

Generalized Bernoulli equation as a source for generating tasks of increased complexity

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Abstract

The technology of creating problems of increased complexity based on generalized ordinary Bernoulli differential equations is considered.

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Problems of increased complexity play a key role in the development of mathematical abilities of students who have a high level of previous training. Such problems stimulate students to creative search, develop their ability to abstract thinking and ingenuity. Successfully solving complex problems increases the student's self-esteem, his confidence in his abilities and motivates him to further study.

When developing such tasks, the teacher must maintain a balance between complexity and accessibility. The tasks should be complex enough to stimulate students' critical thinking, but at the same time understandable for solving. A task that is too complex can demotivate students, and one that is too simple will not produce the desired effect.

Problems of increased complexity are of particular methodological interest when they allow not only varying numerical parameters but also modifying the structure of the problem itself while preserving its core idea. This potential can be fully realized when developing advanced problems in the topic 'Ordinary Differential Equations'.

Scientists are exploring different ways to generalize Bernoulli's ordinary differential equation. Consider the generalization proposed in [Azevedo, Valentino, 2017]:

$$\frac{dy}{dx} + P(x)h(y) = Q(x)g(y), \quad (1)$$

where $P(x)$ and $Q(x)$ are known functions; $h(y)$ and $g(y)$ are known functions from unknown function $y(x)$.

The authors of the aforementioned work demonstrated that given a relationship between known functions:

$$h(y) = g(y) \cdot \int \frac{dy}{g(y)}, \quad (2)$$

equation (1) can be transformed into a linear equation:

$$\frac{du}{dx} + P(x)u = Q(x) \quad (3)$$

by substitution:

$$u = \frac{h(y)}{g(y)}. \quad (4)$$

To create training examples, it is recommended to choose functions such as the function $g(x)$ for which the integral of formula (2) is calculated analytically. For example:

$$(a^2 + y^2), \quad \frac{a^2 + y^2}{y}, \quad \cos^2 y, \quad \frac{1}{\ln y}, \quad \frac{a + \exp(y)}{\exp(y)} \quad etc.$$

The values for the coefficients $P(x)$ and $Q(x)$ should be selected from linear equations found in abundance in various collections of higher mathematics problems. This ensures that we obtain an equation that has an exact solution and can be solved using the aforementioned algorithm.

Example. Solve ODE:

$$2 \cdot \frac{dy}{dx} + \cos x \cdot \sin 2y = \sin 2x \cdot \cos^2 y. \quad (5)$$

Solution process. From equation (5) we find that in this case:

$$P(x) = \cos(x), \quad Q(x) = \frac{1}{2} \sin(2x), \quad h(y) = \frac{1}{2} \sin(2y), \quad g(y) = \cos^2 y.$$

We verify that relation (2) holds:

$$\cos^2 y \cdot \int \frac{dy}{\cos^2 y} = \cos^2 y \cdot \operatorname{tg} y = \frac{1}{2} \sin 2y = h(y).$$

We apply the substitution (4) to the given equation:

$$u = \operatorname{tg} y.$$

and we obtain a linear equation of the form (3):

$$\frac{du}{dx} + \cos x \cdot u = \frac{1}{2} \sin 2x.$$

We solve the last equation in the traditional way and find its general solution:

$$u = \sin x - 1 + C \exp(-\sin x).$$

We return to the desired function $y(x)$ and have the general solution of the given equation:

$$\operatorname{tg} y = \sin x - 1 + C \exp(-\sin x).$$

Note that the transformations omitted from this abstract have a separate educational value for students.

A promising direction for continuing work on the topic of abstract is the creation of an interactive application that facilitates the generation of problems based on the generalized Bernoulli equation. The implementation of this idea together with students who have sufficient programming skills allows for the full realization of interdisciplinary connections and the visual demonstration of the effectiveness of the integration of various sciences.

References

Azevedo D., Valentino, M.C. (2017). Generalization of the Bernoulli ODE. *International Journal of Mathematical Education in Science and Technology*, no. 48(2), pp. 256–260. <https://doi.org/10.1080/0020739X.2016.1201599>