MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
"IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE"

METHODOLOGICAL INSTRUCTIONS FOR LABORATORY WORK NO.4(1)

«STUDY OF THE DYNAMICS OF THE SIMPLEST SYSTEMS USING THE ATWOOD'S MACHINE»

Is recommended by the Methodical Council of I. Sikorsky KPI
as tutorial for students of specialty 163 «Biomedical Engineering»,
of specializations «Biomedical Informatics», «Clinical Engineering», «Rehabilitation Engineering»

Kyiv
I. Sikorsky KPI
2020

Methodological instructions for laboratory work No. 4(1): «Study of the dynamics of the simplest systems using the Atwood's machine» [http://ela.kpi.ua]: tutorial for students of specialty 163 «Biomedical Engineering», of specializations «Biomedical Informatics», «Clinical Engineering», «Rehabilitation Engineering» / I. Sikorsky KPI; compilers: N.O. Iakunina, O.G. Danylevych, O.S. Klymuk, I.M. Ivanova. – online text data (1 file: 0,123 Mb). – Kyiv: I. Sikorsky KPI, 2020. – 8 p.

Classification is provided by the Methodical Council of I. Sikorsky KPI
(protocol № 2 of 01.10.2020 y.)
by presentationof Academic Council of the Faculty of physics and mathematics
(protocol № 3 of 31.08.2020 y.)

Online educational publication

METHODOLOGICAL INSTRUCTIONS FOR LABORATORY WORK NO.4(1)

«STUDY OF THE DYNAMICS OF THE SIMPLEST SYSTEMS USING THE ATWOOD'S MACHINE»

Compilers: Iakunina Natalia Oleksandrivna, Ph.D., Assoc. Prof.
Danylevych Oleksandr Gennadiiovych, Ph.D., Assoc. Prof.
Klymuk Olena Serhiivna, Ph.D., Assoc. Prof.
Ivanova Iryna Mykhaylivna, Ph.D., Assoc. Prof.

Responsible editor Linchevskyi I.V., D. Phys. Math., Prof.

Reviewer: Dzhezherya Yu. I., D. Phys. Math., Prof.

The tutorial outlines a methodology for carrying out laboratory work devoted to the study of physical laws that describe the linear motion of a solid. The proposed method is based on an experimental study of the motion of a system of solids with help of the Atwood's machine.

The publication gives a brief description of the theoretical data, which are needed to describe the phenomenon that is being studied in laboratory work. The theoretical calculations that are necessary for the interpretation of the corresponding experimental data and confirmation of the known theoretical regularities are given. Also the step-by-step instruction for conducting experimental measurements, and corresponding calculations of physical quantities is presented.

The tutorial is created for the students of high educational institutions with technical profile.

Laboratory work № 4(1)

Study of the dynamics of the simplest systems using the Atwood's machine

The aim of the work: to investigate the laws of uniformly accelerated motion by analyzing the kinematic characteristics of the system of bodies.

Brief theoretical information.

Let us consider a system which consists of a block with small friction in the axis, through which a thin rope is thrown over. The masses m_1 and m_2 are hung up on each sides of the rope respectively (Figure 4.1). When analyzing the motion of the system, we assume that the rope is weightless and non-rigid, the air resistance is neglected.

In accordance with Newton's second law, the resultant force of the forces that are applied to each of the bodies can be defined as the product of the mass and acceleration of this body: $\Sigma\vec{F} = m\vec{a}$. If we directed the X axis downwards, as shown in Fig. 4.1, then the equations of motion of each mass in the projection on the X axis could be written as:

$$m_1 a_1 = m_1 g - T_1, \qquad (4.1)$$

$$-m_2 a_2 = m_2 g - T_2, \qquad (4.2)$$

where T_1 and T_2 are forces of tension of the rope, a_1 and a_2 – are absolute values of accelerations of corresponding masses. In the second equation, the acceleration has a negative projection on the X axis if the mass of the second body is less than the mass of the first body and the second body moves with acceleration upward.

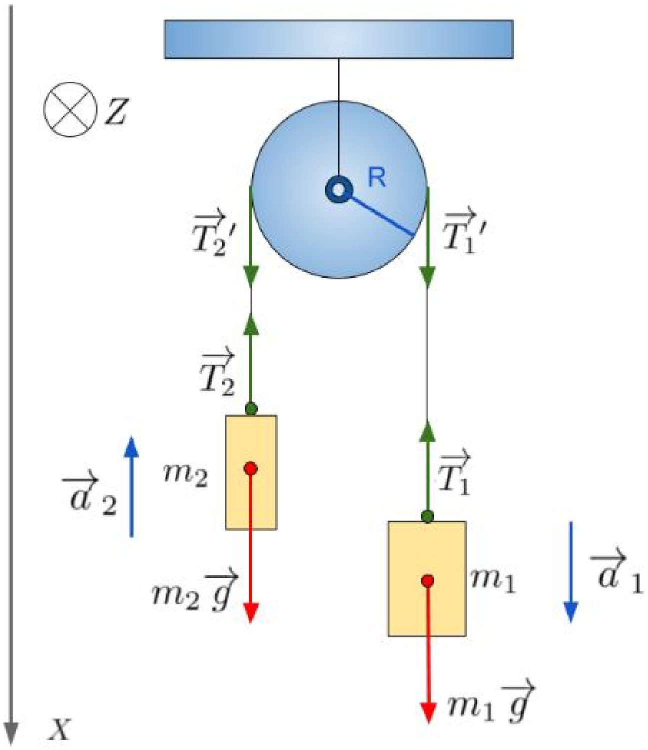

Fig.4.1. General scheme of the Atwood's machine

Due to the fact that the rope is tensile (its length is constant during the movement), the accelerations of both masses are equal in magnitude and are opposite in direction, the equation of kinematic bond can be written as:

$$a_1 = a_2 = a \qquad (4.3)$$

In accordance with Newton's third law, taking into account the weightlessness and stiffness of the rope, we obtain: $\vec{T}_1' = -\vec{T}_1$, $\vec{T}_2' = -\vec{T}_2$, and the absolute values of the tension forces of the rope are equal: $T_1' = T_1$, $T_2' = T_2$. The block is considered to be weightless, so the inertia is not present when it is rotating. In addition, we still neglect the friction in the axis of the block, so the absolute values of the tension forces of the both sides of the rope are identical:

$$T_1 = T_2 = T. \qquad (4.4)$$

We will assume that $m_2 = m,\ m_1 = m + \Delta m$, then, solving the resulting system of equations (4.1) - (4.4), we can obtain the value of acceleration:

$$a = g\frac{m_2 - m_1}{m_2 + m_1} \quad \text{or} \quad a = g\frac{\Delta m}{2m + \Delta m}, \tag{4.5}$$

and the value of the tension force of the rope:

$$T = 2g\frac{m_1 m_2}{m_2 + m_1} \quad \text{or} \quad T = 2gm\frac{m + \Delta m}{2m + \Delta m}. \tag{4.6}$$

Thus, the acceleration of the bodies of this system is always less than the acceleration of free fall and changes with the change of the ratio between the masses of both bodies.

For the chosen system of bodies one can take into account the influence of the mass of the block and the force of friction in its axis. In this case $T_1 \neq T_2$, and the system of equations of motion for moving bodies can be written as:

$$(m + \Delta m)a_1 = (m + \Delta m)g - T_1, \tag{4.7}$$

$$-ma_2 = mg - T_2. \tag{4.8}$$

It must be supplemented with the equation of the rotational motion of the block: the product of the moment of inertia of the body I_Z relative to the fixed axis Z and the angular acceleration β is equal to the total moment of the external forces relative to this axis M_Z:

$$I_Z \beta = M_Z,$$

where $\beta = \frac{d\omega}{dt}$ – the angular acceleration of the body, which is rotating around the axis Z.

Assuming that the moment of force relative to the axis is equal to the product of the absolute value of this force and the smallest distance between the line of force and the axis of rotation (shoulder), and for the forces of tension of the rope the arm is equal to the radius of the block R, we can obtain:

$$I_Z \beta = (T_1 - T_2)R - M_{\text{FR}}, \tag{4.9}$$

where $I_Z = \frac{m_0 R^2}{2}$ – moment of inertia of a cylindrical block with mass m_0 and radius R, M_{FR} – moment of the force of friction in the axis of the block. Moments of the force of friction in the axis and the tension forces T_2 have negative projections on the axis as they prevent the movement. It is worth to note that in case of $I_Z \approx 0$ and $M_{\text{FR}} \approx 0$ we will obtain the equality of forces of tension of the rope (4.4).

Equation of kinematic connection between absolute values of accelerations of bodies and angular acceleration of block β is:

$$a_1 = a_2 = a = a_\tau = \beta R. \tag{4.10}$$

Solving the system of equations (4.7) - (4.10), we obtain the value of acceleration:

$$a = \frac{\Delta mg - M_{FR}/R}{\frac{m_0}{2} + 2m + \Delta m}. \quad (4.11)$$

Obviously, non-zero values of the mass of the block and the force of friction in the axis reduce the acceleration value in comparison with the ideal case (4.5).

Experimental facility.

The Atwood's machine consists of a vertical *rack* attached to the base, on which the *scale* is applied (Fig. 4.2). In the upper part of the rack there is a lightweight *block*, which is rotating with small friction. A light *rope* is thrown through the block, the ends of which are attached to two identical bodies C_1 and C_2. To body C_2 one can add additional cargo in the form of thin plates (overloads), as a result of the inequality of masses the system of bodies begins to move with some acceleration. By changing the mass of additional cargo, you can change the acceleration of the system. After the body C_2 with overload passes some distance L_1, the overload is removed with the help of the bracket G. After that, the loads begin to move uniformly.

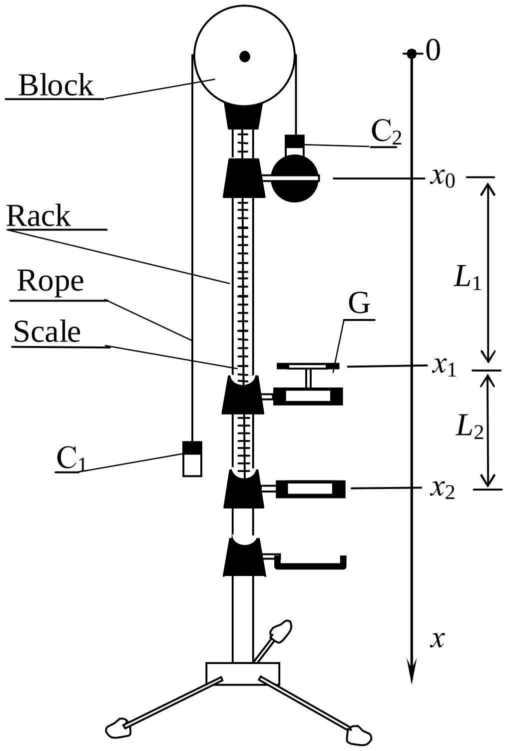

Fig. 4.2. Atwood's machine

Before performing the experiment, it must be ensured that the body C_2 can be lowered freely without touching the bracket G. Otherwise, with the help of the screws that fix the bracket, it is necessary to make the required adjustments.

The method of measurement.

Task 1. Determination of the acceleration of the translational motion.

On the basis of the analysis of the motion of the system of bodies carried out in the theoretical introduction, we can assume that the real motion of bodies on the distance L_1 will be uniformly accelerated. In this case, the law of motion (the time dependence of the coordinates of the body) in the absence of the initial velocity ($v_0 = 0$), will be $x = x_0 + \frac{at^2}{2}$, where x_0 – is the initial coordinate from which the body C_2 begins its movement. In the case of uniformly accelerated motion and $v_0 = 0$, the velocity is changing by law $v = at$, and we can obtain:

$$L_1 = x_1 - x_0 = \frac{v_1^2}{2a}, \quad (4.12)$$

where v_1 – is the velocity of body at the moment of the removing of overload and inclusion of the timer; x_1 – is the coordinate of the upper bracket. If the system does not have forces of friction, then the body C_2 will pass the distance between the photo-sensors with the same speed, and so

$$v_1 = \frac{L_2}{t_2}, \tag{4.13}$$

where $L_2 = x_2 - x_1$ – is the distance between two brackets (x_2 – is the coordinate of the bottom photo sensor); t_2 – is the time of movement on this section of the way.

The measurement procedure.

1. Install the body C_2 to the top position x_0 and put one of the overloads on it. Then set the upper bracket to x_1 so that the distance between the brackets L_2 became 15-20 cm. Do not change the values of L_2 during the whole experiment.
2. Provide the movement to the system of bodies to and determine the time of flight t_2 of body C_2 between the upper and lower brackets. The results of measurements must be entered in the table. 4.1.
3. Change the coordinate of the starting position x_0 of the body C_2. Perform the measurements for five values of L_1 according to p. 2. The results of the measurements must be entered in the table. 4.1.

Table. 4.1

№	L_2, m	L_1, m	t_2, s	v_1, m/s	v_1^2, (m/s)^2
1					
2					
3					
4					
5					

Processing of results.

1. For each value of L_1, determine the velocity v_1 and v_1^2. The results of calculations must be entered in table 4.1.
2. Draw the dependence $v_1^2(L_1)$ by the measurement results, using approximation.
3. Determine the angular coefficient of the straight line $v_1^2(L_1)$, which, in accordance with (4.12), gives the value of the acceleration a. Compare the obtained value with one that is obtained by equation (4.5).

Task 2. The verification of the Newton's second law.

From the equations of motion (4.7), (4.8) taking into account (4.3), (4.4) it follows that:

$$\frac{\Delta m}{2m} = \frac{a}{g - a}. \tag{4.14}$$

In the process of performing this task, an experimental verification of this relation is carried out, which demonstrates the fulfillment of the Newton's second law.

For the experimental determination of acceleration a we can use the relations (4.12) and (4.13), which give:

$$a = \frac{v_1^2}{2L_1} = \frac{L_2^2}{2t_2^2 L_1}. \tag{4.15}$$

The measurement procedure.

1. Install the upper and lower brackets so that the distance L_2 is 15-20 cm.
2. Place one of the available overloads of mass Δm onto the body C_2.
3. Determine 3-5 times the time t_2 of passing of the distance L_2. The results of the measurements must be entered in the table. 4.2.
4. Perform similar measurements, placing other overloads on body C_2.

The results of the measurements must also be included in Table 4.2.

Table. 4.2.

№	m, kg	Δm, kg	t_{2i}, s	a_i, m/s^2	$\langle a \rangle$, m/s^2	A	B
1							
2							
3							
1							
2							
3							
1							
2							
3							

Processing of results.

1. According to experimental data, using (4.15), calculate the acceleration values a_i:

$$a_i = \frac{L_2^2}{2\langle t_{2i}\rangle^2 L_1}$$

and determine the average value of acceleration for each of the three values Δm.
The results of calculations must be entered in Table 4.2.

2. Calculate the values of the left $A = \frac{\Delta m}{2m}$ and right $B = \frac{\langle a\rangle}{(g - \langle a\rangle)}$ parts of the relation (4.14) for each of the experiments. Add to table 4.2 founded values. The experimentally obtained left and right parts of the relation (4.14) for the three different values of Δm should be equal, taking into account the errors of their definition. In this way, the validity of the Newton's second law will be approved.

Control questions.

1. What are inertial and non-inertial reference systems? Formulate Newton's 1st law.
2. What is mass, how can it be measured?
3. What is force, how can it be measured?
4. Formulate Newton's 2nd law.
5. Formulate Newton's 3rd law.
6. Formulate the conditions under which the basic relations of this work are obtained. How these conditions affect the kind of solvable system of equations.
7. Write the system of equations describing the motion of the considered facility without taking into account the mass of the block and the force of friction in the axis.
8. Write the system of equations describing the motion of the considered facility with consideration of the mass of the block and the force of friction in the axis.

Literature.

1. I.V. Saveliev, The course of general physics, vol. 1. M.: Nauka, 1982.
2. D.V. Sivukhin, General course of physics. 1. Mechanics. 3d ed. M.: Nauka, 1989.
3. A.S. Akhmatova, Laboratory practical work on physics. M: Vysshaya shkola, 1980.
4. N.O. Iakunina, O.G. Danylevych, I.O. Yurchenko, T.L. Rebenchuk, V.V. Fedotov, O.S. Klymuk, Methodological instructions for laboratory work No. 5(1): «The rotational motion of a solid body». Kyiv: I. Sikorsky KPI, 2018. http://ela.kpi.ua/handle/123456789/26901
5. N.O. Iakunina, O.G. Danylevych, I.O. Yurchenko, T.L. Rebenchuk, V.V. Fedotov, O.S. Klymuk, Methodological instructions for laboratory work No. 7(1): «Investigation of the rotational motion of a solid body and determination of the velocity of the bullet with the help of a torsion ballistic pendulum». Kyiv: I. Sikorsky KPI, 2019. https://ela.kpi.ua/handle/123456789/30146