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THEORY, ALGORITHM AND CONDITION FOR AGGREGATING ECONOMIC BENEFIT AND HEALTH DAMAGES OF COAL FUEL COMBUSTION

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The purpose of the research is to test a possibility of using the theory of utility function in economics theory [1] for aggregating different kinds of variables such as economic benefit and ecological damages of industrial activities. An example of coal fuel combustion for electricity generation [2, 3] is selected for this test, which produces both economic benefit and human health damages. Several mathematical models for the utility function are tested with the data of the volume of combustions and the amount of air pollutions of twenty seven Oblasts of Ukraine by the regression analysis [4]. Consistent results are obtained upon the theory and the statistical data analysis. It is concluded that the prices take important role to give the weighting factor through the aggregation process of various indicators (independent variables) because in this theory the prices make up the total budget, which gives the constraints for maximizing the utility with given values such as number and volume of the indicators.

INTRODUCTION

Utility function is a theory to indicate the level of wellness of human and/or society, X_i, where $i = 1,2,...,n,$ such as foods, cloth, and utility such as electricity, gas, water, and resources. Human and/or society wishes higher level of utility, $U(X_1, X_2, X_3,......X_n)$, but the constraints are given by the total budget, I, together with the prices P_{x_i} for having different kinds of wellness X_i respectively, where

$$I = \sum_{i=1}^{n} P_{x_i} X_i \,. \tag{1}$$

Under this constraint, the condition for obtaining the maximum utility is to be found, using the Lagrangean Multiplier Technique, as shown bellow.

At first, the Lagrangean is defined as the follow.

$$L = U(X_1, X_2, X_3,....X_n) + \lambda(I - \sum_{i=1}^{n} P_{x_i} X_i)\,. \tag{2}$$

Here, λ is an unknown variable, which is called “Lagrangean multiplier”.

The first order condition to get the maximum utility, $U(X_1, X_2, X_3, ..., X_n)$, is partial derivatives of L by each of $X_1, X_2, X_3, ..., X_n$ and λ are equal to zero, i.e.

$$\partial L / \partial X_i = \partial U / \partial X_i - \lambda P_{x_i} = 0 .$$

$$\partial L / \partial \lambda = I - \sum_{i=1}^{n} P_{x_i} X_i = 0 . \tag{4}$$

For example, by dividing i-th equation by $(i+1)$-th equation of the above $(1)-(4)$, we get the following:

$$\frac{\partial U / \partial X_i}{\partial U / \partial X_j} = \frac{P_{X_i}}{P_{X_j}}, \tag{5}$$

where $i \neq j$.

The above equation (5) means that the marginal rate of substitution (the ratio of these two partial derivatives of utility function by X_i and X_j) should be equal to the ratio of the prices of these X_i and X_j in order to get the maximum utility [1]. In other words, although people wish to possess the higher/bigger utility, the maximum utility is always constrained by the budget and the prices, and the maximum utility is obtained only where and/or when the marginal rate of substitution, $\frac{\partial U / \partial X_i}{\partial U / \partial X_j}$ and the ratio of the corresponding two prices, $\frac{P_{X_i}}{P_{X_j}}$ i.e. the slope of the budget line, are equal. This point is the equilibrium to give people the maximum utility, which is given under the budget constraint. In other words, the utility is at the maximum, and there is enough amount of budget. Therefore, it is expected that if the economy of a region stays for a considerably long time-period, it is reasonable for analysts to think that the utility of the market is at the maximum equilibrium.

METHOD

The mathematical model of the utility function needs to be found. At first, the following three models are assumed, and then empirical analysis is made for testing the fitting of each model to the statistical data:

- Linear model:

$$U = \sum_{i=1}^{n} C_i X_i . \tag{6}$$

- Non-linear model (Cobb-Douglas function [1]):

$$U = \prod_{i=1}^{n} X_i^{C_i} . \tag{7}$$

- Logarithmic model:

$$U = \sum_{i=1}^{n} C_i \mathrm{Log}\, X_i , \tag{8}$$

where

$$\sum_{i=1}^{n} C_i = 1. \tag{9}$$

Here, C_i is a weighting factor to combine various wellness, X_i, to make up a utility U, but it can be also translated as preference or probability to make the weights of different options of the wellness or resources.

In order to make the statistical test, the variable included in the equations $(6)-(8)$ are not enough, but each of these models needs to be transformed to the linear equations, with the Lagrangean multiplier technique as shown bellow, with which each wellness, X_i, can be mathematically indicated as the function of the total budget, I, and the prices of various wellness, $P_{x_1}, P_{x_s}, P_{x_3}, ..., P_{x_n}$, which are available in the actual statistical database. Then, the linear regression analysis can be carried out for the statistical tests.

For the linear model, $U = \sum_{i=1}^{n} C_i X_i$, the Lagrangean is:

$$L = \sum_{i=1}^{n} C_i X_i + \lambda (I - \sum P_{X_i} X_i). \tag{10}$$

Given the budget constraint, the first order condition for maximizing the utility, $\sum_{i=1}^{n} C_i X_i$ is that the partial derivatives of L by each of $X_1, X_2, X_3, ..., X_n$ and λ are equal to zero, i.e.

$$\partial L \big/ \partial X_i = C_i - \lambda P_{X_i} = 0, \tag{11}$$

$$\partial L \big/ \partial \lambda = I - \sum_{i=1}^{n} P_{x_i} X_i = 0, \tag{12}$$

where $i = 1, 2, ..., n$.

From (11)

$$P_{X_i} = \frac{C_i}{\lambda}. \tag{13}$$

From (12)

$$I = \sum_{i=1}^{n} P_{X_i} X_i . \tag{14}$$

Then, replace P_{X_j} of (14) by (13) to get:

$$I = P_{X_i} X_i + \sum_{i=1}^{n-1} \frac{C_j}{\lambda} X_j , \tag{15}$$

where $i \neq j$.

From (13)

$$\frac{1}{\lambda} = \frac{P_{X_i}}{C_i}. \tag{16}$$

Then, replace $\frac{1}{\lambda}$ of (15) by (16) to get:

$$X_i = \frac{I}{P_{X_i}} - \sum_{j=1}^{n-1} \frac{C_j}{C_i} X_j. \tag{17}$$

With the same procedure to get (17) from (6), for the non-linear model, $U = \prod_{i=1}^{n} X_i^{C_i}$ and for the logarithmic model, $U = \sum_{i=1}^{n} C_i \operatorname{Log} X_i$, the following equation is obtained for both of these two models:

$$X_i = \frac{I}{P_{X_i}} \frac{C_i}{\sum_{j=1}^{n} C_j}. \tag{18}$$

The next step is to test which model statistically fits in the actual data, upon (17) and (18).

RESULTS

The economic benefit and the health damages of coal fuel combustion are included together in the models of utility function, and they are empirically analyzed together with the data taken from the National Statistics of Ukraine for 27 Oblasts (Provinces) in 2010 and 2011. From this database, the volume of coal combustion (tons/year) is taken as the surrogate of the economic benefit from the activity of coal combustion, and the emission volumes (tons/year) of Nitrogen oxides and Sulfur compounds from stationary sources, as well as the greenhouse gas emission volume, are selected as the surrogates for health damages of the coal combustion. The prices of these indicators (variables) are set as follows:

- The price of coal combustion: 100 US dollars per ton as the price of coal per ton [5].
- The price of health damage by the emission of Nitrogen oxides and Sulfur compounds are calculated by multiplying the price of one person's life who is dying by the air pollution [2] by the calculated number of long-term mortalities from the nitrate and the sulfate respectively [2] and then divided by the volume of the emission of the Nitrogen oxides and Sulfur compounds respectively from the reference power station, the Tripylska Power Station [2]. Those values are shown in Table 1.
- The price of the greenhouse gas emission, 22 US dollars/ton (CO_2 equivalent) is taken from the average price of the carbon tax of France, 25 US dollars/ton, Ireland, 20 US dollars/ton, and Norway, 21 US dollars/ton [6].

T a b l e 1 . Values used for calculating the price of health damages from the air pollutions

<table>
<tr><th colspan="2" rowspan="2">Value of life, V_L</th><th>Value</th><th>Description</th><th>Reference</th></tr>
<tr><td>18000
US dollars</td><td>The value of one person's death within his or her life time after one year exposure to the air pollutions</td><td>[2]
page 27</td></tr>
<tr><td rowspan="2">Number of the deaths, N_D</td><td>by Nitro gen oxides emission</td><td>992
persons/year</td><td rowspan="2">Calculated number of the long-term mortalities (deaths) in all territory of Ukraine from the emission for one year at the Tripylska Power Station.</td><td rowspan="2">[2]
Page 28
Table 7</td></tr>
<tr><td>by Sulfur compounds emission</td><td>2504 per-sons/year</td></tr>
<tr><td rowspan="2">Emission volume, V_E</td><td>Nitrogen oxides</td><td>11108
tons/year</td><td rowspan="2">Emission from Tripylska Power Station for one year</td><td rowspan="2">[2]
Page 25
Table 3</td></tr>
<tr><td>Sulfur compounds</td><td>40909
tons/year</td></tr>
<tr><td rowspan="2">Price of health dam-age, P_H</td><td>by Nitro gen oxides emission</td><td>4097
US dollars/ton</td><td colspan="2" rowspan="2">Calculated by:
$P_H = V_H N_D / V_E$</td></tr>
<tr><td>by Sulfur compounds emission</td><td>436
US dollars/ton</td></tr>
</table>

The descriptive statistics of the variables selected for this statistical test are shown in Table 2, the correlations between these selected variables are shown in Table 3, and the results of the statistical tests are shown in Table 4 for the linear model, the non-linear model, and the logarithmic model. In Table 2, the total budget is calculated by the equation (1), given that the statistical data is considered at the equilibrium that makes up the maximum utility.

T a b l e 2 . Descriptive statistics of the variables for coal combustion and the health damages

Parameter	**Budget (US$)**	**Coal combustion (tons)**	**Sulfur compounds emission (tons)**	**Nitrogen oxides emission (tons)**	**Greenhouse gas emission (CO_2 equivalent tons)**
Mean	482149,6	2559,024	46,5441	11,9628	7,287157
Median	94856,79	158,2500	4,0085	4,0875	2,928500
Maximum	5917803,	30150,90	381,4940	94,5760	61,47900
Minimum	4590,271	9,5000	0,1610	0,3000	0,400000
Std. Dev.	1014658.	6082,066	86,4365	19,4452	11,75112
Skewness	3,6936	3,6329	2,5260	2,7623	3,618079
Kurtosis	18,0194	16,3432	9,2654	10,5239	16,75077
Obs.	*54*	*54*	*54*	*54*	*54*

Table 3. Correlations between the variables

Parameter	Budget	Coal combustion	Nitrogen oxides emission	Sulfur compounds emission	Green-house gas emission
Budget	1				
Coal combustion	0,9520	1			
Nitrogen oxides emission	0,9452	0,9536	1		
Sulfur compounds emission	0,9201	0,9532	0,9379	1	
Greenhouse gas emission	0,9299	0,9718	0,9346	0,9138	1

Table 4. Statistical test on the coal combustion and health damages by the air pollutions

Model	Dependent Variable	Independent Variable	Coefficient	T-Statistics	R^2	AIC	Schwartz
Linear model, 1	Coal com-bustion	Interception	–966,0035	–5,5818	0,9744	16,7648	16,9490
		Budget/price	0,1018	2,1769			
		Nitrogen emission	20,2686	0,7418			
		Sulfur emission	20,3609	4,1017			
		Greenhouse gas emission	253,0772	6,8217			
Linear model, 2	Nitrogen oxides emission	Interception	2,1496	1,9381	0,9323	6,2464	6,4305
		Budget/price	0,0275	2,8428			
		Coal combustion	0,0006	0,7418			
		Sulfur emission	0,0656	2,3111			
		Greenhouse gas emission	0,2913	1,0949			
Linear model, 3	Sulfur compounds emission	Interception	10,5520	1,9965	0,9219	9,3734	9,5576
		Budget/price	0,0006	0,1102			
		Coal combustion	0,0126	4,1017			
		Nitrogen emission	1,4972	2,3111			
		Greenhouse gas emission	–2,0155	–1,6078			
Linear model, 4	Green-house gas emission	Interception	2,4176	4,8033	0,9479	4,9782	5,1623
		Budget/price	5,51E–06	0,1857			
		Nitrogen emission	0,0820	1,0949			
		Sulfur emission	–0,0249	–1,6078			
		Coal combustion	0,0019	6,8217			
Non-linear/log model, 1	Coal com-bustion	Interception	–192,2655	–0,6776	0,9062	17,9524	18,0260
		Budget/price	0,5706	22,4198			
Non-linear/log model, 2	Nitrogen oxides emission	Interception	3,2290	3,3380	0,8934	6,5897	6,6634
		Budget/price	0,07421	20,8767			
Non-linear/log model, 3	Sulfur compounds emission	Interception	8,7513	1,6968	0,8466	9,9371	10,0108
		Budget/price	0,0341	16,9434			
Non-linear/log model, 4	Green-house gas emission	Interception	2,0946	3,1806	0,8647	5,8208	5,8944
		Budget/price	0,0002	18,2312			

The values of R^2 show that both the linear model and the non-linear model including the logarithmic model well fit in the given database, while the values of R^2 of the linear models indicate better fitting than the non-linear/logarithmic models'. However, the signs, i.e., + and –, of the coefficients of the linear models don't represent signs of the coefficients of the equation (17). To remove this problem and to give the negative signs to the coefficients of the linear model, the following operations are made:

Upon this result, it is observed that the order of magnitude of the prices differ from 22 US dollars/ton to 4097 US dollars, and the volumes (tons) of combustion and the emissions differ from 7 tons/year to 2559 tons/year. From this observation, it is assumed that the order of magnitude of one variable or one price should not be so different from each other. In order to check this assumption, two sub-systems of the dataset are created, i.e., the coal combustion volume, the nitrogen oxides emission as one set, and the coal combustion volume, the sulfur compounds emission, and the greenhouse gas emission as another one set. The descriptive statistics of the variables of each group is shown in Table 5, the correlations of the set of the variables in each group are shown in Table 6 and Table 7, and the results of the regression analysis are shown in Table 8 for the coal combustion and nitrogen oxides emission, and in Table 9 for the coal combustion, sulfur compounds emission and greenhouse gas combustion.

Table 5. Descriptive statistics of the variables for coal combustion and the health damages

Parameter	Total Budget for coal combustion and nitrogen oxides emission (US$)	Total Budget for coal combustion, sulfur compounds and greenhouse gas emission (US$)
Mean	304913,8	276356,0
Median	58034,36	20271,43
Maximum	3389568	3180226
Minimum	2998,500	1264,000
Std. Dev.	684600,9	644479,6
Skewness	3,5428	3,5756
Kurtosis	15,7033	15,9646
Obs.	54	54

Table 6. Correlations between budget, coal combustion and nitrogen oxides emission

Parameter	Budget	Coal combustion	Nitrogen oxides emission
Budget	1		
Coal combustion	0,9994	1	
Nitrogen oxides emission	0,9636	0,9536	1

Table 7. Correlations between budget, coal combustion, sulfur compounds emission and greenhouse gas emission

Variable	Budget	Coal combustion	Sulfur compounds emission	Greenhouse gas emission
Budget	1			
Coal combustion	0,9998	1		
Sulfur compounds emission	0,9584	0,9532	1	
Greenhouse gas emission	0,9710	0,9718	0,9138	1

Table 8. Statistical test on the coal combustion and nitrogen emission

Model	Dependent Variable	Independent-Variable	Coefficient	T-Statistics	R^2	AIC	Schwartz
Linear model	Coal combustion	Interception	0,0002	2,0042	1,0000	–11,6920	–11,5815
		Budget/price	1,0000	19564952			
		Nitrogen emission	– 40,9701	– 2276781			
Non-linear model	Coal combustion	Interception	– 148,2003	– 4,6194	0,9988	13,6157	13,6894
		Budget/price	0,8879	205,7971			
Linear model	Nitrogen emission	Interception	5,90E–06	2,0042	1,0000	–19,1177	–19,0072
		Budget/price	1,0000	2562759			
		Coal combustion	– 0,0244	– 2276781			
Non-linear model	Nitrogen emission	Interception	3,6173	4,6195	0,9285	6,1900	6,2637
		Budget/price	0,1121	25,9916			

Table 9. Statistical test on the coal combustion, sulfur compounds emission and greenhouse gas emission

Model	Dependent Variable	Independent Variable	Coefficient	T-Statistics	R^2	AIC	Schwartz
Linear model	Coal combustion	Interception	5,51E– 06	0,0715	1,0000	–12,9160	–12,7686
		Budget/price	1,0000	20896952			
		Sulfur emission	– 4,3600	–2074635			
		Greenhouse gas emission	– 0,2200	–11923,23			
Non-linear model	Coal combustion	Interception	– 48,5881	–3,0200	0,9997	12,2477	12,3214
		Budget/price	0,9436	408,0275			
Linear model	Sulfur emission	Interception	1,26E-06	0,0715	1,0000	–15,8609	–15,7136
		Budget/price	1,0000	2231241			
		Coal combustion	– 0,2294	–2074635			
		Greenhouse gas emission	– 0,0505	–11939,86			
Non-linear model	Sulfur emission	Interception	11,0232	2,983100	0,9184	9,3056	9,3792
		Budget/price	0,0560	24,1998			
Linear model	Greenhouse gas emission	Interception	2,59E-05	0,074005	1,0000	–9,8875	–9,7402
		Budget/price	1,0001	11928,81			
		Coal combustion	– 4,5459	–11923,23			
		Sulfur emission	–19,8203	–11939,86			
Non-linear model	Greenhouse gas emission	Interception	2,3946	5,688897	0,9427	4,9609	5,0345
		Budget/price	0,0004	29,26192			

As a result after dividing the database into two groups, according to the size of the values of the prices, the results of the regression analysis upon each of two groups in Table 8 and Table 9 show good fitting of the linear model, because all the coefficients of terms for the budget/price are close to 1,0 and the sign of each coefficient from the second term is negative, which satisfy the form of equation (17), while the values of T-statistics of the coefficients show enough statistical significance. The values of R^2, Akaike Information Criterion, and Schwarz Criterion also show statistical well-fitting of the model to each of two databases.

For the non-linear and the logarithmic models, the sum of the calculated coefficients of "budget/price" over the different variables are close to 1,0 in both groups in Table 8 and Table 9, and this result is consistent to the equation (18). The Akaike Information Criterion and Shwartz Criterion don't show statistical well-fitting.

The next step is to estimate the weighting factors, as shown in the equation (17) and (18) as the coefficient C_i, where $i = 1,2,...,n.$ For this purpose, the statistically obtained values for the coefficient of the equation (17) and (18) are used. The coefficients of the linear model are obtained as shown bellow.

When

$$\frac{C_j}{C_i} = \alpha_{ij}, \tag{19}$$

where α_{ij} is the observed value of the coefficient that is obtained by the linear regression analysis, as shown in Table 8 and Table 9.

From (17) and (19),

$$X_i = \frac{I}{P_{X_i}} - \sum_{j=1}^{n-1} \alpha_{ij} X_j, \tag{20}$$

where

$$\frac{\sum_{j=1}^{n-1} C_j}{C_i} = \sum_{j=1}^{n} \alpha_{ij}. \tag{21}$$

From (9)

$$\sum_{i=1}^{n} C_i = C_i + \sum_{j=1}^{n-1} C_j = 1. \tag{22}$$

Then, from (21) and (22),

$$\frac{1 - C_i}{C_i} = \sum_{j=1}^{n-1} \alpha_{ij}, \tag{23}$$

$$1 - C_i = C_i \sum_{j=1}^{n-1} \alpha_{ij}, \tag{24}$$

$$C_i \left(\sum_{j=1}^{n-1} \alpha_{ij} + 1 \right) = 1. \tag{25}$$

Therefore

$$C_i = \frac{1}{1+\sum_{j=1}^{n-1}\alpha_{ij}}. \tag{26}$$

With the equation (26), the following utility functions, (27) from Table 8 and (28) from Table 9, are obtained:

$$U = 0{,}0238X_{\text{coal}} + 0{,}9762X_{\text{nitrogen}}, \tag{27}$$

$$U = 0{,}1792X_{\text{coal}} + 0{,}7814X_{\text{sulfur}} + 0{,}0394X_{\text{greenhouse}}. \tag{28}$$

For non-liner and logarithmic models, the observed coefficient β_i of "budget/price" in Table 8 and Table 9 is given in the equation (29). As observed in Table 8 and Table 9, each sum of the observed coefficients is close to 1,0, and this result indicates the equation (30). With (29) and (30), the observed coefficient β_i is equal to the normalized coefficient C_i of the equation (7) and (8):

$$\sum_{i=1}^{n}\frac{C_i}{\sum_{j=1}^{n}C_j} = 1, \tag{29}$$

$$\frac{C_i}{\sum_{j=1}^{n}C_j} = \beta_i. \tag{30}$$

Thus, the following utility functions are obtained, (31) from Table 8 and (32) from Table 9:

$$U = X_{\text{coal}}^{0{,}8879} X_{\text{nitrogen}}^{0{,}1121}, \tag{31}$$

$$U = X_{\text{coal}}^{0{,}9436} X_{\text{sulfur}}^{0{,}05601} X_{\text{greenhouse}}^{0{,}0004}. \tag{32}$$

For the logarithmic model, the following functions are obtained, (33) from Table 8 and (34) from Table 9.

$$U = 0{,}8879\log X_{\text{coal}} + 0{,}1121\log X_{\text{nitrogen}}, \tag{33}$$

$$U = 0{,}9436\log X_{\text{coal}} + 0{,}05601\log X_{\text{sulfur}} + 0{,}0004\log X_{\text{greenhouse}}. \tag{34}$$

CONCLUSIONS AND RECOMMENDATIONS

It has been demonstrated that the theory of utility function can be used for aggregating various indicators, including both the economic benefits and health damages. For this process, the prices take important role to give the weighting factors for aggregating various indicators (independent variables) because the prices make up the total budget, which gives the constraints for maximizing the utility with given values such as number and volume of the indicators in this theory of utility function.

When the orders of magnitudes of the selected indicators or the values of the prices of those indicators are close to each other, for example within the order of 100, both linear and non-linear/logarithmic models can explain the weighting and/or preference of the various indicators in the database of coal combustion and health damages in 27 Oblasts (Provinces) of Ukraine. Therefore, for constructing the larger system with more number of the indicators (variables), it is necessary to select the indicators, which have the closer orders of magnitudes in the prices.

Further research and analysis are needed for more variety of indicators and for larger system, which includes the indicators of social, ecological and economic impacts of the industrial activities.

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