Pulses' shapes at their scattering on the Dielectric Micro-resonator band-stop filters of the optical communication systems were examined. Most widespread optical pulse envelopes depending on duration, frequency and modulation were studied. The requirements have been formulated for both the pulse parameters, filters and sources of fiber-optic communication systems. Reference 7, figures 7.

Keywords: Gaussian optical pulse, super-Gaussian pulse, rectangular pulse, soliton-like pulse, Lorentzian pulse, exponential pulse, band stop filter, microresonator.

Introduction

It's well known that pulses spreading in the optical communication systems are accompanied by visible changes in their shapes [7]. For the pulse shaping an optical filters with micro-resonators can be used in the infrared wavelength range. Micro-resonator band-stop filters form the coupling with optical transmission lines very well, furthermore have comparatively small dimensions and minor losses of electromagnetic energy [4]. However today their influence on the pulse shapes still has not been explored. The cause of it is the extremely difficult the solutions of Maxwell's equations in the time domain in cases the scattering parameters are calculated for composite shape structures with a large number of boundary surfaces. An approximate solution of this problem can be found from physical justified modeling [5 – 6].

Statement of the problem

The goal of the current article is the investigate the most wide spread types accepted kinds of optical pulses with various envelop shapes scattered by the band-stop filters on ring micro-resonators that can be used in the modern fiber-optic communication systems.

Let's suppose that there is the falling pulse at the input line of band-stop filter consisting of N ring micro-resonators with whispering-gallery oscillations (see fig. 1, a), and has envelop of the electric field $E_{in}(t)$. At the output of the line will be registered, namely a passing pulse with envelop $E_{out}^+(t)$ and also a reflected one with envelop $E_{out}^-(t)$. It's necessary to investigate a passing and reflected pulse envelops depending on the input pulse parameters for a known band-stop filter characteristics.

In order to deal with this problem both S-matrix parameters of band-stop filter and a temporal Green's functions of the band-stop filter were calcu-
lated in present work; also the analytical relationships of the envolos of scattered pulses were found; influence of basic parameters on the structure has been investigated the main demands were formulated and validated for both optical pulse filters and sources in the infrared wave length range.

S-parameters’ matrix calculation

For the purpose of scattering parameters’ calculation of the optical pulses in the time region with help of Green's functions [6], it’s necessary to make an electrodynamic' model of band-stop filter. Let’s examine the simplest band-stop filter consisting of one-dimensional structure of dielectric micro-resonators with whispering-gallery oscillations (fig. 1, a). Up to the present day a scattering model developed in [5] is still used. According to [5], in general case, the filter transmission coefficient \( T \) and the reflection coefficient \( R \) can be found from:

\[
T = T(\omega) = T_0(\omega) + \sum_{s=1}^{N} \frac{A_s^+}{Q_s(\omega)}; \quad (1)
\]

\[
R = R(\omega) = R_0(\omega) + \sum_{s=1}^{N} \frac{A_s^-}{Q_s(\omega)} \quad (2)
\]

Where \( T_0(\omega), \) \( R_0(\omega) \) are define a contribution of non resonance scattering of the normalize waveguide waves, in our case the homogeneous transmission line, without micro-resonators. The second item in \( (1, 2) \) defines a contribution of resonance scattering [3]; here is \( Q_s(\omega) = \omega^2/\omega^2 - 1 - \lambda_s/2) ; \) \( Q^0 = 1/\text{tg} \delta; \) \( \text{tg} \delta \) - is the dielectric loss tangent of the micro-resonator’s material; \( \lambda_s \) - s-th the eigenvalue of the coupling operator \( K \) [3, 5], corresponding to the s-th eigenmode of \( N \) micro-resonator system in the line and with coupling oscillation amplitudes \( b_s = (b_1^s, b_2^s, ..., b_N^s) \), \( s = 1, 2, ..., N \) \[4\]. \( \omega \) - is the current angular frequency; \( \omega_0 \) - is the angular frequency of free oscillation for every insulated micro-resonator in line. All coefficients

\[
A_s^+ = Q^0 \det B_s^+ / \det B; \quad A_s^- = Q^0 \det B_s^- / \det B \quad (3)
\]

depend on amplitudes of each micro-resonator of the system in the transmission line:

\[
B = \begin{bmatrix} b_1^1 & b_2^1 & ... & b_N^1 \\ b_1^2 & b_2^2 & ... & b_N^2 \\ . & . & ... & . \\ b_1^N & b_2^N & ... & b_N^N \end{bmatrix};
\]

\[
B_s^+ = \begin{bmatrix} b_1^s & b_2^s & ... & b_N^s \\ b_1^s & b_2^s & ... & b_N^s \\ . & . & ... & . \\ b_1^N & b_2^N & ... & b_N^N \end{bmatrix}; \quad \text{and}
\]

\[
B_s^- = \begin{bmatrix} b_1^s & b_2^s & ... & b_N^s \\ b_1^s & b_2^s & ... & b_N^s \\ . & . & ... & . \\ b_1^N & b_2^N & ... & b_N^N \end{bmatrix}; \quad (4)
\]

Where

\[
\begin{align*}
K_{sn}^+ = (c_s c_n^*) / (\omega_n w_n) = (\tilde{K}_{sn}) e^{i(z_s - z_n)}; \\
K_{sn}^- = (c_s c_n^*) / (\omega_n w_n) = (\tilde{K}_{sn}) e^{-i(z_s + z_n)};
\end{align*}
\]

\( \Gamma \) - is the longitudinal wave number of the transmission line; \( z_s \) - is the longitudinal coordinate of s-th micro-resonator in the line \( s = 1, 2, ..., N \); \( w_n \) - is the electromagnetic energy, stored in dielectric of s-th micro-resonator; * - is the complex conjugation symbol; \( (\tilde{K}_{sn})_0 \) - is the micro-resonator coupling coefficient for the propagating waves, calculated without phase difference in the line [3]. Every sum in the matrix \( B_s^+ \) (4) is situated in the s-th column.

There is an example of S-matrix frequency response calculation at fig. 1, (b), (c):

\[
|S_{21}| = 20 \log |T|; \quad \text{for the band-stop filter on 15 ring micro-resonators, according to formulas (1 - 4). The frequency of each isolated micro-resonator was supposed to be equal to} \quad f_0 = 200 \text{ THz in the line and the dielectric quality factor of the resonator material was equal} \quad Q^0 = 1/\text{tg} \delta = 5 \times 10^3. \quad \text{It was also supposed that the distance between adjacent micro-resonator centers with whispering-gallery oscillations was equal to} \quad 29 \pi / (2 \Gamma). \quad \text{Let’s assume that the mutual coupling coefficients for non-propagating waves are unequal to zero between adjacent micro-resonators:} \quad k_{ss+1} = 0.01 \text{ as well as the coupling between all nonadjacent micro-resonators are unequal to zero only by propagating waves of the transmission line. All coupling coefficients of the propagating waves are identical and equal:} \quad (\tilde{K}_{sn})_0 = 0.01.
\]

The transmission and reflection coefficient responses from fig. 1 demonstrate that such filter can effectively suppress in frequency band \( \delta f = 5.5 \text{ THz with power level not less than} \quad 40 \text{ dB.} \)
Band-stop Filter Green's functions

Received above relationships (1 - 4) used for temporal Green's functions calculation:

for the passing waves that propagate in the direction of matched termination:

\[
g^+(\tau) = \delta(\tau) + \frac{i\omega_0}{1 + 2Q^2} \sum_{s=1}^{N} A_s^* e^{i\omega_0 \tau}, \tau \geq 0
\]
\[
0, \tau < 0
\]  

(5)

and for the waves reflected in the direction of the source:

\[
g^-(\tau) = \frac{i\omega_0}{1 + 2Q^2} \sum_{s=1}^{N} A_s^* e^{i\omega_0 \tau}, \tau \geq 0
\]
\[
0, \tau < 0
\]  

(6)

Where \(\omega_0\) - is the complex angular frequency of s-th coupling oscillation the micro-resonator system in line. A time-dependence of the solutions of Maxwell's equations is proposed to be \(\exp(i\omega t)\).

Calculation the form of single pulse envelops

The received above relationships (5, 6) are used in order to find general analytic expressions for the pulse shapes scattered on band-stop filter:

1) for the rectangular incident pulse envelops:

\[
E^i(t) = 0(t - t_i) - 0(t - t_2);
\]

\[
E^+_\text{out}(t) = E^i(t) - [E^+\text{out}(t, t_1) - E^+\text{out}(t, t_2)];
\]

\[
E^-\text{out}(t) = -[E^-\text{out}(t, t_1) - E^-\text{out}(t, t_2)],
\]

where is

\[
E^\pm\text{out}(t, t_s) = \frac{\omega_0}{1 + 2Q^2} \delta(t - t_s) \sum_{s=1}^{N} A_s^* e^{i(\omega_0 - \Omega)(t - t_s)} - 1
\]

2) for the Gaussian pulses:

\[
E^i(t) = \frac{1}{\sigma} e^{-\frac{(t - \frac{1}{2})^2}{\sigma^2}}
\]

\[
E^+\text{out}(t) = E^i(t) \cdot 
\left[ 1 - \frac{1}{\sqrt{1 + iC}} \int_2^1 1 + \frac{i\omega_0 \sigma}{\sqrt{2}} \sum_{s=1}^{N} A_s^* e^{i\frac{\omega_0 \sigma}{\sqrt{2}}} \text{erfc}\left[ \frac{z_s(t)}{\sqrt{2}} \right] \right];
\]

\[
E^-\text{out}(t) = \frac{E^i(t)}{\sqrt{1 + iC}} \int_2^1 1 + \frac{i\omega_0 \sigma}{\sqrt{2}} \sum_{s=1}^{N} A_s^* e^{i\frac{\omega_0 \sigma}{\sqrt{2}}} \text{erfc}\left[ \frac{z_s(t)}{\sqrt{2}} \right],
\]

where is

\[
z_s(t) = -\frac{t}{\sigma_c} + i(\Omega - \omega_0)\sigma_c, \quad \sigma_c = \frac{\sigma}{\sqrt{1 + iC}}.
\]

The constant \(C\) defines a chirp as a magnitude of frequency modulation of the pulse carrying [1]; \(\sigma\) - is the half-width of the pulse envelop;

3) for the soliton-like pulses with envelop, that vary by the hyperbolic secant law:

\[
E^i(t) = \text{sech}\left(\frac{t}{\sigma}\right)
\]

\[
E^+\text{out}(t) = \text{E}^i(t) - \frac{i\omega_0 \sigma}{1 + 2Q^2} \int_2^1 \text{e}^{i(\omega_0 - \Omega)\xi} \text{sech}(\xi) d\xi,
\]

\[
E^-\text{out}(t) = \text{E}^i(t) + \frac{i\omega_0 \sigma}{1 + 2Q^2} \int_2^1 \text{e}^{i(\omega_0 - \Omega)\xi} \text{sech}(\xi) d\xi.
\]
Theoretical signals and systems

\[ E_{\text{out}}(t) = -\frac{i\omega \sigma}{1 + 2iQ\sigma} \cdot \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sech}(\xi) d\xi \]

4) for the super-Gaussian pulses:

\[ E_n(t) = e^{-\frac{1}{2\left(\ln^2\alpha\right)} t^2} \]

\[ E_{\text{out}}(t) = E_n(t) - \frac{i\omega \sigma}{1 + 2iQ\sigma} \cdot \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sech}(\xi) d\xi \]

\[ E_{\text{out}}(t) = -\frac{i\omega \sigma}{1 + 2iQ\sigma} \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sech}(\xi) d\xi \]

5) for ultra-short pulses with sinc envelops:

\[ E_n(t) = \sin[N(t / \sigma)] / [N(t / \sigma)] ; \]

\[ E_{\text{out}}(t) = E_n(t) - \frac{i\omega \sigma}{1 + 2iQ\sigma} \cdot \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sinc} (\xi) d\xi \]

\[ E_{\text{out}}(t) = -\frac{i\omega \sigma}{1 + 2iQ\sigma} \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sinc} (\xi) d\xi \]

6) for the exponential pulses:

\[ E_n(t) = e^{\omega_n t} \theta(-t) + e^{-\omega_n t} \theta(t) ; \]

\[ E_{\text{out}}(t) = E_n(t) - \frac{i\omega \sigma}{1 + 2iQ\sigma} \cdot \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sinc} (\xi) d\xi \]

\[ E_{\text{out}}(t) = -\frac{i\omega \sigma}{1 + 2iQ\sigma} \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sinc} (\xi) d\xi \]

7) for the Lorentzian pulses:

\[ E_n(t) = 1/[1 + \left(\frac{t}{\sigma}\right)^2] ; \]

\[ E_{\text{out}}(t) = E_n(t) - \frac{i\omega \sigma}{1 + 2iQ\sigma} \cdot \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sinc} (\xi) d\xi \]

\[ E_{\text{out}}(t) = -\frac{i\omega \sigma}{1 + 2iQ\sigma} \sum_{n=1}^{N} A_n e^{i\left(\omega_n t - \sigma_n t^2\right)} \int_{-\infty}^{t} e^{i\left(\Omega - \omega_n t\right)\tau} \text{sinc} (\xi) d\xi \]

Here \( E_n = E_{\text{out}}(t) \) denotes a temporal dependance of the incident pulse envelopes, and \( E_{\text{out}} = E_{\text{out}}(t) \) is the electric field pulse envelopes passing the filter (sign plus) and reflecting in the opposite direction (sign minus) regarding incident pulse propagation (fig. 1, (a)). Parameter \( m \) stands for front steepness of the super-Gaussian pulse (10), and also their squareness degree [6]; the parameters \( N_i \) and \( \sigma \) denote ultra-short pulse duration (11); \( \Omega \) – is the current circular frequency of the optical pulse. \( \theta(t) \) – is the Heaviside step function; \( \delta(t) \) – is the Dirac delta function; \( \text{erfc}(z) \) – is the Gauss function [2]; \( i = \sqrt{-1} \).

Fig. 3. Result of scattering of the rectangular optical pulse envelops of various durations: (a) - 0,8 ps; (b) - 0,4 ps; (c) - 0,2 ps; (d) - 0,1 ps

The rectangular and super-Gaussian pulse envelops while passing are characterized by visible spikes of power and arise at the moment of the greatest changes of the incident pulse envelopes (fig. 2, (a), (e)). Maximum magnitude of these spikes are proportional to the envelop steepness of incident pulses. Ultra-short pulses (fig. 2, (i) - (n)) are smoothed as a result of their reflections (fig. 2, (j); (l); (n)). According to the following numerical investigations, the decreasing of rectangular and super-Gaussian optical pulses’ durations result in compression and even some peak magnitude increasing (fig. 2, (f); fig. 3, (b)), whereupon the amplitudes decrease (fig. 3, (c) - (d)). Marked phenomenon indicates the possibility of using the band-pass filters for the compression of some pulses.
Fig. 4. Scattered Gaussian pulses ($\sigma = 0.2 \text{ ps}$) of various chirps: (a), (b) - $C = 3$; (c), (d) - $C = 9$

Fig. 4 shows the result of scattering of a Gaussian pulses versus chirp parameter. It obvious that the chirp increasing reduces a reflected pulse width and increases the pulses passing through band-stop filter.

Fig. 5. Rectangular pulse envelops that pass (a); are reflected (b) by filter at various frequencies: 1 - $\Omega = 1.01\omega_0$; 2 - $\Omega = 1.02\omega_0$; 3 - $\Omega = 1.04\omega_0$

While exploring to the sources’ instability it was discovered that it affects pulse scattering. The frequency deviation comparing to central carrier frequency $\omega_0$ substantially deforms as well as passed and reflected pulses (fig. 5). According to that there is possibility of transmitted pulses compression at some "optimal" frequency deviation comparing to the central frequency of the filter (curve 2, fig. 5, (a)).

Hence, according to the conducted calculations for minimal pulses’ changes the indicated frequency deviation would not exceed the value of $0.01\%$ relatively to central frequency of the filter $\omega_0$.

Fig. 6. Mutual influence of rectangular pulses under their reflection from micro-resonator system (Each incident pulse has the duration, is equal $0.2 \text{ ps}$; the temporary interval between adjacent pulse centers is $\delta t = 0.4 \text{ ps}$)

To conclude, the problem of a pulse mutual influence was examined and it appears at consistent scattering in the system of micro-resonators. Such influence is most distinctly of rectangular and also super-Gaussian pulses. Fig. 6 shows a calculation result of envelops for a few scattered rectangular pulses. One can see that spectral component redistribution leads to certain monotonous increases in the maximum reflected amplitudes. The mentioned amplitudes increase or decrease depending on pulses duration or relatively to temporal magnitude between falling pulses’ centers. The Gaussian and soliton-like trains have slight changes under scattering. Thus the bad observable pulses (fig. 7, (a)), or well observable pulses (fig. 7, (b)) remain the same after scattering. These pulse envelops do not change, and their scattering result look as insignificant temporal delay.

Fig. 7. Two Gaussian pulse envelops reflected from micro-resonator filter ($\sigma = 0.2 \text{ ps}$; temporal interval between pulse centers are: (a) - 0.5 ps; (b) - 0.9 ps)

Conclusion

Thus developed analysis enables us to determine a series of demands for pulses, filters and optical sources, used in the communicative systems. Providing of minimal distortions for optical pulses at their scattering on band stop filters requires matching the following conditions:

- The frequency width of a band reflection of the filter must exceed at least $5 - 10$ times comparing to the opposite quantity of minimal duration $\delta t_{\text{min}}$ that is the magnitude of dropped optical pulses.
- Temporary interval between adjacent pulses shouldn’t decrease more than two times of minimally acceptable pulse duration $\delta t_{\text{min}}$. The precise optimal value of this parameter can be determined by band-stop filter’s characteristics.
- The frequency modulation of reflecting pulses can excites the decrease their width.
- Relative instability magnitude of the frequency carrier of the signal sources doesn’t exceed the value of $0.01 \%$. 


The pulses with rectangular envelops can be compressed by the proper selection of optimal parameters of the bandstop filter or by optimal displacement of the carrier frequency. Enumerated maintenance conditions allow asserting acceptable level of entered distortions and therefore the necessary quality of transmission as well as receiving of optical signals at the bandstop filter.

References
Рассеивание инфракрасных оптических импульсов режекторными фильтрами на кольцевых микрорезонаторах

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Приведены результаты исследования огибающих наиболее распространенных оптических импульсов, рассеиваемых режекторными фильтрами, в зависимости от их формы, временной длительности, частоты несущей, модуляции и др.

Рассмотрена возможность сжатия импульсов, а также процессы интерференции, возникающие в результате рассеяния нескольких импульсов. Сформулированы требования к параметрам волоконно-оптических систем связи. Бібл. 7, рис. 7.

Ключевые слова: Гауссовский оптический импульс, супергауссовский импульс, прямоугольный импульс, солитоноподобный импульс, sinc импульс, лоренцевский импульс, экспоненциальный импульс, режекторный фильтр, микрорезонатор.

Литература