

THEORY OF DIPOLE-EXCHANGE SPIN WAVES IN A FERROMAGNETIC NANOTUBE IN THE PRESENCE OF A THERMOELECTRICALLY INDUCED SPIN-TRANSFER TORQUE

V. V. Kulish^{1,a}, S. I. Yeromin^{1,b}

¹ Faculty of Physics and Mathematics of the National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute»

Abstract

A theoretical study of dipole-exchange spin waves in a conducting ferromagnetic nanotube subjected to a longitudinal temperature gradient is presented. The temperature gradient generates an electric current via the Seebeck effect, which becomes spin-polarized in the ferromagnet and produces a spin-transfer torque acting on the magnetization. The magnetization dynamics is described within the Landau-Lifshitz-Gilbert equation augmented by the Zhang-Li torque terms corresponding to adiabatic and nonadiabatic spin transport. An analytical dispersion relation for spin waves in a thin-walled nanotube with uniaxial anisotropy is derived. It is shown that the thermoelectrically induced spin-polarized current causes a Doppler-like shift of the spin-wave spectrum and modifies the effective damping. A critical temperature gradient at which spin-wave generation starts is obtained.

Keywords: spin waves, ferromagnetic nanotube, thermoelectricity, Seebeck effect, spin-transfer torque, dipole-exchange interaction.

Introduction

Spin waves (magnons) in magnetically ordered media remain a subject of significant theoretical and experimental interest due to their fundamental role in magnetization dynamics and their potential applications in magnonics and spintronics [1]. In nanoscale magnetic systems, the properties of spin waves are strongly affected by the geometry of the sample, boundary conditions, exchange interaction, dipole-dipole interaction, magnetic anisotropy, and dissipative effects.

Among nanoscale magnetic systems, ferromagnetic nanotubes represent a promising class of curvilinear waveguiding structures. Due to their cylindrical geometry, they exhibit specific mode quantization and nonreciprocal propagation effects. In recent years, nanotubes have attracted increasing attention as elements of three-dimensional nanomagnetic architectures [2].

A separate important field of study is thermally driven spin transport [3]. In a conducting ferromagnet subjected to a temperature gradient, an electric current arises due to the Seebeck effect. Since transport in a ferromagnet is spin dependent, this thermoelectric current becomes spin-polarized and can interact with the magnetic subsystem through spin-transfer torque. Thus, thermoelectric transport may provide a mechanism for controlling spin-wave propagation and damping.

In previous works, the influence of externally applied spin-polarized current on spin waves in nanotubes was

studied within the Slonczewski-Berger approach [4]. However, in the present case the current is generated inside the same conducting ferromagnet, so the physically consistent description should be based on the Zhang-Li spin-transfer torque model [5].

The aim of the present work is to develop a theory of dipole-exchange spin waves in a uniaxial ferromagnetic nanotube in the presence of a thermoelectrically induced spin-polarized current generated by an axial temperature gradient.

1. Problem Statement

Let us consider a long ferromagnetic nanotube whose length is much larger than its outer radius, so that the system possesses translational symmetry along the axis. The nanotube is made of a uniaxial ferromagnet of the easy-axis type. The easy axis and the external magnetic field are directed along the nanotube axis Oz . Its internal and external radii are denoted as a and b , correspondingly (see Fig. 1).

Under these conditions, the equilibrium magnetization is $\mathbf{M}_0 = M_0 \mathbf{e}_z$. The nanotube is characterized by exchange interaction constant α , the uniaxial anisotropy parameter β , the gyromagnetic ratio γ , and the Gilbert damping parameter α_G . It is placed in an external homogeneous magnetic field $H_0 \mathbf{e}_z$ directed along the z -axis.

A temperature gradient $\partial T / \partial z$ is applied along the nanotube axis. The nanotube is assumed to be a part of a closed thermoelectric circuit, so the Seebeck effect

^akulish_volv@ukr.net

^bs.yeromin.ua@gmail.com

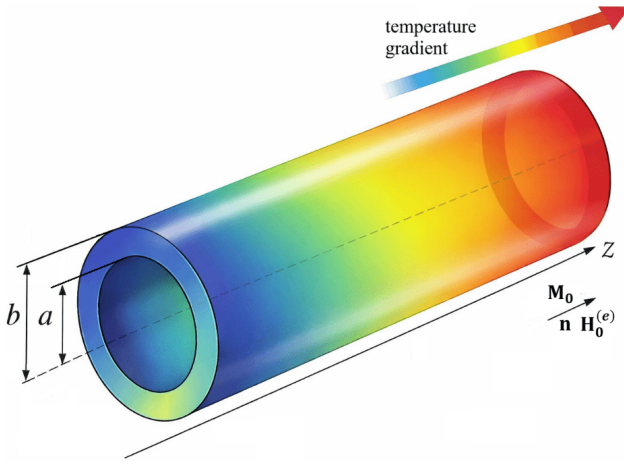


Figure 1. Nanotube investigated in the paper.

generates a stationary electric current.

Within the two-current model, the electric current densities for spin-up and spin-down electrons are

$$j_{\uparrow,\downarrow} = -\sigma_{\uparrow,\downarrow} S_{\uparrow,\downarrow} \frac{\partial T}{\partial z}, \quad (1)$$

where $\sigma_{\uparrow}$, $\sigma_{\downarrow}$ and $S_{\uparrow}$, $S_{\downarrow}$ are the conductivities and Seebeck coefficients for the spin-up and spin-down channels, respectively. The spin-polarized part of the thermoelectric current is then

$$j_s = j_{\uparrow} - j_{\downarrow} = -\kappa_S \frac{\partial T}{\partial z}, \quad (2)$$

where κ_S is the effective thermoelectric spin-polarization coefficient.

2. Basic Relations

The magnetization dynamics is described by the Landau–Lifshitz–Gilbert equation:

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha_G}{M_0} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \mathbf{T}_{\text{STT}}. \quad (3)$$

The effective magnetic field includes exchange, dipolar, anisotropy, and external field contributions. The spin-transfer torque is taken in the Zhang–Li form [5]:

$$\mathbf{T}_{\text{STT}} = -(\mathbf{u} \cdot \nabla) \mathbf{M} + \frac{\xi}{M_0} \mathbf{M} \times (\mathbf{u} \cdot \nabla) \mathbf{M}, \quad (4)$$

where ξ is the nonadiabaticity parameter and $\mathbf{u} = \frac{\mu_B}{eM_0} j_s \mathbf{e}_z$ is the effective drift velocity proportional to the spin-polarized current density.

3. Dispersion Relation

For small deviations from equilibrium, the magnetization $\mathbf{M} = \mathbf{M}_0 + \mathbf{m}$ and the internal field $\mathbf{H}_i = H_{0i} \mathbf{e}_z + \mathbf{h}$ are linearized, with $|\mathbf{m}| \ll M_0$ and $|\mathbf{h}| \ll H_{0i}$. Complementing (3) with the magnetostatic Maxwell equation $\text{div } \mathbf{h} = -4\pi \text{div } \mathbf{m}$, we seek a solution of the form

$$\mathbf{m}, \Phi \sim \exp(i(\omega t - k_{\parallel} z)), \quad (5)$$

where Φ is the scalar magnetostatic potential. After substituting this ansatz and noting that the magnetic potential satisfies a Bessel-type equation, one obtains the following expressions for the real and imaginary parts of the spin-wave frequency:

$$\omega_r = -\kappa k_{\parallel} \frac{\partial T}{\partial z} + \gamma M_0 \sqrt{(\beta_0 + \alpha k^2) \left(\beta_0 + \alpha k^2 + 4\pi \frac{k_{\perp}^2}{k^2} \right)}, \quad (6)$$

$$\omega_i = -\frac{\beta_0 + \alpha k^2 + 2\pi k_{\perp}^2/k^2}{\sqrt{(\beta_0 + \alpha k^2)(\beta_0 + \alpha k^2 + 4\pi k_{\perp}^2/k^2)}} \cdot \left(\alpha_G \omega_r + \kappa \xi k_{\parallel} \frac{\partial T}{\partial z} \right), \quad (7)$$

where $\kappa = P_T \mu_B \sigma S / (e M_0)$, σ is the total electric conductivity, S is the total Seebeck coefficient, $P_T = (\sigma_{\uparrow} S_{\uparrow} - \sigma_{\downarrow} S_{\downarrow}) / (\sigma_{\uparrow} S_{\uparrow} + \sigma_{\downarrow} S_{\downarrow})$ is the thermoelectric spin-polarization parameter, $\beta_0 = \beta + H_{0e} / M_0$, $k = \sqrt{k_{\perp}^2 + k_{\parallel}^2}$ is the total wave number.

Equation (6) shows that the adiabatic current contribution changes only the real part of the frequency, causing a drift-type modification — the current-induced spin-wave Doppler shift [6]. Equation (7) shows that the nonadiabatic contribution changes the effective damping. Depending on the sign of the product $\xi \kappa_S k_{\parallel}$, the thermoelectric current may either increase or decrease the damping.

In the thin-wall approximation $(b - a) \ll a$, the transverse wavenumber is approximately quantized as

$$k_{\perp} (b - a) = \pi n, \quad n = 1, 2, \dots \quad (8)$$

while the fundamental mode $n = 0$ should be treated separately.

In the absence of dissipation and temperature gradient, the resulting dispersion law transforms into the one previously obtained by the authors in [7].

4. Spin-Wave Excitation Condition

The condition for spin-wave amplification or generation is determined by the compensation of intrinsic damping:

$$\alpha_G \gamma M_0 \sqrt{(\beta_0 + \alpha k^2)(\beta_0 + \alpha k^2 + 4\pi k_{\perp}^2/k^2)} - (\alpha_G - \xi) \kappa_S k_{\parallel} = 0. \quad (9)$$

Hence, the critical temperature gradient is

$$\left(\frac{\partial T}{\partial z} \right)_{\text{cr}} = \frac{e \alpha_G \gamma M_0^2}{P_T \mu_B \sigma S (\alpha_G - \xi)} \cdot \frac{\sqrt{(\beta_0 + \alpha k^2)(\beta_0 + \alpha k^2 + 4\pi k_{\perp}^2/k^2)}}{k_{\parallel}}, \quad (10)$$

where the sign of the gradient must be chosen so that the induced current compensates the Gilbert damping. At this threshold the effective damping vanishes and spin-wave generation starts.

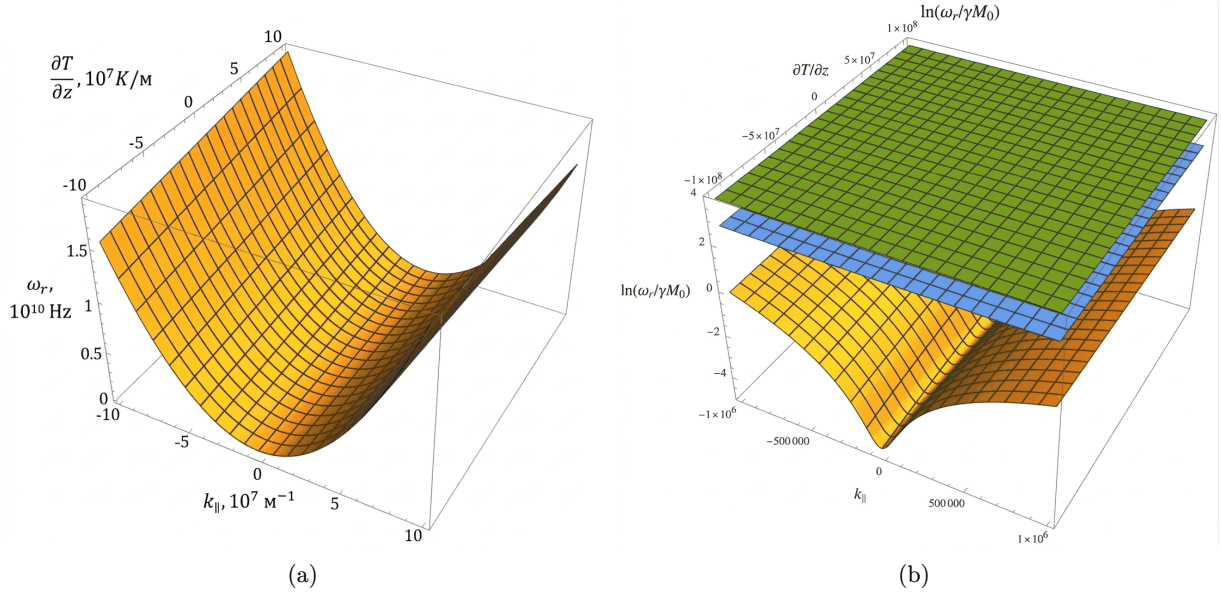


Figure 2. Dependencies of the real part of the frequency ω_r on $k_{||}$ and $\partial T/\partial z$: (a) for the fundamental mode $n = 0$; (b) for the first three modes $n = 0$ (orange surface), $n = 1$ (blue surface), and $n = 2$ (green surface).

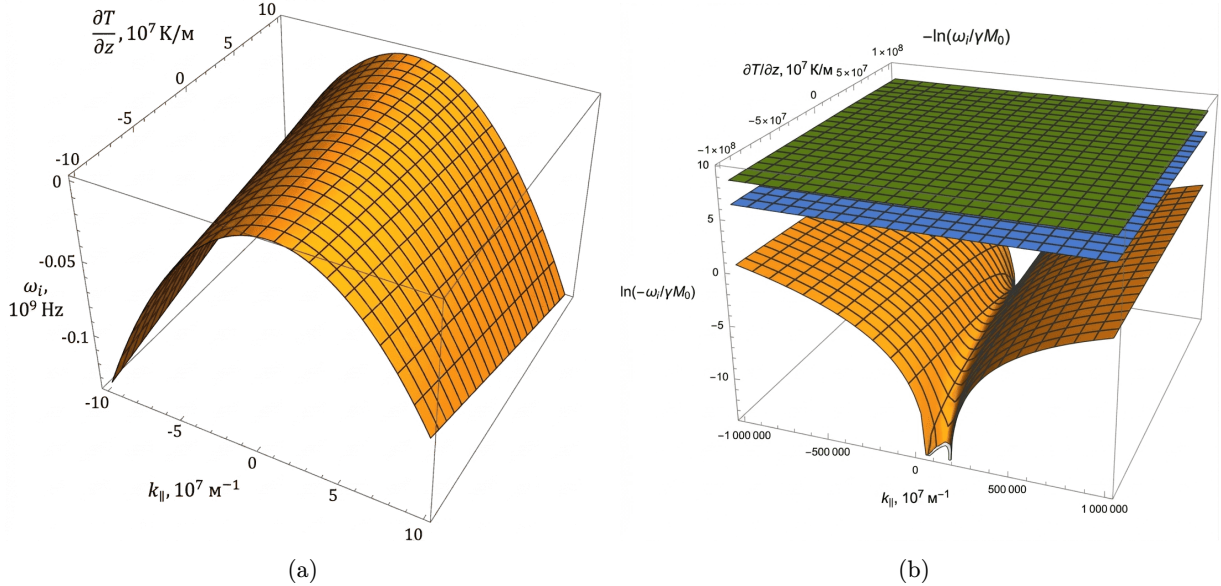


Figure 3. Dependencies of the imaginary part of the frequency ω_i on $k_{||}$ and $\partial T/\partial z$: (a) for the fundamental mode $n = 0$; (b) for the first three modes $n = 0$ (orange surface), $n = 1$ (blue surface), and $n = 2$ (green surface).

5. Results and Discussion

The obtained analytical expressions show that the thermoelectrically induced spin-polarized current affects the spin-wave spectrum in two qualitatively different ways.

First, the adiabatic Zhang–Li term produces a Doppler-like shift of the real part of the frequency:

$$\Delta\omega_r = -\kappa k_{||} \frac{\partial T}{\partial z}. \quad (11)$$

Thus, the symmetry between waves propagating in opposite directions is broken: $\omega(k_{||}) \neq \omega(-k_{||})$. Therefore, the temperature gradient induces nonreciprocity of spin-wave propagation.

Second, the nonadiabatic term modifies the imagi-

nary part of the frequency. Depending on the sign of the temperature gradient, the effective damping may increase or decrease. After passing the threshold (10), the temperature gradient causes spin-wave generation.

Numerical estimates for a permalloy nanotube show that the effect is strongest for the fundamental mode. For $\text{Ni}_{80}\text{Fe}_{20}$ with nanotube thickness $d = 10$ nm, the frequency of the fundamental mode lies in the GHz range, whereas for higher transverse modes it reaches the THz region.

Dependencies of both the real and imaginary parts of the frequency (see Figs. 2, 3) have similar shapes for different transverse modes but on essentially different scales (within two orders of magnitude for the three modes shown). There are pronounced dependencies

on $\partial T/\partial z$ everywhere except in the region where $k_{\parallel}$ is close to zero, which shows that the thermoelectrically induced spin-polarized current can noticeably modify both the real and imaginary parts of the spin-wave frequency.

The transverse mode structure remains mainly determined by geometry, while the temperature gradient governs propagation and attenuation along the nanotube axis. Thus, the thermoelectric current and geometrical quantization can be treated as two relatively independent mechanisms of spin-wave spectrum formation.

Conclusions

A theory of dipole-exchange spin waves in a conducting ferromagnetic nanotube in the presence of a thermoelectrically induced spin-polarized current has been developed.

It has been shown that the Seebeck-induced current affects spin-wave dynamics through the Zhang-Li spin-transfer torque. The adiabatic contribution causes a Doppler-like frequency shift and induces nonreciprocity of propagation. The nonadiabatic contribution modifies effective damping and may lead to spin-wave amplification and generation.

The critical temperature gradient required for the onset of spin-wave generation has been obtained (10).

The results demonstrate that thermoelectric transport can serve as an efficient tool for controlling spin-wave propagation, damping, and excitation in cylindrical magnetic nanostructures.

References

1. The 2024 magnonics roadmap / B. Flebus, D. Grundler, B. Rana, Y. Otani, I. Barsukov, A. Barman, [et al.] // *Journal of Physics: Condensed Matter*. — 2024. — Vol. 36, no. 36. — P. 363501. — DOI: [10.1088/1361-648X/ad399c](https://doi.org/10.1088/1361-648X/ad399c).
2. Ye Y., Geng B. Magnetic nanotubes: Synthesis, properties, and applications // *Critical Reviews in Solid State and Materials Sciences*. — 2012. — Vol. 37, no. 2. — P. 75–93. — DOI: [10.1080/10408436.2011.613491](https://doi.org/10.1080/10408436.2011.613491).
3. Bauer G. E. W., Saitoh E., Wees B. J. van. Spin caloritronics // *Nature Materials*. — 2012. — Vol. 11, no. 5. — P. 391–399. — DOI: [10.1038/nmat3301](https://doi.org/10.1038/nmat3301).
4. Kulish V. V. Spin waves in a ferromagnetic nanotube: Account of dissipation and spin-polarized current // *Ukrainian Journal of Physics*. — 2016. — Vol. 61, no. 1. — P. 59–65. — DOI: [10.15407/ujpe61.01.0059](https://doi.org/10.15407/ujpe61.01.0059).
5. Zhang S., Li Z. Roles of nonequilibrium conduction electrons on the magnetization dynamics of ferromagnets // *Physical Review Letters*. — 2004. — Vol. 93, no. 12. — P. 127204. — DOI: [10.1103/PhysRevLett.93.127204](https://doi.org/10.1103/PhysRevLett.93.127204).
6. Vlaminck V., Bailleul M. Current-induced spin-wave Doppler shift // *Science*. — 2008. — Vol. 322, no. 5900. — P. 410–413. — DOI: [10.1126/science.1162843](https://doi.org/10.1126/science.1162843).
7. Gorobets Y. I., Kulish V. V. Dipole-exchange spin waves in a ferromagnetic nanotube // *Ukrainian Journal of Physics*. — 2014. — Vol. 59, no. 5. — P. 541–548. — DOI: [10.15407/ujpe59.05.0541](https://doi.org/10.15407/ujpe59.05.0541).