

ON THE THEORY OF REPEATED RECORDS

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Let $\{X_n\}$ be a sequence of independent identically distributed random variables. We say that n is a record time if

$$X_n > \max\{X_1, \dots, X_{n-1}\}.$$

We assume that the common distribution function of random variables is continuous. It is known that the number of record times in $\{X_n\}$ is infinite with probability one.

Another random variable of interest is the so called repeated record. Namely, n is said to be a repeated record time if both n and $n - 1$ are record times. In contrast to the the case of records, the number of repeated record times is finite with probability one.

We say that a sequence of independent random variables $\{X_n\}$ is an F^α -scheme if, for aome sequence of positive real numbers $\{\alpha_n\}$, the distribution function of X_n is F^{α_n} , where F is a given distribution function.

Below are two typical results discussed in the talk.

Theorem 1. *Let $\{X_n\}$ be an F^α scheme. If*

$$\sum_{n=1}^{\infty} \frac{\alpha_n}{\alpha_1 + \dots + \alpha_n} \cdot \frac{\alpha_{n+1}}{\alpha_1 + \dots + \alpha_{n+1}} = \infty, \quad (1)$$

then the probability of infinite number of repeated numbers in $\{X_n\}$ equals one.

Denote by $\mu_n^{(2)}$ the number of repeated records in $\{X_1, \dots, X_n\}$. The first of moments of $\mu_n^{(2)}$ are given by

$$\begin{aligned} \mathbb{E} \left[\mu_n^{(2)} \right] &= \sum_{k=1}^{n-1} \frac{\alpha_k}{\alpha_1 + \dots + \alpha_k} \cdot \frac{\alpha_{k+1}}{\alpha_1 + \dots + \alpha_{k+1}}, \\ \text{var} \left[\mu_n^{(2)} \right] &= \sum_{k=1}^{n-1} \left(\frac{\alpha_k \alpha_{k+1}}{A_k A_{k+1}} - \frac{\alpha_k^2 \alpha_{k+1}^2}{A_k^2 A_{k+1}^2} \right). \end{aligned}$$

Note that $E[\mu_n^{(2)}] \rightarrow \infty$ does not imply $\text{var}[\mu_n^{(2)}] \rightarrow \infty$ in general while the converse is true. However, $\text{var}[\mu_n^{(2)}] \rightarrow \infty$ follows, for example, from

$$\limsup_{n \rightarrow \infty} \frac{\alpha_n}{\alpha_1 + \cdots + \alpha_n} < 1.$$

Theorem 2. *If condition (1) holds, then*

$$\lim_{n \rightarrow \infty} \frac{\mu_n^{(2)} - E[\mu_n^{(2)}]}{\text{var}[\mu_n^{(2)}]} = 0$$

almost surely.

The talk is based on the papers [1] and [2].

REFERENCES

- [1] P. Doukhan, O. I. Klesov, and J. Steinebach *Strong laws of large numbers in an F^α -scheme* // in book “Mathematical Statistics and Limit Theorems. Festschrift in Honour of Paul Deheuvels” — Springer — 2015. — P. 287–303.
- [2] K.-H. Indlekofer, O. I. Klesov, J. G. Steinebach *Asymptotic behavior of consecutive records in a F^α scheme.* // in book “Stochastic and analytic structures with applications” — Springer — 2026. — P. 198–220.

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