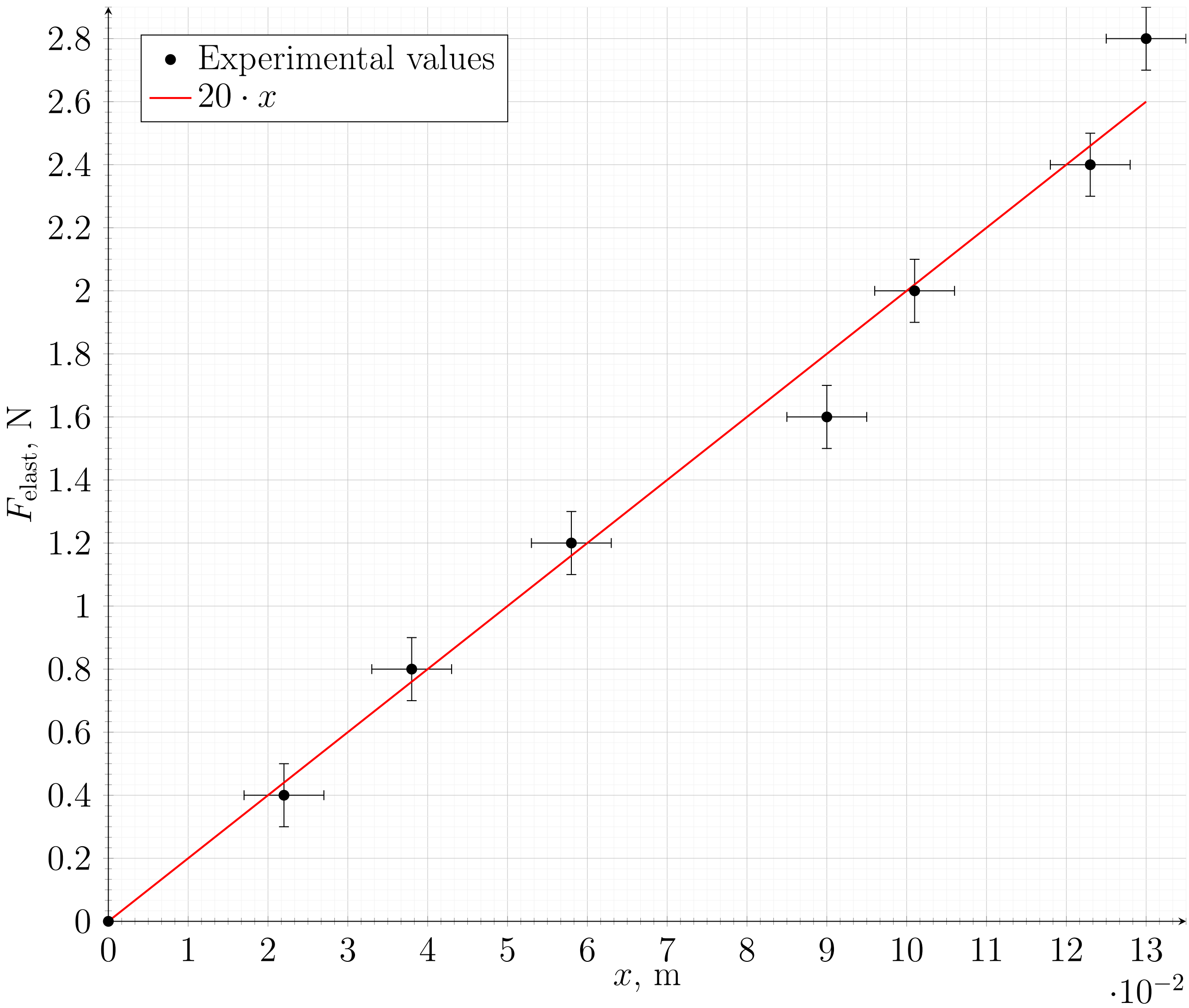
Let's illustrate the experimental data presented in the table in the form of points, putting the value of absolute elongation of the spring x and measurement error Δx (in the form of a segment whose length corresponds to the confidence interval in which the measured elongation x falls into the abscissa) and on the ordinate axis – the corresponding values of the force of elasticity F_{elast} and the

Table T.3

x, 10^{-2} m	$F_{\textbf{elast}}$, N	Δx, 10^{-3} m	$\Delta F_{\textbf{elast}}$, N
0.00	0.00	0	0
2.20	0.40	5	0.1
3.80	0.80	5	0.1
5.80	1.20	5	0.1
9.00	1.60	5	0.1
10.10	2.00	5	0.1
12.30	2.40	5	0.1
13.00	2.80	5	0.1

errors of its measurement ΔF_{elast} (Figure F.1). Since the coefficient of rigidity k does not depend on the elongation of the spring, theoretically, the plot of the dependence of $F_{\text{elast}}(x)$ should be the form of a straight line passing through the origin of the coordinate.

Figure F.1



Let's draw this line, How it can be done, we will describe below later. By selecting an arbitrary point on the line and finding the corresponding values of F_{elast} and x, we determine the mean value of the stiffness of the spring:

$$k = \frac{F_{\text{elast}}}{x} = \frac{1.6\ \text{N}}{8 \cdot 10^{-2}\ \text{m}} = 20\ \frac{\text{N}}{\text{m}}$$

More informations

In fact, when we drew a straight line to the experimental points, we approximated the experimental data by linear dependence $y = ax+b$. The angular coefficient a and the free term b can be determined using the **Least Squares Metod**. According with dthis method:

$$a = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{D\left(x\right)} \tag{E.9}$$

$$b = \langle y \rangle - a \langle x \rangle \tag{E.10}$$

where

$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{E.11}$$

$$\langle y \rangle = \frac{1}{N} \sum_{i=1}^{N} y_i \tag{E.12}$$

$$\langle xy \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i \cdot y_i \tag{E.13}$$

$$\left\langle x^2 \right\rangle = \frac{1}{N} \sum_{i=1}^{N} x_i^2 \tag{E.14}$$

$$D(x) = \left\langle x^2 \right\rangle - \langle x \rangle^2 \tag{E.15}$$

Formulas for estimating the errors in the parameters a and b:

$$\Delta a = \frac{1}{\sqrt{N}} \sqrt{\frac{D(y)}{D(x)} - a^2} \tag{E.16}$$

$$\Delta b = \Delta a \sqrt{D(x)} \tag{E.17}$$

To estimate, how the linear dependence is constructed corresponds to the experimental data, it is possible with the help of linear correlation coefficient R:

$$R = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{D\left(x\right) D\left(y\right)} \tag{E.18}$$

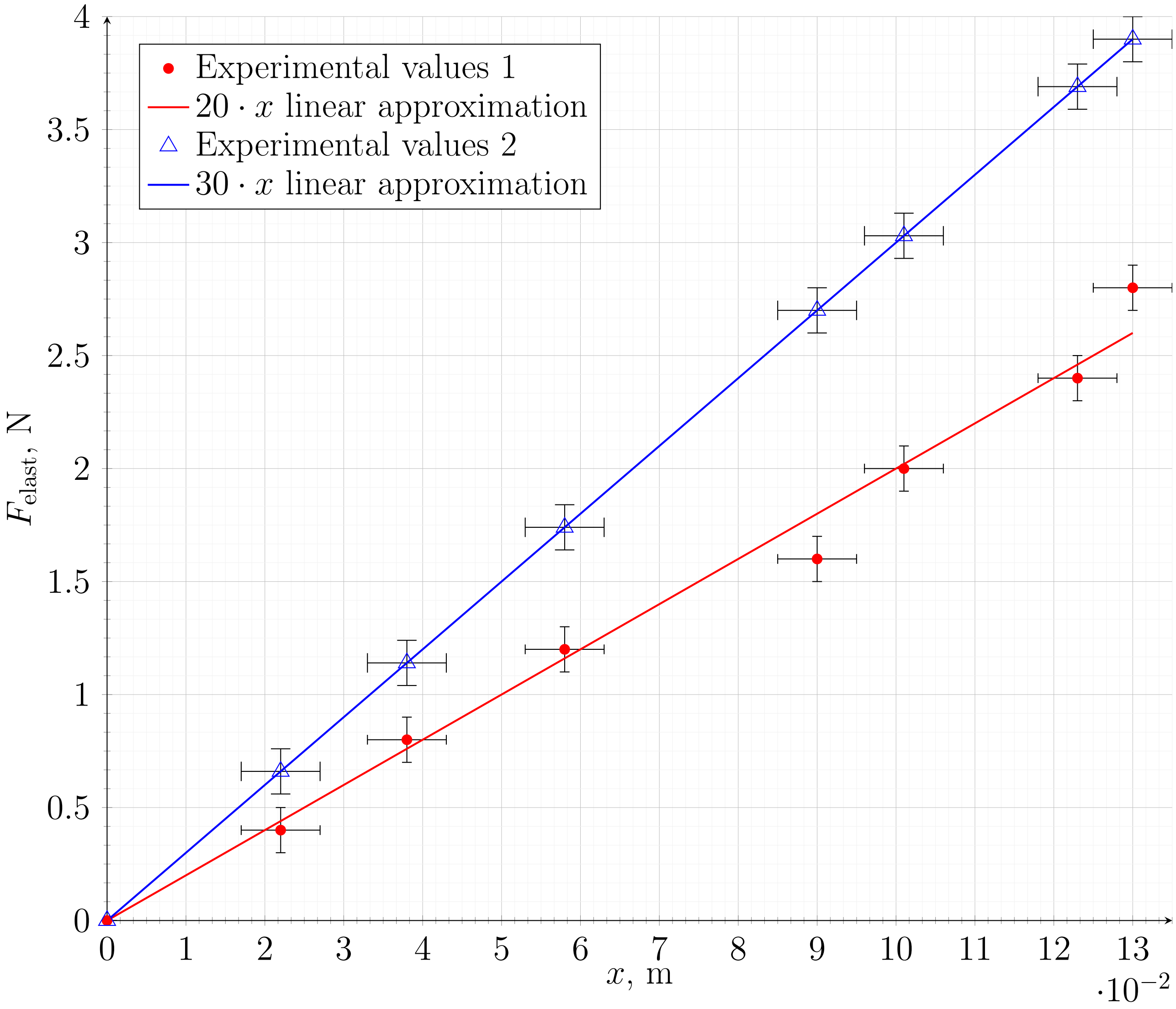
In laboratory and engineering calculations, the relative error with which the function describes the experimental data is determined by the approximate formula:

$$\varepsilon = \frac{\sqrt{\left(\frac{\Delta_x}{\bar{x}}\right)^2 + \left(\frac{\Delta_y}{\bar{y}}\right)^2 + \ldots}}{m} \tag{E.19}$$

In the numerator, under the root, the squares of all types of relative errors of the measured value are summed, m — the number of errors.

If there are several curves on the same graph, then each curve receives its number, and the points on each of them have different markings. Under the figure, write down its number and name, followed by an explanation of the physical parameters that distinguish the numbered curves. The scale and boundaries in which the argument and the function change, must be selected so that the graph occupies the entire allocated area (Fig. F.2).

Figure F.2



Control questions

1. What consecutive operations are performed by measuring any physical quantity?
2. What types of measurement errors do you know?
3. How to find the most probable (average value) of the measured value in the case of direct measurements?
4. How to determine a random measurement error?
5. What is the absolute systematic error determined?
6. What is called a relative measurement error?
7. How to round up and record measurement results correctly?
8. What is the advantage of the graphical method of processing the results of the measurement?
9. How to draw up the graph correctly? How to display the errors on graph?
10. How to approximate the experimental data on the graph with the functional dependence?

Work 1

STUDY OF RECTILINEAR MOTION OF BODIES IN THE FIELD OF GRAVITY USING THE ATWOOD MACHINE

Work purpose:

The purpose of this work is to test Newton's Second Law of Motion by utilizing an Atwood machine apparatus and determination of the acceleration of free fall in the field of gravity of the Earth. The Atwood machine will be used to study the relationship between mass, acceleration and net forces, with the distribution of the mass between the two weights being the independent variable and the time the dependent variable within the experiments.

Apparatus: *Atwood machine consisting of one pulley with string attached over pulley to two weight hangers; sets of gram-weights, meter stick and stopwatch.*

Keywords: *Rectilinear motion of bodies, acceleration, free fall acceleration.*

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Please, read the following sections: Chapter 9 – 12.

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[3] I. E. Irodov. Fundamental Laws of Mechanics. CBS publishers & distributors, 2004. ISBN: 9788123903040
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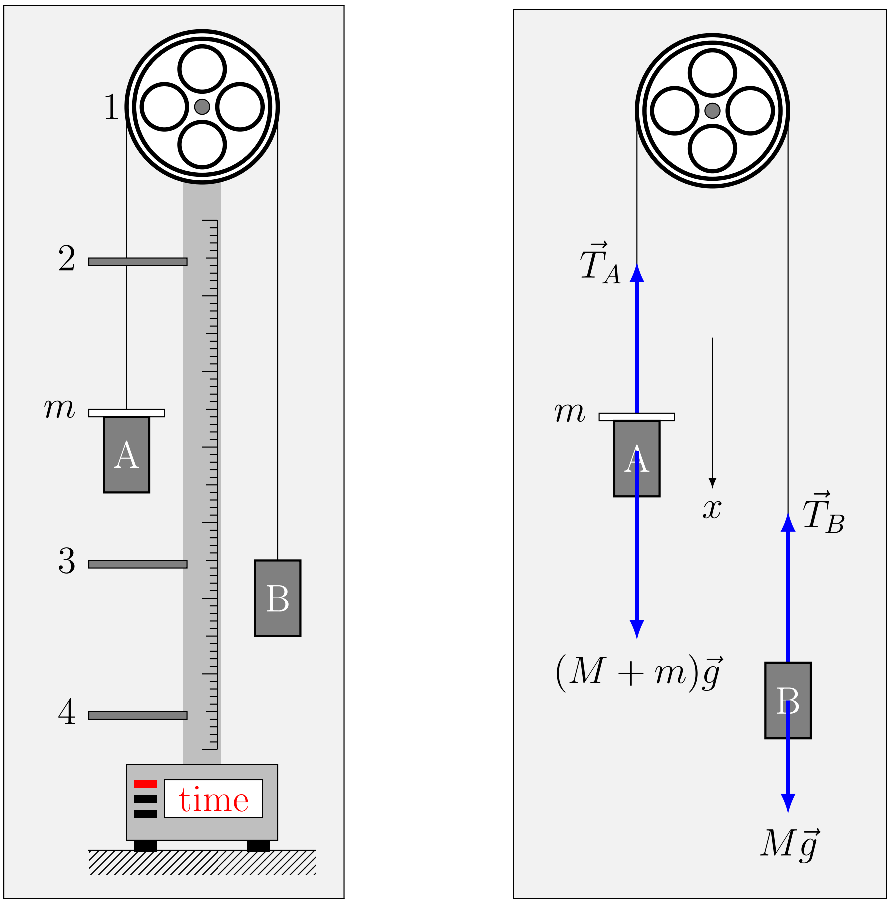
[4] Ch. Kittel et al. Mechanics (Berkeley Physics Course, Vol. 1). McGraw-Hill Book Company, 1973. ISBN: 0070048800
Please, read the following sections: Chapter 3, page 85.

1 Theoretibal background

The Atwood machine is designed to study the laws of motion of bodies in the field of gravity. Of course, it is best to study the field of gravity by exploring the free fall of bodies. But the acceleration of the Earth's gravity is quite large, and therefore the research must be carried out either at a very high device (height from the Pisa Tower), or by means of devices that allow measuring time with high accuracy (fractions of a second). The Atwood machine allows you to slow down the movement of the bodies at convenient speeds and using conventional devices to determine the acceleration of free fall.



(a) The Atwood machine. (b) Forces, acting on bodies.

Figure 1.1

The Atwood machine is depicted in Fig. 1.1a. Lightweight aluminum block 1 rotates freely around the horizontal axis, which is fixed to the top of the riser. A thin thread is thrown through the block, at the ends of which are weights A and B with equal masses M. If the body A loads a load of of m, then the balance weights will be broken and the system will begin to move with acceleration.

On the vertical column are fixed three consoles 2, 3, 4. The console 2 fixes the initial position of the body A. The body should be placed so that its the bottom edge was on one level with a white stripe on the console. On the consoles 3 and 4 there are two photo sensors, which allow to measure the time of fall of the body A. At the beginning of the experiment, the load carrier A with the load was fixed thanks to the friction brake. Turning off the friction brake is released by the weight carrier and the system of weight workers begins to move equally quickly. When the body passes past the console 3, the additional load is removed using a ring located on the same console, and then the system moves evenly. Thus, the photo-sensors capture the time of a uniform motion of the body-taker.

To determine the law of motion of body A, we choose a fixed reference system centered on the axis of the block. Take a look in more detail the forces acting on bodies A and B (Fig. 1.1b). The Ox axis is directed vertically down.

On the payload A there are two forces — the gravity $(M+m)g$, and the force of the tension of the left part of the thread T_A. According to the second law of Newton:

$$(M+m)\,g - T_A = (M+m)\,a \tag{1.1}$$

where a – acceleration of the body A.

Assuming that the thread combining the bodies does not stretch, the acceleration of the body B equals the absolute value of acceleration of the body A and is directed to the opposite side, that is, equals $-a$. Thus, for the body B, the second law of Newton has the form

$$Mg - T_A = -Ma \tag{1.2}$$

where T_B – tension force of the right (in the figure 1.1b) the end of the thread.

The relationship between the forces of tension T_A and T_B can be found from the equation of moments for the block, if you neglect the force of friction in the shafts of the axis of the block:

$$(T_A - T_B)\,r = \frac{Ja}{r} \tag{1.3}$$

where J – moment of inertia of the block, r – its radius. From the equations (1.1) – (1.2) we get the connection between the acceleration of free fall g and the acceleration of the body a:

$$a = g\frac{m}{2M + m + J/r^2} \tag{1.4}$$

Thus, the A carrier moves smoothly according to equation (1.4). If the weight of the load is much less than the mass of the masses $m \ll M$, then the acceleration a is much less than g, and therefore it is easier to measure. Formula (1.4) is much simpler if we neglect the moment of block inertia:

$$a = g\frac{m}{2M + m} \tag{1.5}$$

Using the formula (1.4) or (1.5), we can determine the acceleration of free fall. To do this, measure acceleration a. To determine a, we will use the fact that at the interval h_1 between the consoles 2 and 3 the gravity travels equally rapidly and acquires speed:

$$v = \sqrt{2h_1 a} \tag{1.6}$$

After the carrier A is released from the load near the console 3, it passes the distance h_2 between the consoles 3 and 4 at a constant velocity υ. Measuring the time of a uniform motion t, we find

$$\upsilon = \frac{h_2}{t} \tag{1.7}$$

Comparing (1.6) and (1.7), we finally calculate the acceleration:

$$a = \frac{h_2^2}{2h_1t^2} \tag{1.8}$$

Deriving the formula (1.8) we have neglected the force of air resistance.

2 Experimental details

The travel time of the weight is measured by an electronic stopwatch, which is retracted and disabled when the optical axis of the photosensors 3 and 4 passes, respectively. The stopwatch is ready for the next measurement only after pressing the $\boxed{\text{Reset}}$ button, which sets the stopwatch to zero.

In order for the system of heavy-duty loads to begin to move, it is necessary to press the $\boxed{\text{Start}}$ button (with locking). In this case, the friction brake, which holds the weight in the initial position, is turned off; the stopwatch comes to a standby state and starts when the photo dimension body is transmitted. After passing the photo sensor, the stopwatch switches off and the friction brake is activated.

The drop height is determined by the scale applied to the riser, by the difference in positions of the optical axes of the upper and lower photosensors. The weight of each of the weights and the mass of goods is determined experimentally. The measurement error of the consoles 2, 3, 4 is ± 1 mm. The measurement error of time is determined experimentally. Instrumental measurement error of time – 0.001 seconds.

3 Tasks

Before starting systematic measurements, it is useful to do some experiments with different h and m to ensure that the installation is working properly.

1. Determine the acceleration of the system of connected body with the load (Fig. 1.1a). Fix the console 3. For selected h_1 and h_2 run a series of 10 measurements of the time t of the vehicle system between the 3 and 4 consoles. Is there enough data to determine a?
2. Repeat the experiment for all available loads and their various combinations.
3. Change the position of the console 3 and repeat steps 2 and 3 for new values h_1 and h_2. Do this for the third pair h_1 and h_2.
4. Weigh heavyweights of m and M. Assess the value of g from your data.

5. Make the necessary measurements to estimate the moment of inertia of the dural unit 1. Consider the sources of probable errors. Rate them. If you need experiments for this, make them.

4 Processing the results of the experiment

1. Average time t for each experiment with a separate load m and fixed h_1 and h_2. Determine the random error of time and compare with the error that the stopwatch inserts. Put the data in the table.
2. For each case, using the formulas (1.5) and (1.8), calculate the acceleration a and g. Identify the errors.
3. Construct graphic dependence g from $1/m$. What type is this relationship? Using the obtained dependence, determine the value of g. By which masses the resulting value is consistent with the best g tabulated?
4. Evaluate the error of the experiment. Make conclusions.

Control questions

1. Formulate the basic kinematic quantities and explain their physical meaning.
2. Write down the laws of velocity variation with time and the laws of motion in vector and coordinate forms for uniformly accelerated motion.
3. Formulate the Newton's Laws.
4. What is free fall? What is the acceleration of free fall g?
5. Is the motion of the bodies in the work free fall?
6. At what stage the motion of bodies is uniform (uniformly accelerated)?
7. Apply Newton Second Law for the motion of bodies in work.
8. Why to determine g we plot the dependence graph of $1/m$, but not of m?
9. Is it possible to neglect the moment of inertia and friction? How do they affect the definition of g for different loads? Is it possible somehow to prevent this effect when calculating g?

Work 2

STUDY OF THE LAWS OF ROTATIONAL MOTION ON THE EXAMPLE OF OBERBEK PENDULUM

Work purpose:

study of the rotational motion of the Oberbeck pendulum, depending on the applied torque moment and the moment of inertia of the pendulum.

Apparatus: *Oberbeck pendulum; set of loads; stopwatch; scale ruler, scales.*

Keywords: *Rotational motion of bodies, angular acceleration, torque, moment of inertia*

References

[1] R. Feynman, R. Lejton, and M. Sends. Lectures on physics. Vol. 1. Mainly mechanics, radiation, and heat. New Millenium Edition. Basic Books, 2010. 968 pp.
Please, read the following sections: Chapter 18, Chapter 19.

[2] D. Halliday, R. Resnik, and J. Walker. Fundamentals of Physics. 10th ed. 1450 pp.
Please, read the following sections: Chapter 10 and 11.

[3] I. E. Irodov. Fundamental Laws of Mechanics. CBS publishers & distributors, 2004. ISBN: 9788123903040
Please, read the following sections: § 5.4.

[4] Ch. Kittel et al. Mechanics (Berkeley Physics Course, Vol. 1). McGraw-Hill Book Company, 1973. ISBN: 0070048800
Please, read the following sections: Chapter 8.

1 Theoretibal background

The rotational motion is an example of a simple mechanical motion. To describe the rotational motion, the following categories are used: the moment of inertia of the body and the moment of force (also known as torque). For a single material point, the moment of inertia relative to the axis of rotation is called the product of mass on the square of the distance to this axis. The moment of inertia J of the system of material points with mass m_i is the sum of the moments of individual points.

$$J = \sum_i m_i r_i^2 \tag{2.1}$$

Torque $\vec{M}$ relative to the point is called the vector multiplication of radius-vector $\vec{r}$ from the point to the place of application of force on the active force:

$$\vec{M} = \vec{r} \times \vec{F} \tag{2.2}$$

Module (absolute value) of the moment of force:

$$M = r \cdot F \cdot \sin\alpha \tag{2.3}$$

where α – angle between the vectors $\vec{F}$ and $\vec{r}$. $R\sin\alpha$ – this is shoulder of force, that equals to length of the perpendicular, carried out from the beginning of the radius vector $\vec{r}$ to the line of force. The torque relative to the rotation axis is a scalar value equal to the projection of the vector moment of force on this axis relatively to any point on the axis.

The motion of a body with a moment of inertia J, rotating at an angular velocity ω around the stationary axis, is described by the following equation:

$$\frac{d}{dt}(J\omega) = M \tag{2.4}$$

where M – moment of external forces relative to the axis rotation. When the solid is rotated its moment of inertia does not depend on time and equation (2.4) is simplified:

$$J\frac{d\omega}{dt} = M \tag{2.5}$$

The equation (2.5) is similar to the Newton equation $ma = F$, which describes the motion of a material point. The moment of force M plays the role of force F, the moment of inertia J plays the role of mass m, the angular acceleration $\beta = \frac{d\omega}{dt}$ is analogous to linear acceleration $a = \frac{dv}{dt}$.

Think about it

1. Why does the bicycle wheel have many knitting needles? What would be if the knitting needles were only two?
2. Why, when they throw a stone, they try to take their hand as far away from the body as possible?

2 Theoretical basis of the experiment

The Oberbeck pendulum is schematically represented in Fig. 2.1. Four needles, fixed on the sleeve, form one another with straight angles. The common axis passes through the sleeve and two pulleys with radii r_1 and r_2. The axis is secured in the needle shafts so that the entire system can rotate freely around the horizontal axis. The moment of inertia of the device can be changed by moving the loads m along the needles.

A thin thread is wound on one of the pulleys of the pendulum. The weight of the known mass is tied to the thread. (The set includes bodies of different masses).

Rotating moment is formed by the force of the thread tension T:

$$M_i = r_i T, \tag{2.6}$$

where $i = 1, 2$.

The force T can be found from the mass equation of body with mass m_0:

$$m_0 g - T = m_0 a \tag{2.7}$$

Acceleration a is proportional to angular acceleration β:

$$a_i = \beta r_i, \tag{2.8}$$

where $i = 1, 2$ and is determined experimentally. Indeed, by measuring the time t, for which the body from the state of rest decreases by distance h, we find a by the formula

$$a = \frac{2h}{t^2}. \tag{2.9}$$

The system of equations (2.6) – (2.9) allows determining the moment of inertia of the pendulum and checking the general equation of dynamics (2.5) provided that the moment of friction of the Mfric relative to the pendulum axis is much smaller than the moment of tensile force of the thread T.

In fact, the moment of friction of the Mfric can be quite large and lead to distortion of measurement results. At first sight, it seems that reducing the role of friction forces can be due to an increase in the mass of solid m_0. But such a view is false because

1. an increase in the mass m_0 leads to an increase in the pressure of the pendulum on the axis and, thus, to increase the frictional forces;
2. an increase in m_0 leads to a decrease in the time of the fall of the weight, which reduces the accuracy of the measurement of time.

In the proposed installation, the friction forces are reduced due to the use of needle bearings for attaching the pendulum axis. In spite of this, it is impossible to completely prevent the influence of friction force and this should be taken into account when processing the results of the experiment.

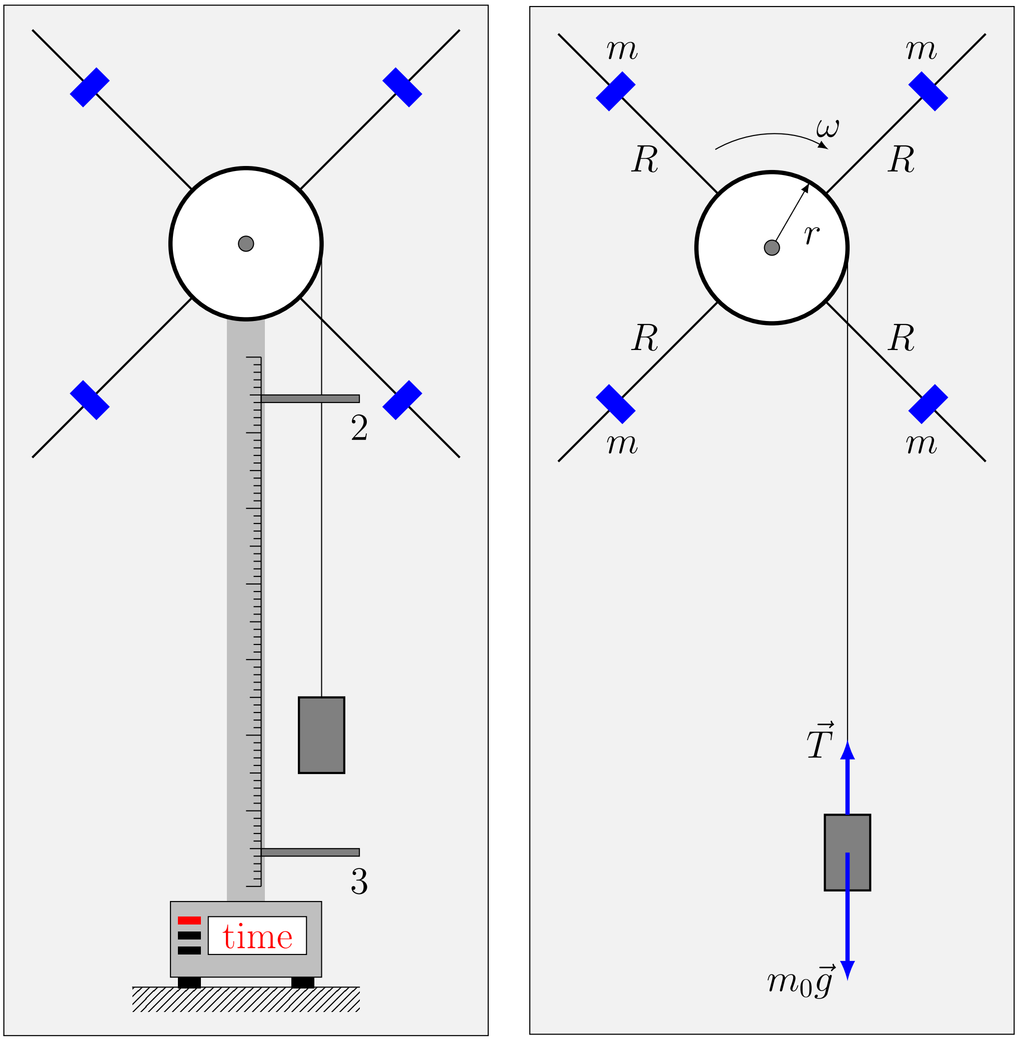
When processing the results of an experiment it is convenient to rewrite the equation (2.5) in such a way that it contains the moment of frictional forces in explicit form:

$$J \frac{d\omega}{dt} = M - M_{fric}. \tag{2.10}$$

The moment of inertia of the system is determined by the formula:

$$J = J_0 + 4mR^2 \tag{2.11}$$

where J_0 – moment of inertia of the system without loads, R – distance from the axis of rotation to the center of mass of loads.



(a) (b)

Figure 2.1

3 Experimental details

Time of movement of the body is measured by an electronic stopwatch, which is activated and deactivated by signals from photo sensors. The beginning and end of the motion of the body is recorded by its passage of the optical axis of the photodetector, so before the experiment begins, the lower end of the body should be located directly above the optical axis of the upper photosensor.

The photosensors and the digital display are activated when the device is switched on. This also includes a friction brake that holds the pendulum in a given position. The brake is deactivated if you press the START button and hold it in the pressed state.

The height of fall is determined by the scale applied to the riser, by the difference in positions of the optical axes of the upper and lower photosensors.

4 Tasks

1. Investigate the rotational motion of the pendulum under the influence of various weights at a constant moment of inertia of the system.

(a) Place the loads m at a certain distance R from the axis of rotation and achieve the indifferent equilibrium of the pendulum. Measure and record the distance R.

(b) Conduct an experiment with the weight of the mass m_0, measuring the time of fall of the weight. The experiment should be repeated $8-10$ times, and then t should be averaged.

(c) Repeat the experiment of point 1b on both pulleys for 3-4 different values of m_0.

2. Repeat measurements 1b, 1c for $3-4$ different values of the moment of inertia of the system.
3. Repeat measurements 1b for $3-4$ different values of the masses m_0 for the system without loads m.

5 Processing the results of the experiment

1. Using measurements 1b and 1c find the angular acceleration β and the torque moment M corresponding to the movement of each of the bodies on both pulleys for all values of the moment of inertia of the system J. Determine the error.
2. For each value of R, graphically represent the dependence of the angular acceleration β on the rotating torque M. Determine the moments of inertia of the system J and moments of the friction forces of the M_{fric}. What errors have these values?
3. Compare the obtained values of M_{fric}. Does the value of Mfric depend on the moment of inertia of the system? Averaging the value of M_{fric}.
4. The results of the definition of J at different values of R, are presented graphically as the dependence of $J(R^2)$. Consider why it is proposed to build such a dependence. After processing the results, determine the moment of inertia of the system without loads J_0. How do the results of the experiment with formula (2.11) consist? How does the magnitude of the experiment error with J_B? What are the possible sources of experimental errors?

Control questions

1. Formulate the basic kinematic quantities of rotational motion and explain their physical meaning.
2. Between what quantities does the basic law of dynamics of rotational motion establish a relationship?
3. How is the magnitude and direction of the moments of forces (torque) determined? In what units is this value measured?
4. What determines the moment of inertia, in what units is it measured? How to understand that the moment of inertia is an additive quantity? How it was used in the work?
5. Formulate and prove the parallel axes theorem.

Work 3

STUDY OF THE OF CONSERVATION LAWS OF ENERGY AND LINEAR MOMENTUM FOR COLLISION OF THE BALLS

Work purpose:

On the example of collision of balls check the conservation laws; calculate the energy dissipation coefficient and the mass correlation.

Apparatus: *experimental installation, to which the balls are fastened; set of balls of different weights and different materials; electronic stopwatch, scales.*

Keywords: *Total mechanical energy, linear momentum, conservation laws, collisions, absolutely elastic collision, absolutely inelastic collision.*

References

[1] R. Feynman, R. Lejton, and M. Sends. Lectures on physics. Vol. 1. Mainly mechanics, radiation, and heat. New Millenium Edition. Basic Books, 2010. 968 pp.
Please, read the following sections: §12.

[2] D. Halliday, R. Resnik, and J. Walker. Fundamentals of Physics. 10th ed. 1450 pp.

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Please, read the following sections: § 2.4.

[4] Ch. Kittel et al. Mechanics (Berkeley Physics Course, Vol. 1). McGraw-Hill Book Company, 1973. ISBN: 0070048800.

1 Theoretibal background

When moving, the bodies often collide each other. During the colission, both bodies are deformed, and as a result, the kinetic energy of the body before the collision, partially or completely transforms into the potential energy of the elastic deformation and the internal energy of the bodies. There are two limiting types of collision – *absolutely elastic* and *absolutely inelastic*.

Consider these processes on an example of an elastic and inelastic collision in a one-dimensional space. This will greatly simplify mathematical calculations, without changing the essence. Simplification refers to the velocity, which in a one-dimensional space is a scalar. Of course, in general, velocity is a vector.

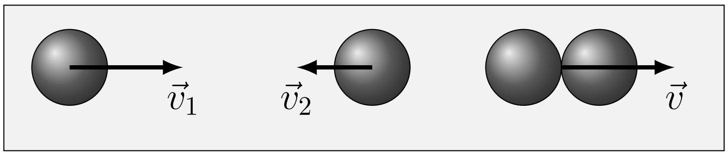


Figure 3.1. Inelastic collision

With a completely inelastic collision, one body stick to another one. In this case, the potential energy of deformation does not arise; kinetic energy is completely or partially converted into internal energy; after the collision, both bodies move at the same speed.

Suppose two bodies with masses m_1 and m_2 move towards each other (Fig. 3.1) with velocities v_1 and v_2, respectively. After an inelastic collision, they form one body of net mass $m_1 + m_2$, which moves at a velocity v. From the linear momentum conservation law:

$$m_1 v_1 - m_2 v_2 = (m_1 + m_2)v \tag{3.1}$$

From here we find the speed of the bodies after the collision:

$$v = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}. \tag{3.2}$$

In the case of an inelastic collision there is a law of conservation of momentum. Mechanical energy is not stored. Indeed, the total mechanical energy of the system before the collision (initial energy) is equal to the sum of the kinetic energies of each of the bodies:

$$E_{\text{initial}} = \frac{1}{2}\left(m_1 v_1^2 + m_2 v_2^2\right). \tag{3.3}$$

Mechanical energy after an collision (final energy) is defined as

$$E_{\text{final}} = \frac{1}{2}\left(m_1 + m_2\right) v^2 = \frac{1}{2}\frac{\left(m_1 v_1 - m_2 v_2\right)^2}{m_1 + m_2}. \tag{3.4}$$

When one writing the second equality in formula (3.4), we used the correlation (3.2). It is convenient to characterize the recoverment of mechanical energy using the coefficient k, which is defined as the ratio of $E_{\text{final}}/E_{\text{initial}}$. Taking into account (3.3), (3.4), we obtain

$$k = \frac{E_{\text{final}}}{E_{\text{initial}}} = 1 - \frac{m_1 m_2 (v_1 + v_2)^2}{(m_1 + m_2)(m_1 v_1^2 + m_2 v_2^2)}. \tag{3.5}$$

Formula (3.5) indicates that the coefficient of mechanical energy recovery at a non-elastic collision is always less than one. In the case when one of the bodies, say, the first, before the collision was immovable (that is $v_1 = 0$), k is determined only by the mass ratio of the bodies:

$$k = \frac{m_2}{m_1 + m_2} \tag{3.6}$$

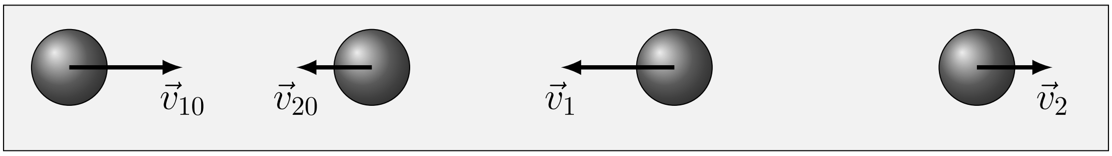


Figure 3.2. Absolutely elastic collision

In the case of equal masses ($m_1 = m_2$) and nonzero initial velocities:

$$k = 1 - \frac{(v_1 + v_2)^2}{2(v_1^2 + v_2^2)}. \tag{3.7}$$

and depends only on the initial velocities.

Is called an *absolutely elastic collision*, in which the mechanical energy of the system is stored. When the elastic collision of the body first deformed and their kinetic energy passes into potential energy of elastic deformation. Then the bodies restore their shape and push away each other, while the energy of the elastic deformation again becomes kinetic. Body movement after elastic collision is determined by laws conservation of momentum and kinetic energy. Consider the central collision of two bodies moving toward each other with velocities v_1 and v_2 (Fig. 3.2).

If the bodies move only translationally and do not rotate, then the equations of conservation of energy and momentum have the following form:

$$\frac{1}{2}\left(m_1 v_{10}^2 + m_2 v_{20}^2\right) = \frac{1}{2}\left(m_1 v_1^2 + m_2 v_2^2\right), \tag{3.8}$$

$$m_1 v_{10} - m_2 v_{20} = -m_1 v_1 + m_2 v_2, \tag{3.9}$$

where v_1 and v_2 – body speed after the collision and it is believed that after the collision of the body move along the same line as before the collision. Rewrite equations (3.8), (3.9) in the following form:

$$m_1\left(v_{10} + v_1\right)\left(v_{10} - v_1\right) = m_2\left(v_{20} + v_2\right)\left(v_2 - v_{20}\right), \tag{3.10}$$

$$m_1\left(v_{10} + v_1\right) = m_2\left(v_{20} + v_2\right). \tag{3.11}$$

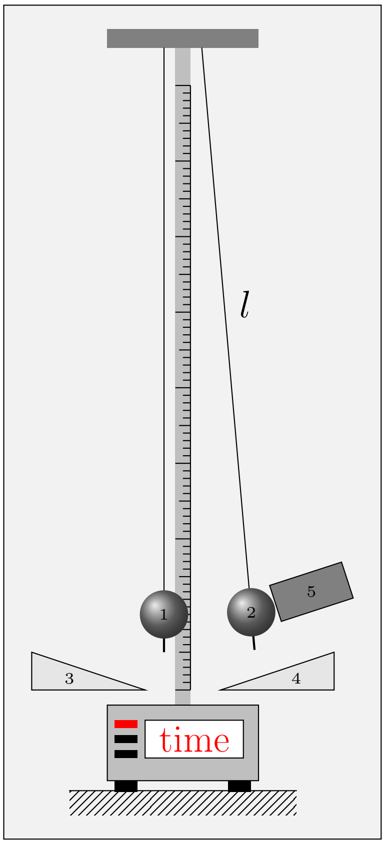
Comparing (3.10) and (3.11), we arrive at the conclusion that

$$v_{10} - v_1 = v_2 - v_{20}. \tag{3.12}$$

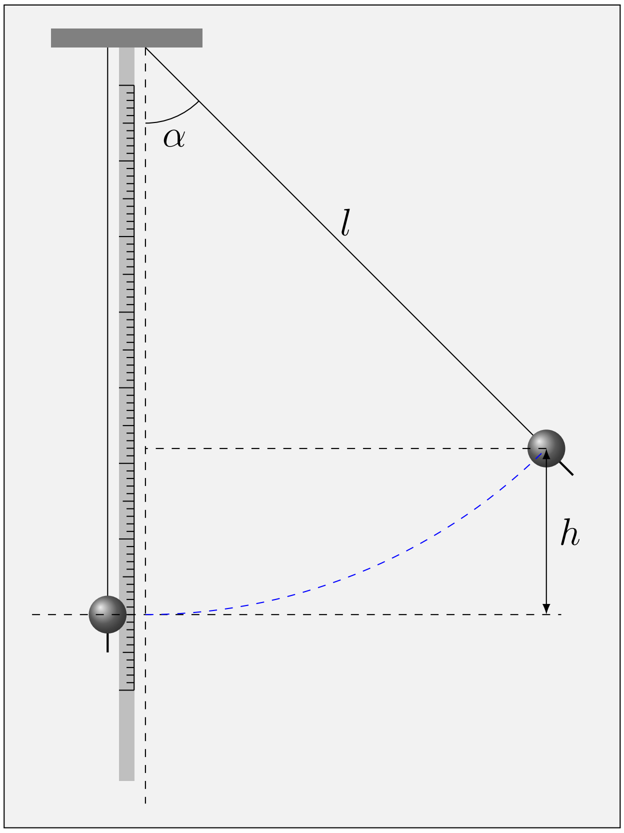
From (3.11) and (3.12) it is easy to determine the velocity of both bodies after the collision:

$$v_1 = \frac{\left(m_2 - m_1\right) v_{10} + 2m_2 v_{20}}{m_1 + m_2}, \tag{3.13}$$

$$v_2 = \frac{\left(m_1 - m_2\right) v_{20} + 2m_1 v_{10}}{m_1 + m_2}. \tag{3.14}$$



(a) Experimental installation.



(b) Calculation of the ball's lifted height.

In the case when the first body before the collision was in a state of rest ($v_{10} = 0$), formula (3.13) – (3.14) takes the form:

$$v_1 = \frac{2m_2}{m_1 + m_2} v_{20}, \tag{3.15}$$

$$v_2 = \frac{m_1 - m_2}{m_1 + m_2} v_{20}. \tag{3.16}$$

Formula (3.15) – (3.16) indicates that in case of equal masses ($m_1 = m_2$) the bodies after the collision exchange of the speedes, namely, after the collision, the second body stops, and the first body moves with the speed v_{20}, which is the second body before the collision. The greater the difference between body masses, the less the speed of the first and the greater speed of the second body after the collision.

2 Theoretical basis of the experiment

Experimental installation is shown in Fig. 3.3a. Two balls, 1 and 2, are suspended to a riser on conducting threads of length l. In the bottom of the riser there are two scales 3 and 4, by which the deviations of balls from the equilibrium position are measured.

At the beginning of the experiment, the ball 1 is in equilibrium, and the ball 2 is deviated at an angle α from the vertical axis and fixed with the help of an electromagnet 5. After the electromagnet is switched off, ball 2 begins to move (initial ball speed is zero). The velocity of the ball 2 before the collision is determined at the initial angle of deviation α, based on the law of conservation of mechanical energy.

$$mgh = \frac{1}{2}mv_{20}^2 \tag{3.17}$$

where h – height at which the ball was lifted, g – acceleration of free fall, v_{20} – the velocity of the ball 2 at the point of equilibrium. For geometric reasons (See Fig. 3.3b):

$$h = l(1 - \cos\alpha) = 2l\sin^2\frac{\alpha}{2} \tag{3.18}$$

Thus, if the maximum angle of deviation of a ball is equal to α then its velocity at the equilibrium point is determined by the formula:

$$v_{20} = 2\sqrt{gl}\sin\frac{\alpha}{2}. \tag{3.19}$$

Similarly, by measuring the angle of deviation of the ball 1, we can find its velocity v_{20} immediately after the collision.

In the theoretical guide, we considered two limiting cases of an absolutely elastic and absolutely inelastic impact. In real experiments, during impact, energy is partially dissipated. In this paper, the energy dissipation coefficient β, which is defined as the ratio of energy loss during impact with the initial energy, is measured:

$$\beta = \frac{E_{\text{initial}} - E_{\text{final}}}{E_{\text{initial}}}. \tag{3.20}$$

Consider the case when the first ball before the impact is at rest ($v_{20} = 0$). Then the laws of conservation of energy and momentum (3.8), (3.9) taking into account (3.20) will look like:

$$(1 - \beta)m_2v_{20}^2 = m_1v_1^2 + m_2v_2^2, \tag{3.21}$$

$$m_2v_{20} = m_1v_1 - m_2v_2. \tag{3.22}$$

(in the formula (3.22) the modules of velocities are inserted!).

An experimental setup allows you to measure v_{20} and v_1 speeds. Consequently, by excluding from equations (3.21), (3.22) v_2, we can find the coefficient β. For identical balls:

$$\beta = \frac{2v_1\left(v_{20} - v_1\right)}{v_{20}^2} = 2R\left(1 - R\right),\ R = \frac{v_1}{v_{20}} \tag{3.23}$$

On the other hand, according to the known coefficient β of equations (3.21), (3.22) we can find the relation of the masses of the impacting balls. Indeed, solving these equations for v_2, m_1/m_2 we obtain:

$$\frac{m_1}{m_2} = \frac{2 - R \pm \sqrt{(2-R)^2 - 4\beta}}{2R}. \tag{3.24}$$

3 Experimental details

The electromagnet is switched off by pressing the $\boxed{\text{RESET}}$ button. In the intervals between experiments, it is desirable to switch off the electromagnet!

The time of contact of the balls is fixed by an electronic stopwatch, which is attached to the electrical circuit formed by balls and threads of the suspension. When impact balls electrical circuit is closed and the stopwatch is triggered.

4 Tasks

1. Take two steel balls with the same numbers. Weigh them and carefully fasten them on the hanging. Ensure that the contact is impact-centered.
2. Deviate the ball 2 (Fig. 4.3.) to some angle and release. Measure the angle of deviation of the ball 1 after the first impact. Measurement should be repeated at least 10 times.
3. Repeat measurement for point 2 for different initial angles of deviation of the ball (we recommend $5°$, $10°$, $15°$).
4. Repeat steps 2, 3 for another pair of identical balls (you only need to explore three pairs of balls).
5. Take a couple of balls with different numbers. Weigh them. Repeat steps 2 and 3. Why do you think the balls have different and identical numbers?
6. Select two more pairs of balls with different numbers and repeat experiment 5. So you have to get data for 6 pairs of balls at the three angles for each pair (only 18 measurements of the angle of deviation).
7. Think about what kind of mistakes make the devices? Can they evaluate how to do it? What are they: random or systematic?

Control questions

1. What are elastic and inelastic collisions?
2. Formulate and derive the laws of conservation of momentum and total mechanical energy for elastic and inelastic collisions. In what conditions these laws can not be applied?
3. What are central and noncentral collisions? Explain how the balls will move for a noncentral collision.
4. Calculate the fraction of the energy of the balls passing into the internal energy during an inelastic collision, using the results of the laboratory work.
5. How precisely are the laws of conservation of momentum and mechanical energy in the experiments carried out? What leads to deviations from conservation laws?

Work 4

STUDY OF PHYSICAL PENDULUM

Work purpose:

Study of physical pendulum and determination of the acceleration of free fall by means of a reversible pendulum.

Apparatus: *Physical pendulum (homogeneous steel rod with a pair of loads and prisms); tripod for pendant suspension; mathematical pendulum; oscillation counter; stopwatch; scale ruler.*

Keywords: *Physical pendulum, mathematical pendulum, harmonic oscillations, axis of rotation, proper oscillation frequency*

References

[1] R. Feynman, R. Lejton, and M. Sends. Lectures on physics. Vol. 1. Mainly mechanics, radiation, and heat. New Millenium Edition. Basic Books, 2010. 968 pp.
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[4] Ch. Kittel et al. Mechanics (Berkeley Physics Course, Vol. 1). McGraw-Hill Book Company, 1973. ISBN: 0070048800.

1 Theoretibal background

A physical pendulum is called any solid which can freely fluctuate around the horizontal axis under the action of gravity. The motion of this pendulum is described by the equation:

$$J\frac{d^2\varphi}{dt^2} = M. \tag{4.1}$$

where J – moment of inertia of the pendulum, φ – angle of deviation of center of mass of pendulum from equilibrium position, M – torque acting on pendulum, t – time. For example, for a homogeneous rod of length l, according to Parralel Axes theorem, the moment of inertia is equal to:

$$J = ms^2 + \frac{ml^2}{12} \tag{4.2}$$

where m – mass of pendulum, s – distance between the center of mass and the axis of rotation.

Torque of gravity force acting on the pendulum is based on the formula:

$$M = -s \cdot mg \sin \varphi$$

If the angle φ is small, then $\sin \varphi \approx \varphi$, and

$$M \approx -s \cdot mg\varphi.$$

A well-tuned pendulum can make a few hundred oscillations without noticeable extinction, so moment of frictional force in first approximation can be neglected. Substituting expression for M in (4.1), it is easy to obtain an equation for oscillations

$$\ddot{\varphi} + \omega^2 \varphi = 0 \tag{4.3}$$

with frequency

$$\omega = \sqrt{\frac{mgs}{J}} \tag{4.4}$$

Equation (4.3) describes the harmonic oscillations that occur under the law $\varphi(t) = A \sin(\omega t + \delta)$.

The amplitude of the oscillations A and their phase δ depends on the mode of oscillation excitation, that is, on the initial conditions. The proper oscillation frequency ω, according to (4.4), is determined only by the parameters of the pendulum J and s.

The period of oscillations of the physical pendulum $T = 2\pi/\omega$, as well as its frequency, does not depend on the phase and amplitude of the oscillations and is equal to

$$T = 2\pi \sqrt{\frac{J}{mgs}} \tag{4.5}$$

The motion of the pendulum is described by the equation of harmonic oscillations (4.3) only in the case of small amplitudes, namely, when $\sin \varphi \approx \varphi$. The suitability of this assumption can be verified experimentally, ensuring that the period of oscillation is independent of amplitude.

If you type designation

$$L = \frac{J}{ms} \tag{4.6}$$

then formula (4.5) will have the same form as the formula for the period of oscillations of a mathematical pendulum with a length L:

$$T = 2\pi \sqrt{\frac{L}{g}} \tag{4.7}$$

Therefore, the value of L is called the reduced length of the physical pendulum (Fig. 4.1b). Point O' that is remote from the support point O at a distance L is called the center of the oscillation of the physical pendulum. The oscillation point of the pendulum and the pivot center of the pendulum are reciprocal, that is, when the pendulum is oscillated around the point O', the oscillation period must be the same as in the case of the oscillation around point O. We propose to prove this fact on your own.

2 Theoretical basis of the experiment

In this paper we use the method of finding the acceleration of free fall by determining the period of free oscillations of the physical pendulum by the formulas (4.5), (4.7).

Here J – the moment of inertia of the pendulum relative to the oscillation axis, m – its mass, s is the distance from the center of mass to the axis of oscillation, L – the reduced length of the physical pendulum.

The mass of the pendulum and the period of its oscillations can be determined with a sufficiently high accuracy. But it is difficult to do this for the moment of inertia. Avoiding these difficulties is helped by the method of a reversible pendulum. In it, instead of J, the reduced length of the pendulum is measured (4.6).

This method is based on the fact that the period of oscillations of the physical pendulum will not change, if you move it so that the new point of the suspension is the former center of rotation. This point is located at a distance equal to the reduced length of the physical pendulum from the oscillation axis and on one straight line with the axis of oscillation and the center of mass.

The reversible pendulum used in this work consists of a steel rod on which two loads of B_1 and B_2, each with a mass m, and two supporting prisms P_1 and P_2 (Fig. 4.1a).

Assume that we found such position of loads in which the periods of oscillations of the pendulum T_1 and T_2 coincide around the prisms of P_1 and P_2 that is,

$$\sqrt{\frac{J_1}{mgs_1}} = \sqrt{\frac{J_2}{mgs_2}} \tag{4.8}$$

This equality is possible provided that combined lengths L_1 and L_2. are equal.

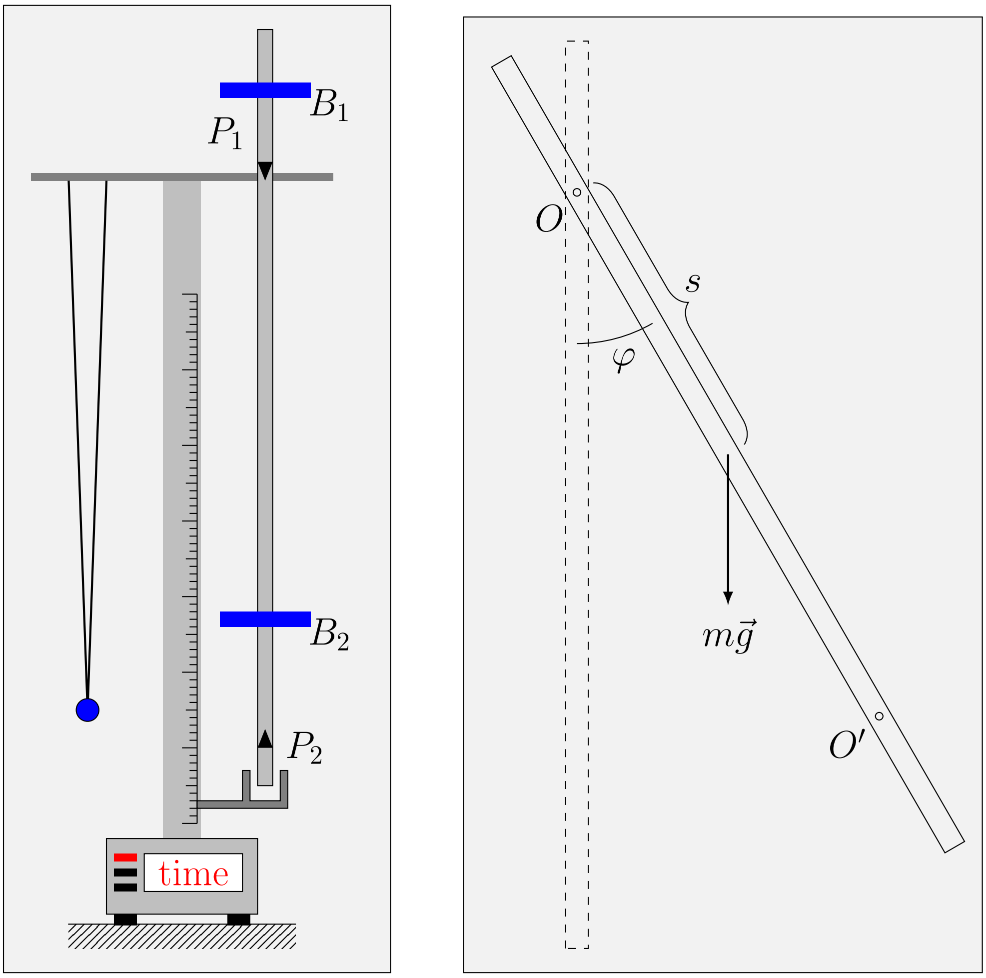
On the other hand, by Huygens-Steiner theorem

$$J_1 = ms_1^2 + J_0 \quad J_2 = ms_2^2 + J_0 \tag{4.9}$$

where J_0 – moment of inertia of the pendulum relative to the axis passing through the center of mass in parallel with the oscillation axis. Deleting from formulas (4.8), (4.9) J_0 and m, and using (4.5) we find that

$$g = \frac{4\pi^2 (s_1 + s_2)}{T^2} \tag{4.10}$$

Thus, according to (4.7), $L = s_1 + s_2$. Note that formula (4.10) is derived from formulas (4.8), (4.9) provided $s_1 \neq s_2$, otherwise the formulas (4.8) and (4.9) are satisfied identically.



(a) (b)

Figure 4.1

In deriving formula (4.10), we neglected the difference between periods T_1 and T_2. In fact, it is impossible to ensure that the periods mentioned are not the same, because

$$T_1 = 2\pi\sqrt{\frac{J_0 + ms_1^2}{mgs_1}}, \quad T_2 = 2\pi\sqrt{\frac{J_0 + ms_2^2}{mgs_2}} \tag{4.11}$$

Where we have

$$T_1^2 gs_1 - T_2^2 gs_2 = 4\pi^2\left(s_1^2 - s_2^2\right)$$

Taking this into account, we obtain more accurate formula for g:

$$g = 4\pi^2 \frac{s_1^2 - s_2^2}{T_1^2 s_1 - T_2^2 s_2} = 4\pi^2 \frac{L}{T_0^2} \tag{4.12}$$

where the period entered

$$T_0^2 = \frac{T_1^2 s_1 - T_2^2 s_2}{s_1 - s_2} \tag{4.13}$$

Let's analyze the limits of the application of our theory. For this we consider the error of the definition of T_0, which itself depends on the errors of measuring periods σ_{T_1} and σ_{T_2}:

$$\sigma_{T_0} = \sqrt{\left(\frac{\partial T_0}{\partial T_1}\right)^2 \sigma_{T_1}^2 + \left(\frac{\partial T_0}{\partial T_2}\right)^2 \sigma_{T_2}^2} \tag{4.14}$$

With good equipment when $\frac{\sigma_{T_1}}{T_1} \approx \frac{\sigma_{T_2}}{T_2} \equiv \varepsilon_T \ll 1$ we have

$$\sigma_{T_0} = \sqrt{\frac{s_1^2 T_1^4 + s_2^2 T_2^4}{(s_1 - s_2)\left(s_1 T_1^2 - s_2 T_2^2\right)}} \varepsilon_T \tag{4.15}$$

Note that with $s_2 \approx s_1$ error significantly increases and this is reflected in the accuracy of determination g.

Therefore, the values of s_2 and s_1 should not be very close. On the other hand, if these values are very different, then the period of oscillations increases substantially, hence, the time of observation increases and, as a result, the role of the force of friction also is increasing. Thus, while performing the experiment, it is necessary to ensure that the ratio of s_1/s_2 is not very large and not very small, the recommended interval

$$1,5 < s_1/s_2 < 3 \tag{4.16}$$

3 Experimental details

The system for determining the oscillation period consists of an electronic stopwatch and period counter. The period counter works like this. Near the position of the equilibrium of the pendulum is a photosensor. During oscillation, the rod crosses the axis of the photosensor and thus sends signal to the counter. The counter registers every second pulse. Since during the period the pendulum passes through any position twice, in this way the display of the counter corresponds to the number of periods. The period counter is activated by the $\boxed{\text{RESET}}$ button, which simultaneously resets the counter indication. Simultaneously with the counter the electronic stopwatch is switched on. On the digital display of the counter of periods you can observe the number of oscillations made by the pendulum (from the moment the counter is turned on). Turning on of the $\boxed{\text{STOP}}$ button causes the counter of the periods and the stopwatch to stop after pendulum passes another equilibrium position. Therefore, if necessary, to investigate a certain number of periods (suppose 10), the $\boxed{\text{STOP}}$ button should be pressed at the moment when the digital display shows the number of periods per unit less than necessary (in the example given, 9).

Thus, the recommended measurement procedure is as follows:

1. deflecting the rod from the equilibrium position, excite the oscillations;
2. turn on the $\boxed{\text{RESET}}$ button;
3. if necessary, to measure the time of n oscillations (for rough measurements $n = 10 - 15$, for exact $n = 20 - 25$ periods) turn on the $\boxed{\text{STOP}}$ button at a time when the number of periods appears on the display of the number $n - 1$;
4. determine the period of one oscillation, dividing the time recorded by the stopwatch on the number of periods.

Own error of measurement of electronic stopwatch ± 0.001 sec.

In this paper, for the independent estimation of the reduced length of the physical pendulum, a mathematical pendulum model is used, which is a massive ball suspended on two threads. The length of the threads is changed by winding them on the axis. Turning the upper console on which the pendulum is mounted at an angle 180° so that the pendulum ball crosses the optical axis, one can measure the period of this pendulum. (The risk on the ball should be on the same level as the photosensor).

4 Tasks

1. Define a range of amplitudes in which the oscillation period of the pendulum T is independent of the amplitude. To do this, deviate the pendulum from the equilibrium position at some angle φ_1 (about 10°) and measure the time at which the pendulum will make 50 oscillations. According to the results of the experiment, find the period of oscillation T.
   Repeat the experiment by reducing the initial deviation by $1.5 - 2$ times, and then again reducing the amplitude again. If the periods coincide with the limits of the measurement error, then for further measurements, you can choose any initial deviation, less than φ_1. If the periods are significantly different, one must study the behavior of the pendulum with smaller deviations.
   Find out what makes the biggest mistake in the definition of the period and try to reduce it.
2. Fix the loads on the rod asymmetrically, so that one of them is located near the end of the rod, and the other one is near the middle of the rod. Place the support prisms on both sides of the center of the masses of the system. Measure the periods of oscillation of the pendulum T_1 and T_2 around the prisms of P_1 and P_2.
3. Investigate the dependence of T_1 and T_2 oscillation periods on load positions B_1 and B_2. It is enough to measure the time of $10 - 15$ oscillations. Find out:
   - which of the load has a greater effect on the size of the periods;
   - which of the load significantly affects the difference between periods.

4. Moving loads, which significantly affects the difference in periods, reach a rough coincidence of periods. Define periods for $10-15$ oscillations. Measure the distance between the loads and their position on the rod, find the position of the center of the masses of the pendulum. Estimate the distances s_1 and s_2. As noted above, they must differ by at least 1.5 and not more than 3 times.
5. Changing the position of the load, which has less effect on the periods, achieve a coincidence of T_1 and T_2 periods with an accuracy of not less than 1%. Check whether the s_1 and s_2 inequalities (4.16) in this case are satisfied. The final measurement of the period of oscillation of the pendulum should be done in $20-30$ full oscillations. It is also necessary to make sure that the effect of friction with such a number of oscillations is insignificant (i.e, the amplitude of oscillations is not noticeably diminished).
6. Find the reduced length of the physical pendulum. Check this value using the mathematical pendulum model. By changing the length of the pendulum with a coil, achieve the coincidence of periods of mathematical and physical pendulums within the accuracy of measurements.
7. Repeat the measurement for several (not less than $4-5$) values of the reduced length of the physical pendulum (the distance between the reference prisms).

5 Processing the results of the experiment

1. Plot the dependence graph L on T^2, and then determine the acceleration of free fall (according to formula (4.10)).
2. Determine the error of the calculations at each step of the experiment and estimate the overall error.

Control questions

1. Types of pendulums. Give the definition of a physical pendulum?
2. What are oscillations? What oscillations are harmonic?
3. What are period and amplitude of oscillations?
4. With which simplifying assumptions the formula is obtained (4.5)?
5. What is the period of oscillations of mathematical pendulum? What is the reduced length of a physical pendulum?
6. Explain how the physical pendulum will move if the support point is located in the center of the mass of the pendulum.
7. What is the absolute measurement error of the period of the oscillation of the pendulum in this experiment?
8. Get (4.5), (4.10), (4.12).