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LABORATORY PRACTICE

PHYSICAL FUNDAMENTALS OF FLUID MECHANICS

for foreign students of higher technical educational institutions

Faculty ______________________________
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LABORATORY PRACTICE
PHYSICAL FUNDAMENTALS OF FLUID MECHANICS

LABORATORY PRACTICE. PHYSICAL FUNDAMENTALS OF FLUID MECHANICS: роб. зошит для студентів технічних спеціальностей/*V.Moiseenko, O. Pugach,V. Uzhva, F. Gareeva, A. Pugach, S. Kulieznova, O. Shtofel*; *IgorSikorskyKyivPolytechnicInstitute*. – Kyiv: IgorSikorskyKyivPolytechnicInstitute, 2019. – 33p.

This publication provides 4 laboratory works from the section "Fluid mechanics" in accordance with the working curriculum of the credit module "Physics 1" of the discipline "Physics". The purpose of the educational publication is to provide foreign students with materials for laboratory work in physics that correspond to the module's curriculum for all technical specialties. The tasks of the work are to make laboratory works more accessible and understandable to foreign students.

Laboratory work № 1-5.Determination of the viscosity coefficient using the Stokes' method

Objectives: study of the motion of a material point under the force, which is proportional to velocity; determination of the glycerin viscosity coefficient.
Materials: glass cylinder with the fluid under the study (glycerol), thermometer, hydrometer, micrometer, stopwatch, scale ruler, small balls.

Theoretical basis

A moving body in a viscous fluid is affected by the resistance force, which depends on many factors: the geometric shape of the body, the nature of the flow, the coefficient of fluid viscosity, etc. The nature of the liquid flow around a body is determined by the Reynolds number (Re).

If the values of Re are big, the flow becomes turbulent with the specific formation of vortices behind the body. In the vortex region, the pressure is low, resulting in pressure difference between the front and back surfaces of the body which induces the resisting force. Thus, the total resistance force is composed of frictional resistance and pressure resistance, and their relative contribution is determined by the value of Re. The flow will be laminar under the following condition:

$$Re < Re_{\mathrm{cr}}, \qquad (5.1)$$

whereRe_{cr} is the critical value of Reynolds number, which may be volatile between tens and several thousands, depending on the flow pressure. If an infinitely viscous liquid, the density of which (ρ_1) is flowing around a ball, and the fulfillment of the following condition takes place

$$Re = \frac{\upsilon r \rho_1}{\eta} << 1, \quad (5.2)$$

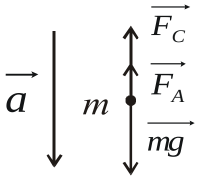
then, the resistance force F_C may be determined with the help ofStokes' formula

$$F_C = 6\pi r \eta \upsilon, \qquad (5.3)$$

whereη is fluid viscosity coefficient, υ is ball velocity, r is ball radius.

Criterion 5.2 is provided not only by the usage of Stokes' formula, but also by flow laminarity, because in this case, certainly, the condition (5.1) is fulfilled. Practically, it corresponds to a slow viscous liquid flow around a ball or ball's slow motion.

Let’s consider the nature of motion of a ball during a slow fall in an infinitely viscous liquid. In this case the ball is affected by three forces: force of gravity $\overrightarrow{mg}$, Archimedes' buoyant force $\overrightarrow{F_A}$ and resistance force $\overrightarrow{F_C}$, as shown on fig.5.1. According to Newton second law: $\overrightarrow{ma}=\overrightarrow{F_C}+\overrightarrow{F_A}+\overrightarrow{mg}$, where $\vec{a}$ is ball’s acceleration.



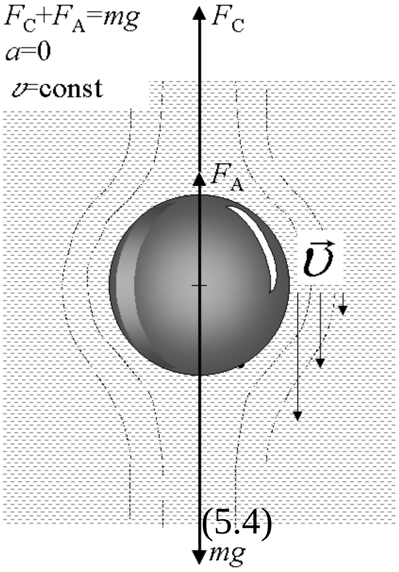


Fig. 5.1. The ball is affected by three forces

After projecting this equation on acceleration direction, we will obtain:

$$m\frac{d\upsilon}{dt}=mg-F_A-F_C\,,$$

or $\rho V\dfrac{d\upsilon}{dt}+6\pi r\eta\upsilon=Vg(\rho\text{-}\rho_1),$ (5.4)

where ρ is ball material density, V – its volume.

To solve the equation (5.4)let’s rewrite it as follows:

$$\frac{d\upsilon}{dt}=-\frac{6\pi r\eta}{\rho V}\left[\upsilon-\frac{Vg(\rho\text{-}\rho_1)}{6\pi\eta r}\right], \tag{5.5}$$

taking into account, that the value

$$B=\frac{Vg(\rho\text{-}\rho_1)}{6\pi r\eta}, \tag{5.6}$$

does not depend on time and has a dimension of velocity. This allows us to rewrite the equation (5.5) as a differential equation with separated variables

$$\frac{d(\upsilon-B)}{dt}=-\frac{6\pi r\eta}{\rho V}(\upsilon-B)\,, \tag{5.7}$$

or

$$\frac{d(\upsilon-B)}{\upsilon-B}=-\frac{6\pi r\eta}{\rho V}dt\,. \tag{5.8}$$

After integrating we obtain

$$\ln(\upsilon-B)=-\frac{6\pi r\eta}{\rho V}t+\ln \boldsymbol{C}, \tag{5.9}$$

where arbitrary constant is written in the form $\ln C$.

Finally $\upsilon(t)=Ce^{-\frac{6\pi r\eta}{\rho V}t}+B$. (5.10)

As a result, the relation between ball motion velocity and time $\upsilon(t)$ is defined by the following formula: $\upsilon(t)=Ce^{-\frac{6\pi r\eta}{\rho V}t}+\dfrac{Vg(\rho\text{-}\rho_1)}{6\pi r\eta}$.(5.11)

An arbitrary constant C is defined by the initial velocity at which the ball is entering the liquid under the following condition:

$$\upsilon(t=0)=\upsilon_0, \tag{5.12}$$

(time-marking, naturally, begins from the moment of ball cutting off piercing the liquid surface).

Using the general solution (5.11) and the condition (5.12), we determine that

$$C=\upsilon_0-\frac{Vg(\rho\text{-}\rho_1)}{6\pi r\eta}. \tag{5.13}$$

Finally $\upsilon(t)=\frac{Vg(\rho\text{-}\rho_1)}{6\pi r\eta}-\left[\frac{Vg(\rho\text{-}\rho_1)}{6\pi r\eta}-\upsilon_0\right]e^{-\frac{6\pi r\eta}{\rho V}t}$. (5.14)

Let's analyze the solution (5.14). At$t\to\infty, v\to v_{st}=\frac{Vg(\rho-\rho_1)}{6\pi r\eta}$ is the steady-state motion velocity. The condition $t\to\infty$ from the physical standpoint means that $t>>\tau$, where $\tau=\rho V/6\pi r\eta$ is so called relaxation time, in other words, time at which the motion will reach a steady-state character (a = 0).

Let's write down the solution (5.14) in a simplified form using the agreed notation:

$$v(t)=v_{st}-(v_{st}-v_0)e^{-\frac{t}{\tau}}. \tag{5.15}$$

The function graph, depicted on fig.5.2, gives a vivid indication of the ball motion character. Therefore, irrespective of the velocity υ_0, at which the ball is entering the liquid after the time$t>>\tau$, it is possible to state with the specified degree of accuracy the steady motion of a ball with the velocity υ_{st}.

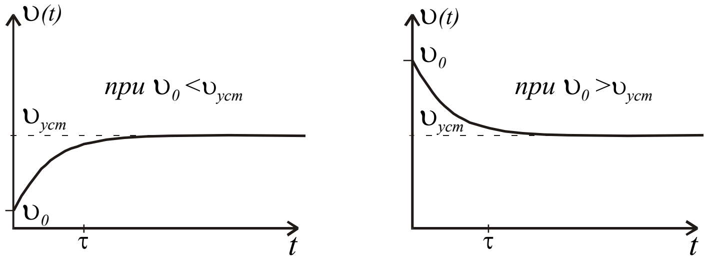


Fig.5.2.

Task 1. Prove that at $t=3\tau$ the deviation of ball velocity from the steady velocity is ~ 5%, in other words, calculate$\frac{v_{st}-v(3\tau)}{v_{st}}$ considering that the ball is entering the liquid at a starting velocity υ_0=0.

After measuring the steady fall velocity of the ball υ_{st} and magnitudes r, ρ, ρ_1, the code of viscosity of the liquid can be calculated using the following formula:

$$\eta=\frac{2}{9}\cdot gr^2\frac{\rho-\rho_1}{v_{st}}. \tag{5.16}$$

That is the basic idea of the Stokes' method.

The task of this workisto determine the code of viscosity of glycerin. Glycerin is assigned to that kind of liquid the viscosity code of which depends on the temperature value when it is close to the room temperature values.

In addition, it is better to use glycerin in real experiments because it adsorbs the aqueous vapor in the air. The physical presence of water has a significant impact on its density and viscosity values. Data presented in table 5.1 show the dependence of viscosity degree on the temperature and glycerin percentage in the solution. There is no point in measuring glycerin viscosity without knowing its temperature and density (water-percentage).

Experimental technique

The equipment for the experiment is pretty simple: cylindrical glass container filled with glycerin. The diameter of the container is ≈ 5 cm and the length ≈ 1 m. The wall of the container is marked and the distance between these marks *l* is measured with the help of scale ruler. The marks are applied on the cylinder wall; the distance between them *l* is measured with the help of scale ruler. The upper mark is located slightly lower than the opened glycerin surface. By the time of the ball passing this mark, its velocity manages to reach a steady value.

Table 5.1

Glycerin water solution			
Glycerin solution composition, weight percentage	Viscosity η, 10^{-3}Pa·s		
	20^0 C	25^0 C	30^0 C
100	1495,0	942,0	622,0
99	1194,0	772,0	509,0
98	971,0	627,0	423,0
97	802,0	521,0	353,0
96	659,0	434,0	295,0
95	543,0	365,0	248,0

The time of ball's motion between the marks *t* is measured with the stop-watch which allows finding the value of υ_{st} using formula:

$$v_{st} = \frac{l}{t}. \qquad (5.17)$$

Let's find out what is the optimal size of the ball for the measurements.

Task 2. Using the Stokes' formula conditions (5.2) prove that the measurements can be done with those balls whose radius is:

$$r^3 << \frac{9}{2}\frac{\eta^2}{(\rho-\rho_1)\rho_1 g}. \qquad (5.18)$$

Take the lead pellet for the measurements ($\rho=11,3\cdot10^3$ kg/m^3) with the radius *r*=1 mm. Can the viscosity coefficient of 95% glycerin solution be measured with its help at 20^0 C? Put $\rho_1=1,1\times10^4$ kg/m^3.

It is also worth measuring how much lower the opened glycerin surface should be located from the upper mark so that $\upsilon(t) \approx \upsilon_{st}$by the time it is passed by the ball (correct to 5%, for instance). To do that, find out which distance S the ball is making within time 3τ with the starting zero velocity. If we integrate the equation (5.15) in the range from 0 to 3τ, we will obtain:

$$S(3\tau) = \int_0^{3\tau} v(t)dt = v_{st} \cdot \tau \left(\frac{t}{\tau} - 1 + e^{-\frac{t}{\tau}} \right) \Big|_0^{3\tau} \approx 2v_{st} \cdot \tau \approx \frac{8}{81} g r^4 \frac{\rho(\rho - \rho_1)}{\eta^2}$$

Calculate this distance for the previous example.

Naturally, the following question occurs: how should we carry out the experiment to determine the viscosity coefficient of an “unknown liquid” if we have no idea about its viscosity coefficient? As a rule, students accomplish some series of measurements η_1, η_2, ...η_n and determine the average viscosity coefficient $<\eta>$, not even thinking about the legitimacy of such averaging.

What would be your attitude toward the obtained in such a way experimental value $<\eta>$, when it turns out that in this case Stokes' criterion of applicability is not fulfilled? Probably - skeptical. This situation may seem strained, but it isn’t. If the experiment is carried out improperly, the result can be exactly as described above.

What would be your attitude toward this result, when it turns out that the values η_1, η_2, ...η_n had systematic dependence (conformity) on the ball radius r, for instance?

This is the example of an experiment that was carried out inaccurately, even though all the measurements and calculations were done perfectly. In this case further measurements should be done, using smaller balls, and increasing the distance between the upper mark and the opened glycerin surface.

The reliability criterion in this experiment is the absence of systematic dependence of η on r; this dependence may have only random character, coming from random errors. Only in such a case it is allowable to average out the results of the experiment and make conclusions regarding validity of the theorems.

The order of work procedure

1. Take 5-10 balls of different diameter and measure their mean diameters using micrometer. The values of the balls density ρ and glycerin ρ_1 are indicated on the desk.
2. If the density ρ_1 isn’t indicated, measure it using aerometer, previously mixing glycerin with the mixer. Also, measure glycerin temperature.
3. Use the data from table 5.1 and analyze the applicability of Stokes’ formula. Find out at which distance from the opened glycerin surface the upper mark should be fixed. For your calculations use “the worst” values of r and η, in other words, maximum value of radius of the balls and 95% viscosity of glycerin solution at room temperature. If this distance is too small, take into consideration practical understanding.

4. Take the ball with pincers and carefully put it down in the middle of the opened glycerin surface. Closely watch its motion and measure the time of this ball passing between two marks, using the stopwatch. While counting, it is important that the eye should be at the level of the mark. The distance between the marks should be measured with the scale ruler. All of the measurements results should be written down in table 5.2.

5. Determine steady velocity values of the ball and calculate the viscosity coefficients of glycerin using formula (5.16). Make sure that obtained values of η don't show systematic dependence on ball's radius. Plot a graph $\eta(r)$ according to table 5.2.

Table 5.2

n	d (mm)	t (s)	υ_{st} (m/s)	η(Pa·s)	$\eta_i - \langle\eta\rangle$(Pa·s)	$(\eta_i - \langle\eta\rangle)^2$(Pa·s)^2
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

6. Find the mean value of glycerin viscosity coefficient $\langle\eta\rangle$. Considering η_1, η_2, ... ,η_n as the results of direct measurements, calculate the value of sample mean standard $S_{\langle\eta\rangle}$, using table 5.2.

7. Derive formula for systematic error η calculations:

$$\left(\frac{\sigma_\eta}{\eta}\right)^2 = 4\left(\frac{\sigma_r}{r}\right)^2 + \left(\frac{\sigma_g}{g}\right)^2 + \frac{\sigma_\rho^{\,2}+\sigma_{\rho_1}^{\,2}}{(\rho-\rho_1)^2} + \left(\frac{\sigma_t}{t}\right)^2 + \left(\frac{\sigma_l}{l}\right)^2 .$$

(5.19) Calculate the error σ_η, using table 5.3.

8. Evaluate the error $\langle\eta\rangle$ with dependence on the values σ_η and $S_{\langle\eta\rangle}$. Write down the final result, indicating density and temperature of glycerin.

9. Using table 5.1 evaluate water percentage in the experimental glycerin.

Calculations parameters:

Density of balls' matter	$\rho = 11{,}3 \cdot 10^3$ kg/m^3
Glycerin density	ρ_1=…………………
Distance between the marks	l =…………………
Glycerin temperature	t =…………………

Write down in table 5.3 values of systematic errors of parameters that are included in formula 5.19.

Table 5.3

Systematic errors of parameters

$\sigma_{\rho 1}$=……………	(kg/m^3)	σ_r =…………	(mm).
σ_t =……………	(s)	σ_g = ………	(m/s^2)
σ_e =……………	(m)	σ_ρ =………..	(kg/m^3)

Calculations formulas:

1) $\sum_{i=1}^{20}\eta_i = \dots\dots\dots\dots\dots\dots..$ (Pa·s)

2) $<\eta> = \frac{\sum_{i=1}^{20}\eta_i}{n} = \dots\dots\dots \quad \dots\dots\dots\dots\dots\dots$ (Pa·s)

3) $\sum_{i=1}^{20}(\eta_i - <\eta>)^2 = \dots\dots\dots\dots..$ (Pa·s)^2

4) $S_{<\eta>} = \sqrt{\frac{\sum_{i=1}^{n}(\eta_i - <\eta>)^2}{n(n-1)}} = \dots\dots\dots\dots$ (Pa·s)

5) $\frac{\sigma_\eta}{\eta}\cdot 100\% = \dots\dots\dots\dots\dots\dots$

6) $\sigma_{<\eta>} = \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..$(Pa·s)

1.

Questions to answer

1. Coefficients of viscosity. Newton’s formula for the force due to viscosity.
2. Laminar and turbulent flow. Reynolds number.
3. Stokes' formula. Conditions of its use.
4. Derive the differential equation of motion of a ball in an infinitely viscous liquid.
5. Obtain its solution $\upsilon(t)$ and make the corresponding analysis.
6. The idea of measuring the liquid viscosity coefficient using Stokes' method.
7. Which balls should be used for the measurements?
8. At which distance from the opened glycerin surface should the upper mark be fixed?
9. What is the reliability criterion in this experiment?
10. How are the errors calculated in this work?
11. Give answers to the questions given in the main text.

Laboratory work№ 1-6.Determination of theratio between gas heat capacity andits heat capacity atconstant pressure

Objectives: study the possibility of determination of the ratio between gas heat capacities at constant pressure and constant volume based on the first law of thermodynamics
Tools and equipment: oneballoon with two tubes and taps; liquid pressure gage (manometer), onerubber bulbor pump.

Theoretical basis

Heat capacity C is the ratio between an infinitesimal changeof heat quantityδQ,obtained by a bodyand the corresponding temperature increment of that bodydT:

$$C = \frac{\delta Q}{\delta T}. \tag{6.1}$$

Heat capacity that is taken relative to the mass unit is called specific heat capacity; it is designated by letter c (small). But more preferred variable is molar heat capacity – a heat capacity that is taken relative to the mole of a substance; it is designated by letter C (capital). Both of the above mentioned heat capacities are correlated in the following formula:

$$C = c\mu, \tag{6.2}$$

were μ – molar mass.

Gas heat capacity depends on theconditions under which the heat is transferred to a body. Let’s use the first law of thermodynamics in order to calculate the value of heat capacity:

$$\delta Q = dU + \delta A = dU + PdV, \tag{6.3}$$

wheredU – internal energy change, δA – work executed by a gas, P – gas pressure, V –gas volume.

By substitutingusing(6.3) in (6.1) we obtain:

$$C = \frac{dU}{dT} + P\frac{dV}{dT}, \tag{6.4}$$

or considering the fact that for a mole of an ideal gas

$$U = \frac{i}{2}RT, \tag{6.5}$$

we obtain:

$$C = \frac{i}{2} R + P \frac{dV}{dT} \cdot \tag{6.6}$$

Where R=8,314 J/(mol*K) – universal gas constant, and i – the number of degrees of freedomof a gas molecule. For a monoatomic gas $i = 3$, for a diatomic gas $i = 5$ and for a triatomic gas $i = 6$. In this case for a diatomic and a triatomic gas a strong bonding between the atoms in the molecule is assumed to be.

From the equation (6.6), taking into consideration the Clapeyron-Mendeleev equation, we can derive equations for the ideal gas heat capacity at a constant volume and at a constant pressure:

$$C_V = \frac{i}{2} R, \; C_P = \frac{i+2}{2} R. \tag{6.7}$$

The ratio of specific heats (heat capacities at a constant pressure and volume) $\gamma = \frac{C_P}{C_V}$ plays a key role in thermodynamics. In particular,besidesit is a part of Poisson's equation, which governs the adiabatic expansion of gas (the one that occurs without heat exchange with the environment).

$$PV^{\gamma} = \text{const}. \tag{6.8}$$

For an ideal gas γ can be easily derived from (6.7):

$$\gamma = \frac{i+2}{i}. \tag{6.9}$$

However, the chemical composition of a gas isn’t always identified, which means that the value of $i$is unknown in sucha case. Due to this fact such experiments that allow to define the ratio ${C_P}/{C_V}$ for any gases that are close to the ideal gas by their properties are currently central. One of the easiest ways to determine the ratio ${C_P}/{C_V}$ is the Clement and Desormes' method. In this method the adiabatic expansion and compression of gas areused.

A gas that was placed into a vessel is experiencing three states consequently. Each of those states is thus characterized by pressure P,volume $V$and temperature T. The first state has parametersP_1, V_1, T_1, second – P_2, V_2, T_2 , third – P_3, V_3, T_3.

The first state stands for a gas placed in a closed glass vessel; it has room temperature T_1 and pressure P_1 that is slightly greater than atmospheric pressure.

If we combine the vessel with an atmosphere for a short period of time the adiabatic expansion of gas will take place. The gas pressure will become equal to atmospheric pressure P_2, and temperature T_2 will decrease due to the rapid gas expansion. This transition follows the Poisson's equation (6.8), which with the help of Clapeyron-Mendeleev equation can be transformed to the following equation form

$$\left(\frac{P_1}{P_2}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)^{\gamma}. \quad (6.10)$$

Because of the heat exchange with the environmentthe gas in a closed vessel will transfer from the second state to the third. As this occurs, the temperature of gas and room temperature will become equalized $T_3 = T_1$, volume will remain constant, and pressure P_3 will increase. Such a transition follows the Charles law:

$$\frac{P_3}{P_2} = \frac{T_3}{T_2} = \frac{T_1}{T_2}. \quad (6.11)$$

After solving both (6.10) and (6.11) we obtain:

$$\gamma = \frac{\ln P_1 - \ln P_2}{\ln P_1 - \ln P_3}. \quad (6.12)$$

In the formula (6.12) according to the conditions of the experiment P_2 is atmospheric pressure, and pressures P_1 and P_3 are greater than P_2 correspondingly by a value of hydrostatic pressurein a manometer with the height of h_1 and h_2, so that $P_1 = P_2 + \rho g h_1$ and $P_3 = P_2 + \rho g h_2$, where ρ is liquid density.

Since $\rho g h_1$ and $\rho g h_2$ are small values compared to P_2, then the logarithms of pressures may be expressed as:

$$\ln(P_2 + \rho g h_1) = \ln P_2 (1 + \frac{\rho g h_1}{P_2}) \approx \ln P_2 + \frac{\rho g h_1}{P_2}, \quad (6.13)$$

$$\ln(P_2 + \rho g h_2) = \ln P_2 (1 + \frac{\rho g h_2}{P_2}) \approx \ln P_2 + \frac{\rho g h_2}{P_2}. \quad (6.14)$$

After having substituted (6.13) and (6.14) in (6.12), we will obtain:

$$\gamma = \frac{h_1}{h_1 - h_2}. \quad (6.15)$$

Instrument description

The instrument used in this experiment is a glass balloon (fig.6.1) filled withair and tightly corked. The size of the balloon should be big enough to neglect the value of changes in the gas volume in the leg of manometer. Two tubes pass through the cork: one of them is connected to the liquid pressure gage, in other words – manometer (the liquid inside the manometer is water), another tube is connected to the combined tap. In one of the orientations (*K)* the volume of the balloon is connected to the environment, in another orientation – it is connected to the rubber bulb or pump. The apertures in the tap and tube connected to the tap *K* should allow a quick air-out, as otherwise the process can’t be considered as adiabatic.

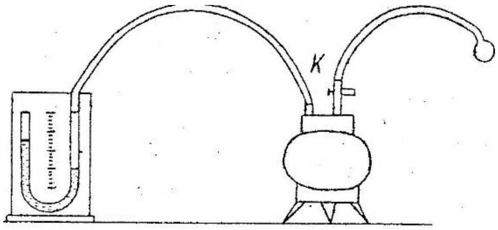


Fig. 6.1.

The order of work procedure

1. While studying the instrument composition a lot of attention should be paid to tap*K*. At the beginning, the tap is in such position that the volume of the balloon is isolated from the atmosphere and connected to the pump.

2. Using the pump, inflate the air until the difference between pressure inside manometer *h* reaches 7-8 cm. Due to the work of compression, caused by external force, the temperature inside the balloon will slightly increase. Compressed and heated air inside the balloon will start to cool down to the room temperature. Within 3-5 minutes, at the moment when liquid levels inside the manometer will stop changing, write down the difference between liquid levels inside manometer h_1.

3. Quickly, using the tap, connect the volume of the balloon to the atmosphere andcover the exit to the atmosphere at the moment when the levels of liquids inside the manometer will become equal. If you perform all of the steps quickly, the heat exchange between the air inside the balloon and the surrounding air won't occur. Thus, the adiabatic expansion of air will take place here. The air inside the balloon, while expanding,does work against the pressure of external forces through its internal energy. After some time, the air inside the balloon will warm up to the room temperature and its pressure will increase.Duetothis, the liquid column inside the manometer attached to the balloon will start to decrease.

4. After the changes in levels of liquid inside the manometer stabilize, calculate and write down the difference between these levels h_2.Repeat the experiment (point1-3) 8 times. Write down the results of measurements in table 6.1.

5. Usingformula (6.15), calculate γ for each of the experiments and find their mean value.

6. Calculate the value of standard deviation$S_{<\gamma>}$ and relative error ε. Write down the results in table 6.1.

7. Write down the final result as the following $\gamma = \langle\gamma\rangle \pm t_{\alpha,n} S_{\langle\gamma\rangle}$; α=0, 8.

8. Take thevalueof Student's coefficient $t_{\alpha n}$ for α=0,8from the table in the "Supplement" to work 1.1.

Table 6.1

N	Difference between levels inside the manometer		γ	$(\gamma_i - <\gamma>)^2$
	h_1, cm	h_2, cm		
1				
2				
3				
4				
5				
6				
7				
8				
$t_{\alpha,n} =$		$\sum_{i=1}^{8} \gamma_i =$		$\sum_{i=1}^{8} (\gamma_i - <\gamma>)^2 =$
$\varepsilon = \frac{S_{\langle\gamma\rangle}}{\langle\gamma\rangle} \cdot 100\% =$		$<\gamma> = \frac{\sum_{i=1}^{8} \gamma_i}{8} =$		$S_{\langle\gamma\rangle} = \sqrt{\frac{\sum_{i=1}^{8} (\gamma_i - <\gamma>)^2}{8 \cdot 7}} =$

The net result: $<\gamma>=$………………

Answer the following questions:

1. What is the connection between the specific heat capacity and molar heat capacity?
2. Howmuchgreateris the molar heat capacity at a constant pressure than molar heat capacity at a constant volume?
3. Explain the first law of thermodynamics.
4. Give a definition of the terms “equilibriumisoprocess” and “adiabatic process”. Depict them on a thermodynamic diagram.
5. Derive ratio 6.10from ratio 6.8.
6. How will the late covering of tap *K* exit influence the result of the experiment?

7. Howdoesthe presence of water vapor in the air inside the balloon influence the result of the experiment?

Laboratory workNo 1-7. Study of gas laminar flow through thintubes

Objectives:experimental check of Poiseuille equation; determination of air viscosity coefficient.
Equipment: capillary, gas meter (flow meter), dehumidifier (desiccator, desiccant), manometer, stopwatch.

Theoretical basis

Let us consider some steady flow of viscous incompressible liquid (gas) along the linear cylindrical tube with radius R[1]. Under the small flow velocities we can observe laminar (layered) flow: the liquid is seemed to be divided into some layersthat are sliding (slipping) passing each other without intermixing. In this case the layers are the set of indefinitely thin cylindrical surfaces inserted one into another with the same axis, and this axis coincides with the tube's axis.Under the condition of incompressibility we can assume that the velocity of each layer is constant. Thus, liquid (gas) flow velocity υ can only depend on the distance r from the tube's axis.

Now let's define hypothetical cylindrical volume of liquid with radius r andlength$L$**,**as shown on Pic.7.1. The pressure at its ends will be correspondingly marked as P_1iP_2. Under the condition of steady flow the force of pressure on the cylinder $F = (P_1 - P_2)\pi r^2$ is balanced by the force of internal friction F_{mp} that effects on the side surface of the cylinder from the external layers of the liquid. Thus, the condition of the stationarity of the defined (chosen) flow volume is the following:

$$F - F_{mp} = 0. \tag{7.1}$$

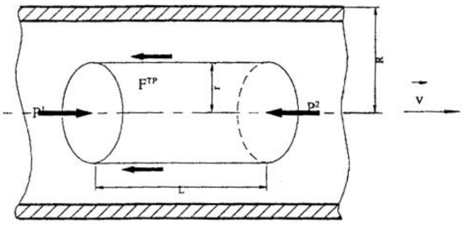
The internal friction force is calculated (determined) according to the Newton's formula:

$$F_{mp} = \eta \left| \frac{d\upsilon}{dr} \right| S, \tag{7.2}$$

---

[1] With the view of more evidence we examine liquid while derivation Poiseuille equation, though all the given above can be applied to the gas flow as well.

Where:η–liquid (gas) viscosity coefficient); $\frac{d\upsilon}{dr}$ – velocity (speed) module gradientthat definesvelocity (speed) changes in direction r (tube radius); S – the square of the surface of contacting liquid layers.



Pic. 7.1

Velocity (speed) $\upsilon(r)$falls off while distancing from the tube's axisthat is $\frac{d\upsilon}{dr}$ < 0,thus force quantity:

$$F_{mp} = -2\pi r \eta L \left| \left(\frac{d\upsilon}{dr} \right) \right|.$$

In this case the condition of stationarityhas the following characteristics (Pic. 7.1):

$$\pi r^2 \left(P_1 - P_2 \right) + 2\pi r \eta L \left(d\upsilon / dr \right) = 0. \tag{7.3}$$

If we integrate this equation, we can find $\upsilon(r) = \frac{\left(P_1 - P_2 \right) r^2}{4L\eta} + C$,

where C – integration constant that are calculated (determined, found) by the boundary conditions of the problem. It is worth mentioning that the velocity must be equal zero when $r = R$,as the liquid 'sticks' to the tube's side; it allows us to determine C .At the result we have:

$$\upsilon(r) = \frac{P_1 - P_2}{4\eta L} \left(R^2 - r^2 \right). \tag{7.4}$$

Thus, liquid velocity (speed) changes in the squared numbers with the radius ant it has its maximum on the tube's axis where it equalsto $\upsilon(r = 0) = \frac{P_1 - P_2}{4\eta L} R^2$.

Now let's determine liquid consumption Q that is the volume which every second flows through the diametrical (transversal) cross-cut of the tube. The circular square of the internal radius r and external $r + dr$ is flowed every second by the liquid flow with the volume of $dQ = 2\pi r dr \, \upsilon(r)$.Then the consumption of the liquid will be:

$$Q=\int_0^R 2\pi r\upsilon(r)\cdot dr=\pi\frac{P_1-P_2}{2\eta L}\int_0^R\left(R^2-r^2\right)\cdot rdr$$

$$\text{or } Q=\frac{P_1-P_2}{8\eta L}R^4 \qquad (7.5)$$

Formula 7.5 is calledPoiseuille equation and it allows us to experimentally determine the liquid viscosity (gas).

Poiseuille equation was derived proceeding on the assumption that the flow of the incompressible liquid (gas) is laminar.But when the flow speed accelerates, the movement becomes turbulent and the layers migrate. Under the condition of turbulent movement, the velocity and direction at every point change very quickly and only the average velocity is kept the same.

The movement character of liquid or gas in the pipe is defined by the dimensionless Reynolds number:

$$Re=<\upsilon>\frac{R\rho}{\eta}, \qquad (7.6)$$

where $<\upsilon>$ is the average flow velocity, ρ is the liquid or gas density. In smooth pipes with the round cut the change from the laminar to the turbulent flow takes place when $Re_{кр}\approx 1000$.That is why, if we want to apply Poiseuille equation, first of all, it is necessary to have enough margin for solving inequality $Re < 1000$.Besides, the experiment should be conducted in such a way that the gas compression can be neglected. As for the liquid, as a rule,this assumption always works,but for the gas it is correct only whenpressure differential along the tube (pipe) is significantly lower than the pressure itself. In our case the gas pressure is equal to the atmospheric one (10^3smwg.)and the pressure differential is ~ 10 smwg., that is ~ 1% of the atmospheric one.

Equation 7.5 is valid for those sections of the tube where flow is steady with the characteristic (typical) law of speed (velocity) distribution (7.4) along their cut. Laminar movement of the gas while going from the wide vessel to the tube is set up not at once but when the gas has passed the following distance:

$$a\approx 0{,}2R\cdot Re. \qquad (7.7)$$

Poiseuille equation gives reliable results only when the tube’s length is $L>>a$. To satisfy this condition we must (have to) use extremely thin tube – capillary.

To check experimentally Poiseuille equation it is necessary to study dependence of consumption Q from $\Delta P=P_1-P_2$.As a rule for measuring pressure difference U-like manometer is used. In this case $\Delta P=\rho_0 g\Delta h$, where ρ_0 is liquid density in the manometer; Δh is the difference of its levels in the manometer's legs.

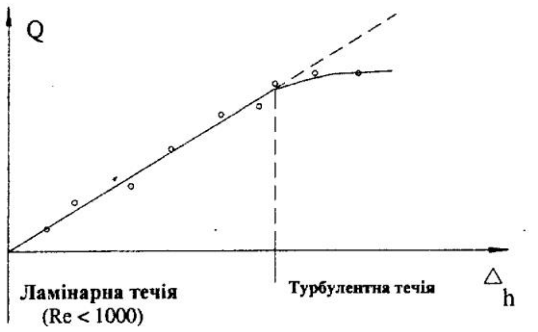
Having applied Poiseuille equation we will have:

$$Q=\left(\pi\rho_0 g\frac{R^4}{8\eta L}\right)\Delta h \; . \qquad \text{(7.8)gas}$$

As we can see in case of laminar flow dependence $Q(\Delta h)$ has linear character.Whenturbulenceoccurs, the linearity is disturbed (broken down): pressure differentialthat is proportional to Δh rises quicker than the consumption (pic.7.2). The angular coefficient of the linear section of the graph:

$$k=\pi\rho_0 g\frac{R^4}{8\eta L} \qquad (7.9)$$

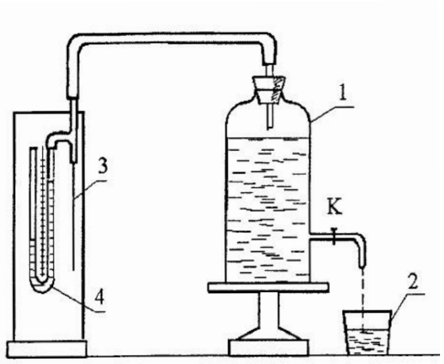
By this coefficient we can define gas viscosity η.The point of the graphic bend gives the critical meaning (value) of number$Re_{\text{кр}}$,which corresponds to the transition from laminar to turbulent flow. If on the section of flow laminarity the experimental values $(Q,\Delta h)$(taking into account the experiment error) fall on a straight line, then this proves the validity of Poiseuille equation.



Pic 7.2

Experimental installation description

Gas (flow) meter used in this work is a glass vessel (pic.7.3) partially filled with water and thoroughly covered. Water from the gas meter can flow (flows) into measuring glass 2 after turning on tap K. Air in the gas meter combines with the atmosphere by means of capillary tube 3. Pressure differential ΔP at the capillary tube ends is measured by manometer 4 filled with water.



The order of work fulfillment

1. Prepare the installation for the measuring procedures. While turning on tap K slowly, watch carefully the manometer's

indicators. Adjust the water run-off from the gas meter by tap K in such a way that $\Delta h \cong 1см$. Measure air discharge (consumption) Q .To do this you have to measure time t by the stopwatch V=250mlwater. Air discharge (consumption) you have to define according to equation $Q = V/t$. It is better to measure volume Vnot at once but in some time after the beginning of water running-out when the difference of liquid levels in the manometer's leg becomes stable. Pic.7.3

2. Deliver the same measuring gradually increasing the pressure differences. For example with the step of 0,5 cm.w.t., that is define the experimental dependence $Q(\Delta h)$, thatmust have not less than 10 –15 experimental points. Then write down the results in table 7.1

3. Write down the equipment parameters R , L , ρ_0 indicated atthe working place as well as the air temperature in the room. Air density $\rho =$ 1,293 kg/m^3.

Experiment data analysis

1. On the squared paper build dependence graph $Q(\Delta h)$.
2. According to angular coefficient $k_{гр}$ .of the linear section of the graph define air viscosity η (equation 7.9).
3. With equation 7.6 determine the value of number $Re$for the air of transition between laminar and turbulent flows. The average flow speed is determined by the following equation: $< \upsilon >= Q/S_k$. Where $S_k$isthe square of the capillary diametrical cut. Compare your result with the critical value of Reynolds number$Re_{кр}$given above.
4. Deliver the correlation analysis (processing) of those experimental data $Q(\Delta h)$ thatcorrespond to the laminar flow section (Appendix "Correlation analysis"). Draw the conclusion on the correctness of Poiseuille equation based on the laboratory work results. Compare values of angular coefficient $k_{гр}$ received graphically with the result obtained under the calculation using the method of the smallest squares $k_{кор}$ .Define η using $k_{кор}$.

Questions for self-control

1. Viscosity coefficient. Explain Newton’s formula for the internal friction force.
2. Draw out Poiseuille equation.
3. Laminar and turbulent flow.Reynolds number.
4. Analyze the conditions of the Poiseuilleequationcorrectness.
5. In which way (How) can we support the experimental check of Poiseuille equation?
6. How are air viscosity coefficient and critical value of Reynolds numberdefined in this laboratory work?
7. Experimental equipment, what the elements are used for, measurement techniques (methods)?

Calculations parameters

Capillary radius $R =$ .... Capillarylength $L =$ ....

Water density $\rho_0 =$ .... Air temperature $t^{\hat{\imath}}\,C =$ ....

Таблиця 7.1

Test number	*t*, s	Q, ml/s	Q, m^3/s	Δh, m
1				
2				
3				
4				
5				
6				
7				
8				

Angular coefficients $k_{гр}$та$k_{кор}$ calculation (defining). a) According the graph
б) According МНК

$k_{гр} = \Delta Q / [\Delta(\Delta h)] =$........................ $k_{кор} =$ ....

(We designate) Let’s designate $\pi\rho_0 g R^4 / 8L = \text{const} =$
....

Calculation of viscosity coefficient η

1)with the graph angular coefficient 2) with correlation coefficient $k_{\hat{e}\hat{\imath}\eth}$.

$\eta = \text{const}/k_{\tilde{a}\eth} =$............................. $\eta = \text{const}/k_{\hat{e}\hat{\imath}\eth} =$..............................

Calculation (Determination) of Reynolds number

$\mathrm{Re} = <\upsilon> R\rho/\eta = (Q/S_k) \times (R\rho/\eta)$ =...

Appendix

Correlation analysis.The smallest squares method.

While solving numerous scientific and technical problems it often turns out to be important to describe some functional dependence between two measured quantities ***x***, ***y***. If values)**x**, **y**are measured accurately enough, then each value **x**corresponds to only one (practically completely defined) value ***y***, so in such a case experimental measurement gives us some random data about the dependence under the consideration.

Then for the description we can use some methods of mathematical analysis. For example, we can buildinterpolating Lagrange polynomial. It acquires in the determined points$\boldsymbol{x_i}$ (***i*** = 1, 2,***n***- measurement numbers) determined values $\boldsymbol{y_i}$and thus we can try to approximatethe dependence under the consideration by the power function or the exponential one, the cosine function, etc.

But in practice we sometimes encounter the situation when experimental data have considerable uncontrolled variations, and consequently we cannot state that each value of *x*correspond only one determinedvalue of *y*.This spread can be the result of the measurement errors or can be caused by the statistic nature of the phenomenon under the study (the classical example is the measuring of the radioactive background intensity).

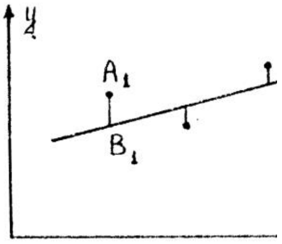
Pic 1.

In some cases the spread is so considerable that the displacement of points $\boldsymbol{A_i(x_i, y_i)}$on the plane ***x,y***seemto be absolutely chaotic. But this does not exclude that the displacement of points$\boldsymbol{A_i}$hassome regularity. Let’s consider as an example Pic. 1. The points shown in this picture are dispersed almost uniformly in some rather wide area. But it can be easily seen that <u>in average</u> bigger $\boldsymbol{x_i}$are corresponded to bigger y_i.

This trend is depicted by the solid line on the same picture.Obviously, this line explainsthe correlation between **x**and***y***abstracting the details of ***A*** points displacement. These average dependences are called <u>statistic</u> or <u>correlational</u> contrary to the mentioned above functional ones.

The description of such dependences needs solving of two problems:

1) to define the degree of dependence, 2) to determine the force power between**x**and***y***. We are not going to consider in details the first question, we will assume that we have managed to find some function***y* =*f* (*x*)**thatcan be applied for the description of correlation between **x**and***y***. In this case it is often worth linearize the proposed dependency ***y* =*f* (*x*)**, that is to bring it to the linear one. In simple functions it is quite easily to do it.For



example, to do the degree function $y = ax^n$ linear you have to logarithm it with any basis(for example e), then this dependence will be of this kind $\ln y = \ln a + n \cdot \ln x$, linearrelatively to the variables $\ln y$ and $\ln x$.Indicating (indicated) dependence $y = ca^x$ is also linearized with the help of taking logs: $\ln y = \ln C + x \ln a$ and we receive the linear dependency in variables $\ln y$ and x etc. Now let’s consider that the initialvariables x, y are already modified in such a way that the dependence under measurement has linear form:

$$y = kx + b. \tag{1}$$

But our choice of dependence in the form of 1 does not determine its values of parameters k, b. It is obviously desirable to select parameters k, b in themostoptimalway, and thus they will provide the best adjustment of correlation dependence 1 to points Рис.2 $A_i\ (x_i, y_i)$. To do this, as a rule, the method of the smallest squares (МНК) is used. According to this methodthe sum of distances squares (on the vertical) of A_i points from the straight line (Pic.2) is taken as the F deviation of A_i points from the straight line (1):

$$F = \Sigma\ |A_iB_i|^2 = \Sigma\ [y_i - (kx_i + b)]^2$$

(here and later sign Σ meansadding for i from 1to n). Value F is function of k, b: $F = F(k, b)$.

Parameters k and b are selected with value F to beminimal. Writing down the conditions of function $F(k,b)$ extremum it is easy to prove that this extremum corresponds to the minimum: $\frac{\partial F}{\partial k} = 2k\sum x_i^2 + 2b\sum x_i - 2\sum x_i y_i = 0;$

$$\frac{\partial F}{\partial b} = 2k\sum x_i + 2bn - 2\sum y_i = 0.$$

Thus, we have the system of two equations relatively to k, b, from which we will find

$$k = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum x_i^2 - (\sum x_i)^2},$$

$$b = \frac{(\sum x_i^2)(\sum y_i) - (\sum x_i)(\sum x_i y_i)}{n(\sum x_i^2) - (\sum x_i)^2} \tag{2}$$

If we introducedesignations $< x > = \frac{1}{n}\sum x_i$ $< y > = \frac{1}{n}\sum y_i$, $< x^2 > = \frac{1}{n}\sum x_i^2$

$< y^2 > = \frac{1}{n}\sum y_i^2$, $< xy > = \frac{1}{n}\sum x_i y_i$, $S_x^2 = < x^2 > - < x >^2$,

$S_y^2 = < y^2 > - < y >^2$, $S_{xy} = < xy > - < x >< y >$,

Then equation 2for parameters k, b andequation 1 can be written in this symmetricalform:

$$k=\frac{S_{xy}}{S_x^2},\quad b=\frac{S_x^2<y>-S_{xy}<x>}{S_x^2},\qquad (3)$$

$$y-<y>=\frac{S_{xy}}{S_x^2}(x-<x>).\qquad (4)$$

Now let’s consider the question of force or<u>degree of density</u> dependence (1). This notion is illustrated by pic.3.It is obvious that in pic.3а the linear dependence between $x$and$y$is maximally dense (functional); inpic.3в the points spread ischaoticand there isno obvious connection between $x$and$y$because the dependence force presented as the straight line equals to zero; in pic.3б the intermediate case is depicted.

Thus, the degree of correlation density is an important quality that definesthe practical value of dependence: if the correlation density is big, then applying it we can assume$y$of the given $x$quite accurately; otherwise, the accuracy of the assumption (prognosis) is not so important and we can assume according to the given $x$only the average value of y.

In mathematical statistics the degree of correlation density is usually characterized by <u>correlation coefficient</u> which is defined by the following:

$$\boldsymbol{r}=\frac{S_xS_y}{S_{xy}},\qquad (5)$$

where$r$is dimensionless value. We can demonstrate that $r$changes in the limits from –1 to +1. If$x_i$and$y_i$are connected by the linear dependence (1), then$/r/$ =1 (prove this independently, without assistance). If $x$and$y$are statistically independent, then while calculating the average $<xy>$, values $x$and$y$can be averaged independently and we will have:

$<xy>\approx<x><y>,\ S_{xy}\approx0,\ r\approx0.$

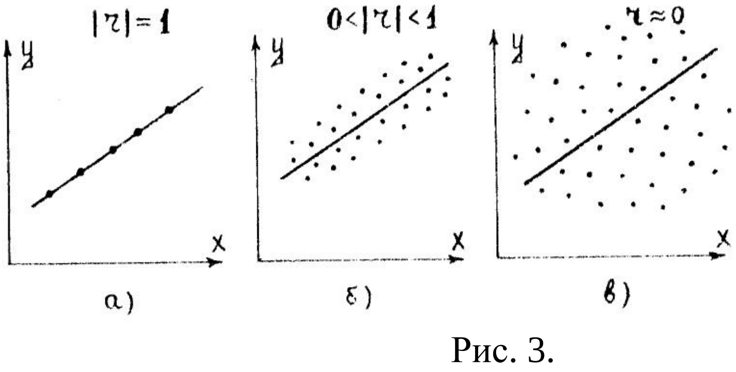


Рис. 3.

If $n$goes to∞,thenthese approximate equalities become accurate and thecorresponding rational correlation coefficient $r_{icm.}$=lim$r$when$n$approaching∞,turns into zero. In the intermediate case when$x_i$and$y_i$are bound by the nonlinear dependence or when we have some data spread, then coefficient$r$has the intermediate value: $0</r/<1$ (seepic. 3). Thus, $/r/$isthe proof of how much the correlation between $x$and$y$is close to the linear: both in relation to the degree of data spread and to the nonlinearity of this correlation; the closer $/r/$ to1, the closer is the connection between $x$and$y$to the linear.

Let’s determine which value **/*r*/**is to be considered sufficient for the statistically proved derivation on the existence of the linear correlation dependency between ***x***and***y***. For this we will consider the so-calledstatistical hypothesis $\boldsymbol{H_0}$: $\boldsymbol{r_{icm.}=0}$, it means that we study hypothesis assumption that $\boldsymbol{r_{icm.}=0}$.Performing this assumption means that ***x***and***y***are not bound by the correlation dependency, and its refutation means that there is no proof to neglect the existence of such dependence.Thus while analyzing experimental data, as a rule, it is known not $\boldsymbol{r_{icm.}}$but only optional correlation coefficient ***r*** ; with this optional coefficient we can examine performing or not performing hypothesis $\boldsymbol{H_0}$onlystatistically, that is with some predetermined probability confidence α (most often it is taken α= 0,95). For this it is necessary to know the statistic distribution of value ***r***. As it is shown in mathematical statistics, correlated with it value $R = r\frac{\sqrt{n-2}}{\sqrt{1-2^2}}$ is described by Student’s spread with ***n-2***degrees of freedom (seeitems1-4 in Literature). From this we can conclude that under the predetermined probability of confidence value $\boldsymbol{R_{icm}}$is in the interval $\boldsymbol{R_{icm} = R\pm t_{\alpha};\ _{n-2}}$. We are interested in value $\boldsymbol{R_{icm}}$ = 0 (that corresponds to$\boldsymbol{r_{icm}}$= 0). It enters this interval under the fulfillment of the condition$\boldsymbol{/R/\leq t_{\alpha;\ n-2}}$. If the opposite condition is fulfilled $\boldsymbol{/R/>t_{\alpha;\ n-2}}$, the true value $\boldsymbol{R_{icm}}$(consequently $\boldsymbol{r_{icm}}$ as well) under the determined degree of confidence probability it is impossible to consider it equal to zero and hypothesis $\boldsymbol{H_0}$ should be neglected. Value 1-α = 0,05is called error probability or significance degree, and α=0,95 is called statistic trustworthiness. So, if we use inequality $\boldsymbol{/R/>t_{\alpha;\ n-2}}$ **,**thenwith the statistictrustworthiness0,95 (or 95%) we can state that ***x***and***y***are bound by the correlation dependence. To simplify the examination of this inequality it is convenient to include intermediate value ***R***andto apply it in the form of inequality for value ***r*** (do these calculations yourself); then we will have:

$$/r\ /> r_n,\ \text{де}\ r_n = \frac{\tau}{\sqrt{1+\tau^2}},\quad \tau = \frac{t_{0.95;n-2}}{\sqrt{n-2}}$$

Necessary for calculating values τcoefficient values$t_{0.95;n-2}$are given in table 1.

Table 1

Numbern	5	6	7	8	9	10	11	12
$t_{0,95;n-2}$	3,18	2,78	2,57	2,45	2,36	2,31	2,26	2,23
Number n	13	14	15	17	18	20	30	∞
$t_{0,95;n-2}$	2,20	2,18	2,16	2,13	2,12	2,10	2,04	1,96

Notice, that in the given Appendix the simplest scheme of correlation analysis is given. For more accurate analysis systematic errors should be taken into account calculating S_x, S_y, S_{xy}according to theequations of the selected average (that is from*n*-1 instead of*n*under the sign of square root)etc. But all these complications are either not so important in most physical or engineering spheres of correlation analysis application, or their description would be inappropriate in these methodological instructions.

And finally, we give the short scheme of the correlation analysis of experimental data.

With the experimental values of x_i and y_i the following values are calculated:

$$< x >= \frac{1}{n}\sum x_i, \quad < y >= \frac{1}{n}\sum y_i, \quad < x^2 >= \frac{1}{n}\sum x_i^2,$$

$$< y^2 >= \frac{1}{n}\sum y_i^2, \quad < x\,y >= \frac{1}{n}\sum x_i y_i; \quad S_x^2 =< x^2 > - < x >^2,$$

$$S_y^2 =< y^2 > - < y >^2, \qquad S_{xy} =< x\,y > - < x > < y > \cdot$$

1. Parameters of correlation dependence and correlation coefficient are determined:

$$k = \frac{S_{xy}}{S_x^2}, \qquad b = \frac{S_x^2 < y > - S_{xy} < x >}{S_x^2}, \qquad r = \frac{S_{xy}}{S_x S_y}.$$

2. According to the table $t_{0.95;n-2}$ isdetermined and value r_n is defined:

$$r_n = \frac{\tau}{\sqrt{1+\tau^2}}, \quad \text{де}\ \tau = \frac{t_{0.95;n-2}}{\sqrt{n-2}}$$

3. Inequality performance $/ r/ > r_n$is checked. Under the condition of this inequality fulfillment it is concluded that there is the existence of (with 95% probability)the linear dependence between *x*and y . When it is not fulfilled we conclude that there is no linear dependence.While calculating it is helpful to use table 2:

Table 2.

	x_i^2	x_i	$x_i\,y_i$	y_i	y_i^2
1					
…					
n					
	$< x^2 >=$	$< x >=$	$< xy >=$	$< y >=$	$< y^2 >=$

Laboratory work№ 1-9. StudyofBoltzmann distribution

Objectives: experimental verification of Boltzmann distribution for small particles weighed in liquid and evaluation of Boltzmann constant.
Tools and equipment:narrow cylindrical vessel with transparent liquid inside; small particles, weighed in that liquid; light source; photodetector; photocurrent meter; scale ruler.

Theoretical basis

At the thermal equilibrium state distribution of equal particles by velocity and coordinates is determined only by particles' energy E and system's temperature T (where T is absolute temperature, measured in Kelvin). In order to write down this distribution, which is called Maxwell-Boltzmann distribution, let's introduce a system of coordinatesx, y, z, that provides the basis for determination of particle's position. We will define the particle's velocity by componentsv_x, v_y, $v_z$and axes x, y, z.

In this case the distribution can be written as:

$$dn_{x,y,z,v_x,v_y,v_z} = Ae^{-\frac{E}{kT}} dx\,dy\,dz\,dv_x dv_y dv_z, \qquad (9.1)$$

where dn_{x,y,z,v_x,v_y,v_z} – amountofthoseparticleswhosecoordinateslieintherangefrom$x$, y, $z$to$x+dx$, $y+dy$, $z+dz$, andvelocitycomponentslieintherangefromv_x, v_y, $v_z$to$v_x + dv_x$, $v_y + dv_y$, $v_z + dv_z$,***A***– normalizingmultiplier (the typeofwhichwewon'tspecify), ***k***– constant, which is termed as Boltzmann constant, in the Si-system, ***k***= 1,38·10^{-23} J/K, ***E***– particle's energythat depends on its coordinate and velocity.

Ifweareinterestedinthe distribution$dn_{x,y,z}$ ofparticlesonlybytheircoordinates, while their velocities aren't significant to us, we will have to integrate the equation (9.1)with respect to the velocities. Inordertoperform this task, weshould assumethatexternalforces, whichaffect the particles, areconservative. Let'swritedownenergy***E***asasum of kinetic and potential energies:

$$E = E_{\text{ê}}(v_x, v_y, v_z) + E_{\text{ï}}(x, y, z)$$

and insert (plug) this distribution into the formula (9.1), after splitting up those terms of this equation that depend on the coordinates and velocity components:

$$dn_{x,y,z,v_x,v_y,v_z} = Ae^{-\frac{E_{\text{ï}}(x,y,z)}{kT}}\; dx\; dy\; dz \cdot e^{-\frac{E_{\text{ê}}(v_x,v_y,v_z)}{kT}}\; dv_x\; dv_y\; dv_z.$$

Let's perform the integration with respect to all of the velocities. In other words, let's take a triple integral with respect to v_x, v_y, v_z in the range from $-\infty$ to $+\infty$:

$$dn_{x,y,z} = \int\int_{-\infty}^{\infty}\int dn_{x,y,z,v_x,v_y,v_z} =$$

$$= Ae^{-\frac{E_{\text{ï}}\,(x,y,z)}{kT}}\, dxdydz \cdot \int\int_{-\infty}^{\infty}\int e^{-\frac{E_{\text{ê}}(v_x,v_y,v_z,)}{kT}}\, dv_x dv_y dv_z.$$

Thetripleintegralfromtherightsideofthisformuladoesn't dependoncoordinates; so let's introduce it to the normalizing multiplier (constant) $A$and rewrite the integration result as:

$$dn_{x,y,z} = A^1 \cdot e^{-\frac{E_{\text{ï}}\,(x,y,z)}{kT}}\, dx\, dy\, dz\,, \tag{9.2}$$

whereA^1– new normalizing constant that equals to:

$$A^1 = A\int\int_{-\infty}^{\infty}\int e^{-\frac{E_{\text{ê}}(v_x,v_y,v_z)}{kT}}\, dv_x dv_y dv_z,$$

and$dn_{x,y,z}$ stands for the amount of particles with random velocities. The coordinates lie in the range from***x, y, z***to ***x+dx, y+dy, z+dz***. The distribution expressed in formula (9.2) is called Boltzmann distribution. Let's apply the distribution (9.2) to an ensemble of equal particles with mass ***m***weighed in liquid at temperature ***T***. Let's assume, that the liquid is poured into a narrow vertical vessel. We will mark the position of particle in the liquid column by height ***h***counted from the vessel'sbottom. Let's place the origin of Cartesian coordinate system ***x, y, z*** at the vessel's bottom. Also, let's direct axis ***z***upward vertically **(*z* = *h*)**, while axes ***x*** and ***y*** will be directed horizontally, as shown in fig. 9.1.

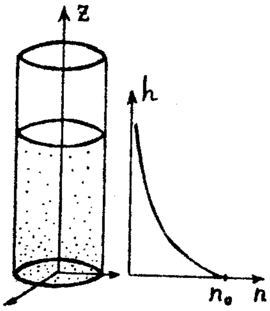


Рис. 1

The particles are affected by the force$F = F_T - F_{\text{À}}$, which equals to the difference between the force of gravity and Archimedes' force:

$F_T = mg = \rho\tau g$, (ρ – particle's density, τ - particle's volume,g - acceleration of gravity);

$F_A = \rho^1\tau g$ (ρ^1 – density of the liquid).

Aslongasforce$F$ isconstant, it is conservative and corresponds to the potential energy:

$E_{\text{ï}} = F \cdot z = \tau(\rho\text{-}\rho^1)gh.$

Thus, distribution (9.2) will acquire the following form:

$$dn_{x,y,z} = A^1 e^{-\frac{\tau(\rho\text{-}\rho^1)gh}{kT}}\, dx\, dy\, dz\,. \tag{9.3}$$

Boltzmanndistributionistrueforanyparticles, thus, itcanbeappliedtoheavyparticlesaswell. Ifwetakegrainsofsandin the capacity ofas suchparticles, obviously, theywillplacethemselvesinacertainlayer near the vessel's bottom. That would This will be the resultof Boltzmann distribution, because heavy particles mean high level of potential energy. The value of this potential energy would be will be so big ($E_{\text{Ï}} >> kT$), that the exponent index in formula (9.2) will change very rapidly with the height, and beyond the layer of sand grains the distribution function (9.2) almost equals to zero.

In order to elude precipitation of heavy particles at the bottom of vessel and to achieve distribution of heavy particles within a thick layer, it is required that their potential energy must be quite small. It can be achieved by placing those particles in a liquid with such density that is close to the density of particles' material. To make this effect noticeable, one should take small-sized particles.

Let's introduce a variable *n*, which stands for volumetric density of particles and equals to the amount of particles per unit volume or to the ratio $dn_{x,y,z}/dV$, between the amount of particles $dn_{x,y,z}$ in an elementary volume $dV = dxdydz$ and the value of this volume. In this case, formula 9.3 after dividing by dV can be rewritten in this form:

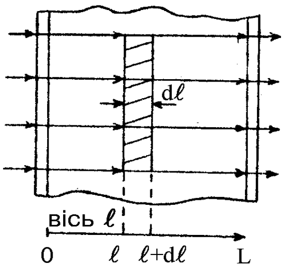
$$n(h) = A^1 e^{-\frac{\tau(\rho-\rho^1)gh}{kT}} . \qquad (9.4)$$

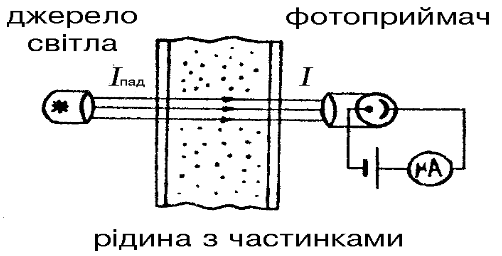
Let's find the value of *A'*. In order to do that, let's introduce a value n_0– the density of particles near the bottom of vessel. If we consider that in formula 9.4 $h=0$,we will obtain $A^1=n_0$. Thus, the distribution of particles density with height *h* should be written in the final form:

$$n(h) = n_0 e^{-\frac{\tau(\rho-\rho^1)gh}{kT}} . \qquad (9.5)$$

This formula is the direct corollary of Boltzmann distribution, thus, its confirmation will mean the experimental confirmation of Boltzmann distribution. In order to perform such confirmation, we ought to use one of the particles' density determination methods and examine the measured density height-dependency.

In this laboratory work the optical method is used to determine particles' density. The principle of this method lies in the fact that the more particles there will be in a certain liquid, the less the intensity of light, that passes through a transparent layer of this liquid with opaque particles inside will be. This happens due to the absorption and dispersion of light by the particles. Thus, through this decrease of light intensity we can identify particles' density. Due to the fact, that this particles density determination method is indirect (we don't determine the density itself, instead we determine the light intensity, which is connected to the density) we require a certain formula, that depicts the connection between the particles' density *n* and the light intensity I. For this purpose, let's have a look at fig.9.2a. In this picture we can see a parallel light beam passing through a flat layer of liquid with particles inside. The layer itself is located between two transparent walls.





a) *b*)

Fig.9.2

I_{imp} stands for the intensity of light that is impinging, I stands for the intensity of light, that passed through. First of all, the light has to pass through the first wall; afterwards its intensity decreases due to the partial retro reflection (reflection in backward direction), possible dispersion in different directions and some absorption by material in wall. Let's introduce value α_1, which will stand for the correspondent attenuation coefficient; then, the intensity of light, that entered the liquid, will be equal to $\alpha_1 \cdot I_{\text{imp}}$. In order to calculate the attenuation coefficient in the liquid let's introduce coordinate ℓ, which we will count following the normal line from the first wall to the second wall (fig.9.2, b). Let's regard the light intensity as a coordinate function – $I(\ell)$. Obviously, $I(0) = \alpha_1 \cdot I_{\text{imp}}$. On a short path length from ℓ to $\ell + d\ell$ the light disperses on particles that are located within a volume, shown by hatching (fig. 9.2, b). The size of this volume equals $S \cdot d\ell$ (S – cross-sectional area of the light beam), and the amount of particles, located within this volume equals to $n \cdot S\,d\ell$; this amount is pretty small due to the small value of $d\ell$. During the light dispersion over a single particle its intensity I decreases by a small portion, which we will express with|as α. Thus, the intensity of dispersed light will be equal to $I\alpha$; it is obvious, that during the light dispersion by $nSd\ell$ particles, that scarcely overlap each other in layer $d\ell$ (due to the fact that the amount of particles is quite small), the total intensity of dispersed light will be equal to the result of multiplication of light intensity, dispersed by a single particle, by the amount of particles: $I\alpha \cdot nSd\ell$. This value is nothing but a decrease of light intensity on the interval $d\ell$:

$$dI = -I\alpha nSd\ell, \qquad (9.6)$$

(sign "minus" is present here because I decreases with the increase of ℓ, in other words, $dI < 0$, if $d\ell > 0$). Ratio (9.6) is a differential equation with variables I, ℓ. These variables are easy to separate. In order to do that, let's divide both parts of ratio (9.6) by I :

$$\frac{dI}{I} = -\chi d\ell,$$

where $\chi = \alpha nS$.

After integrating both parts of this equation we will obtain:

$$\ell n I = -\chi\ell + C,$$

where C is integration constant. From here we find the dependency of I on ℓ:

$$^2(\ell)=\tilde{N}^1\cdot e^{-\chi\ell}, C^1 = e^c.$$

Under the condition $I(0)=\alpha_1 I$ imp, while $\ell=0$ we find that $C^1=\alpha_1 I$ imp, and from here we obtain:

$$I(\ell)=\alpha_1 I_{\text{imp}}e^{-\chi\ell}.$$

This formula governs the exponential attenuation of light intensity during its passage through a liquid. Let’s use this formula in order to determine the light intensity near the inner edge of the second wall. In this case $\ell=L$ (L - liquid layer thickness): $I(L)=\alpha_1 I_{\text{imp}}\, e^{-\chi L}$.

The light needs to pass through the second wall as well; upon that, the light loses some of its intensity again. Let’s express the correspondent attenuation coefficient through α_2 and, with that, obtain the final equation for the intensity of light that passed through: $I=\alpha_1\alpha_2 I_{\text{imp}}\, e^{-\chi L}$.

Let’s express this formula through the dependency of *I* on *n*:

$$^2={}^2{}_0 e^{-\beta n}, \tag{9.7}$$

where $^2{}_0=\alpha_1\alpha_2{}^2{}_{\text{imp}}$, $\beta=\alpha SL.$ The physical sense of value I_o isn’t hard to determine: if we consider that in this formula $n=0$, we obtain $I_o = I(0)$. Therefore, I_o stands for the intensity of light that passed through a layer of pure (particle-free) liquid.

In order to measure the intensity of light, that passed through, we can use light emission photo detector, connected to a measuring instrument, for instance, micro ammeter (fig. 9.2 a). Let’s consider instrumental indication $\mathbf{I}$ proportional to the light intensity I. Let’s introduce a correspondent conversion coefficient a.

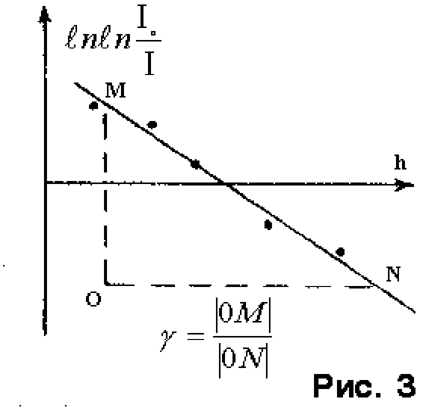


Fig. 9.3

If $\mathbf{I_0}=aI_0$, $\mathbf{I}=aI$ – instrumental indications, that correspond to the intensities $^2{}_0$ and I. After multiplying both sides of equation (9.7) by a, we will obtain the final formula, that connects $\mathbf{I}$ and *n*:

$$\mathbf{I}=\mathbf{I_0}\, e^{-\beta n}. \tag{9.8}$$

From here we obtain the following ratio:

$$n=\frac{1}{\beta}\ell n\frac{\mathbf{I_0}}{\mathbf{I}}.$$

If we put *n* in formula (9.5), we obtain:

$$\ell n\frac{\mathbf{I_0}}{\mathbf{I}}=\beta n_0 e^{-\frac{\tau(\rho-\rho^1)gh}{kT}}.$$

This dependency can be easily expressed in the form that is linear in h. In order to do that, let’s take the logarithm of this ratio:

$$\ell n\ell n\frac{I_o}{I}=\ell n\beta n_o-\frac{\tau(\rho\text{-}\rho^1)gh}{kT}. \qquad (9.9)$$

Therefore, the graph representing dependency of $\ell n\ell n\frac{I_o}{I}$ on h (fig.9.3) should take the form of a straight line with a slope ratio $\gamma=\frac{\tau(\rho\text{-}\rho^1)g}{kT}$. After determining γ, as shown in fig.9.3, we can find Boltzmann constant:

$$k=\frac{\tau(\rho\text{-}\rho^1)g}{\gamma T}. \qquad (9.10)$$

Experimental apparatus description

A simplified scheme of the apparatus is shown in fig.9.4.

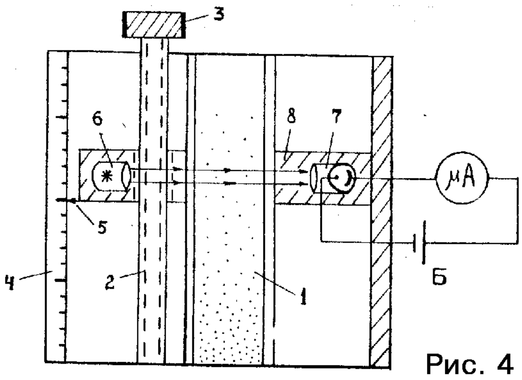


Fig.9.4

1- vessel filled with liquid, that contains small particles inside, 2-micrometer screw, 3 - handle for micro meter screw rotation, 4-scale, 5 - pointer, 6 - light source, 7 - photo detector, 8 - metallic frame, Б - power source.

Light source and photo detector (in order to simplify the scheme, photo detector is shown as a photocell) are fixed on the metallic frame 8. During the rotation of handle 3 for micro meter screw 2 – metallic frame dislocation occurs. During this process we can examine the dependency of photocurrent I on height h. Height h should be measured with the help of scale 4 against pointer 5. At the top of the vessel the particles are nearly absent, which allows to detect I_o, which is proportional to the intensity of light that passed through the layer of pure liquid.

The order of work procedure

1. Turn on and adjust the experimental apparatus. Carry out all of the measurements, indicated in the instructions, which are located on the work table.

2. Write down the results of measurements in table 9.1. Also, write down in the protocol record the temperature of surrounding environment t^o (in ^{o}C) and experimental apparatus parameters: liquid density ρ^1, particles density ρ and particles diameter *d*.

Data analysis

1. Calculate the double natural logarithm of I_o/I. Write down the calculation results in table 9.1. I_o – the biggest value of current I.
2. Using data in from table plot a graph (on a cross-section paper), that depicts the dependency of $\ln\ln I_o/I$ on *h*. In other words, plot the experimental points and, using a ruler, draw a straight line as close as possible to all of the experimental points.
3. Calculate the value of the slope ratio γ of a straight line using formula 9.10 and find the value of Boltzmann constant ($T = t^o + 273$ K, $g = 9{,}8$ m/s^2).
4. Apply the formulas from the supplemental document “correlation analysis” (laboratory work 1-7) to the experimental data obtained in this laboratory work and make conclusions about the validity of Boltzmann distribution.

Questions to answer

1. What is the particle coordinates and (or) velocity distribution function? Indicate the dimension of this distribution function.
2. Write down Boltzmann distribution.
3. Derive the formula for the distribution of particles’ density with height.
4. Derive the law for the decrease of light intensity in an opaque liquid.
5. What is the main point of the liquid density determination method used in this laboratory work?
6. How is Boltzmann distribution verified in this laboratory work?
7. How can Boltzmann constant be determined using experimental data from this laboratory work?
8. Explain, why we should use small particles, the density of which is close to the liquid density in order to verify Boltzmann distribution.
9. Derive the formula for the height *h*, at which particles density decreases by (a) - 50%; (b) - 95%; (c) - 99,9% compared to n_o.In case (b) calculate the mass and size of sand grains in water, considering $\rho=2\rho^1$, needed to obtain the column of weighed particles with the height *h*= 10 cm.

Table 9. 1.

Height h, 10^2m														

I														
I_o/I														
$\ln\ln I_o/I$														

t (°C)=	$\rho_1\left(\frac{kg}{m^3}\right) =$	d(m)=	$I_0 =$	$k_{exp.}$=
T(K)=	$\rho\left(\frac{kg}{m^3}\right) =$	$\tau = (m^3)$	$\gamma(m^{-1}) =$	k_{table}=

Recommended literature

1. PhysicsforScientistsandEngineerswithModernPhysics,eighthedition, 2010Raymond A. Serway, John W. Jewett, Jr., ISBN-13: 978-1-4390-4844-3.
2. Mechanics and Oscillations: University Physics I: Notes and exercises, first edition, 2015, Daniel Gebreselasie, ISBN: 978-87-403-1164-8.
3. Modern Introductory Mechanics, second edition, 2015, Walter Wilcox, ISBN: 978-98-403-0855-6.

Electronic resources: http://zfftt.kpi.ua/en/education/online-library