

DETERMINATION OF MATERIAL CONSTANTS OF MAGNETOSTRICTIVE MATERIALS

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Abstract. A mathematical model of a toroidal inductor with a flat ring core made of a polycrystalline magnetostrictive ferromagnet capable of oscillations was developed, as well as a method of measuring the material constants of a polycrystalline ferromagnetic core based on the resonance frequencies of radial oscillations of a ring placed in a toroidal coil. Material constants are understood as a set of the following quantities: components of the tensor of the modulus of elasticity of a demagnetized ferromagnet, components of the tensor of magnetostrictive constants, and components of the tensor of the magnetic permeability of a ferromagnet.

Keywords: *magnetostriction, magnetomechanical resonance, material constants*

Introduction

When magnetized, ferromagnets (FM) change their size and shape due to magnetostriction (MS). On the other hand, during mechanical deformation of pre-magnetized FMs, their magnetization also changes - this is the so-called Villari effect, or the reverse MS effect. Experimentally, MS is investigated using known methods of direct measurement of small displacements, namely: strainometric, interferometric, capacitive and other methods. These methods are characterized by insufficient accuracy and sometimes sensitivity, as well as some limitation of the frequency range. Magnetization of FMs under various types of their deformation is usually performed by observing FM hysteresis loops. These methods are time-consuming and cannot give accurate results. But the majority works are of an applied nature and are focused on the engineering calculation of specific types of MC converters and the construction of their equivalent electrical circuits. Thus, the question of determining the material constants of FM (MS) of materials requires the search for simple, but at the same time sufficiently accurate methods.

Mathematical model of an inductor with a ferromagnetic core that oscillates radially

The design of the studied model of the inductance coil L_k is shown on Figure 1.

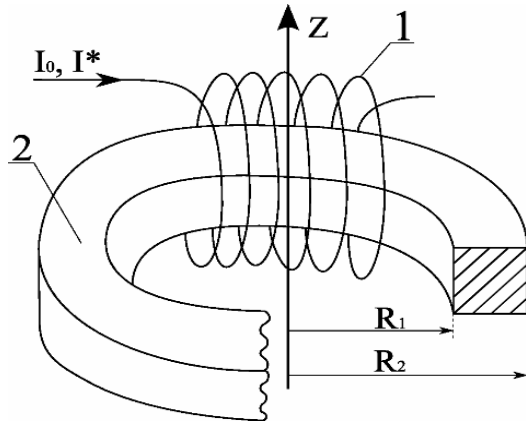


Figure 1. The design of the inductor L_k .

Coil 1 contains N turns of wire. Core 2 is made in the form of a toroid of polycrystalline FM (MS) material and is located in the middle of the sleeve (the sleeve is not shown), touching it at several points. Core dimensions: R_1 , R_2 - inner and outer radii, h - thickness.

A feature of the coil design is that the core is not compressed by the turns of the coil, so as not to prevent the occurrence of elastic mechanical oscillations in the core.

A direct current I^0 is supplied to the coil under study, which creates a circular permanent magnetization field with a voltage in the volume of the core $H_g^0 = NI^0/R_0$, g - circular coordinate (polar angle) of the cylindrical coordinate system (ρ, g, z) , the beginning of which is connected to the center of the coil core L_k . $R_0 = (R_1 + R_2)/2$ - the middle radius of the core. At the same time, an alternating current is supplied to the coil I^* .

From the general definition of inductance, for the case of a winding uniformly applied to the ring core [9], we obtain the calculation formula:

$$L_K = \frac{2\pi}{(I^*)^2} \int_{R_1-h/2}^{R_2} \int_{-h/2}^{h/2} \rho B_g^* \cdot H_g^* d\rho dz, \quad (1)$$

where ρ - the current value of the radial coordinate of the cylindrical coordinate system (ρ, g, z) , the beginning of which is connected to the center of the coil core, I^* , \vec{B}^* i \vec{H}^* - amplitudes of electric current, magnetic induction and magnetic field strength, which change harmonically in time.

For the occasion $I^0 \gg I^*$, which is equivalent to the condition $|\vec{H}^0| \gg |\vec{H}^*|$, magnetic induction is defined as follows [1]:

$$B_g^* = m_{rsnm} H_r^0 \varepsilon_{nm}^* + \mu_{sl}^\varepsilon H_l^*, \quad (2)$$

where B_g^* - vector component of the resulting magnetic induction, m_{pqij} - component of the tensor of magnetostrictive constants, ε_{kl}^* - component of the strain tensor, μ_{sl}^ε -

component of the magnetic permeability tensor, in the mode of constancy of deformation. Tensor components ε_{kl}^* and tension vector H_q^* are interconnected by Hooke's law for elastic media with complicated properties [2], which in a linear approximation takes the form:

$$\sigma_{ij}^* = c_{ijkl}^H \varepsilon_{kl}^* - m_{pqij} H_p^0 H_q^* \quad (3)$$

where σ_{ij}^* – the amplitude value of the tensor component of the resulting mechanical stresses, c_{ijkl}^H – component of the tensor of the modulus of elasticity of the demagnetized FM.

Within the framework of the solved problem, the components of the intensity vectors of the constant and alternating magnetic fields are determined from the full current law, and the elastic deformations are determined from Newton's second law in the differential form:

$$\sigma_{ij,j}^* + \rho_0 \omega^2 u_i^* = 0 \forall x_k \in V, \quad (4)$$

where ρ_0 – FM density; u_i^* – amplitude of the i -th component of the displacement vector material particles of FM, which changes harmonically in time; a comma between indices denotes the operation of differentiating the expression written before the comma by the coordinate whose index is placed after the comma.

The uniqueness of the solution of the system of differential equations (4) is ensured by the boundary conditions:

$$n_j \sigma_{ij}^* = 0 \forall x_k \in S \quad (5)$$

where n_j – component of the external normal to the surface S , which limits the volume V of the core. Since all physical fields in the ring core have axial symmetry, the boundary value problem (4) – (5) in the cylindrical coordinate system is written as follows:

$$\sigma_{\rho\rho,\rho}^* + \sigma_{\rho z,z}^* + (\sigma_{\rho\rho}^* - \sigma_{\theta\theta}^*)/\rho + \rho_0 \omega^2 u_\rho^* = 0 \forall (\rho, z) \in V, \quad (6)$$

$$\sigma_{z\rho,\rho}^* + \sigma_{zz,z}^* + \sigma_{z\rho}^*/\rho + \rho_0 \omega^2 u_z^* = 0 \forall (\rho, z) \in V, \quad (7)$$

$$\sigma_{\rho\rho}^* \Big|_{\rho=R_1, R_2} = 0, \quad \sigma_{\rho z}^* \Big|_{\rho=R_1, R_2} = 0, \quad (8)$$

$$\sigma_{z\rho}^* \Big|_{z=\pm h/2} = 0, \quad \sigma_{zz}^* \Big|_{z=\pm h/2} = 0. \quad (9)$$

Resulting mechanical stresses σ_{ij}^* we determine from the equation of state (3):

$$\sigma_{\rho\rho}^* = c_{11}^H \varepsilon_{\rho\rho}^* + c_{12}^H \varepsilon_{\theta\theta}^* + c_{13}^H \varepsilon_{zz}^* - m_{211}^0 H_\theta^*, \quad \sigma_{\theta\theta}^* = c_{21}^H \varepsilon_{\rho\rho}^* + c_{22}^H \varepsilon_{\theta\theta}^* + c_{23}^H \varepsilon_{zz}^* - m_{222}^0 H_\theta^*,$$

$$\sigma_{zz}^* = c_{31}^H \varepsilon_{\rho\rho}^* + c_{32}^H \varepsilon_{g,g}^* + c_{33}^H \varepsilon_{zz}^* - m_{233}^0 H_g^*, \quad \sigma_{\rho z}^* = c_{55}^H \varepsilon_{\rho z}^*, \quad (10)$$

where $m_{kij}^0 = m_{pkij} H_p^0$ - piezomagnetic constants.

In the demagnetized state, the polycrystalline ferromagnet is isotropic in elastic and magnetostrictive properties, i.e. material constants and are components of isotropic tensors of the fourth rank. Since the isotropic tensor of the fourth rank is completely determined by two constants [3], the regular notations of the following form:

$$c_{ijkl}^H = \lambda \delta_{ij} \delta_{kl} + G(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad m_{pqij} = m_2 \delta_{pq} \delta_{ij} + \frac{m_1 - m_2}{2} (\delta_{pi} \delta_{qj} + \delta_{pj} \delta_{qi}), \quad (11)$$

where λ i G – Lamé constants (G – shear modulus); $\delta_{i,j}$ – the Kronecker symbol; m_1 and m_2 – constants determined experimentally.

From the definitions of material constants (11) it follows that $c_{11}^H = c_{22}^H = c_{33}^H = \lambda + 2G$, $c_{12}^H = c_{13}^H = c_{21}^H = c_{23}^H = \lambda$, $c_{55}^H = G$, a $m_{21} = m_{23} = m_2$ and $m_{22} = m_1$. Voigt's matrix indices were used when recording the modulus of elasticity and tensor components of magnetostrictive constants, i.e: $c_{\beta\lambda}^H \Leftrightarrow c_{ijkl}^H$ и $m_{\beta\lambda} \Leftrightarrow m_{pqij}$. Ratio (2) takes the following form:

$$B_g^* = m_{211}^0 \varepsilon_{\rho\rho} + m_{222}^0 \varepsilon_{g,g} + m_{233}^0 \varepsilon_{zz} + \mu_2^e H_g^*, \quad (12)$$

where $m_{211}^0 = m_{233}^0 = m_2 H_g^0$, $m_{222}^0 = m_1 H_g^0$.

Consider the case of a flat core, that is, when a strong inequality holds $h/R_0 \ll 1$, and the width of the heart $d = R_2 - R_1$ satisfies a weak inequality $d/R_0 < 1$. In this case $\sigma_{zz} = \sigma_{\rho z} = 0$, whence it follows that $u_z^* \approx 0$, and deformation ε_{zz}^* is determined from relation (10) as follows:

$$\varepsilon_{zz}^* = -\frac{\lambda}{\lambda + 2G} (\varepsilon_{\rho\rho}^* + \varepsilon_{g,g}^*) + \frac{m_{22}^0}{\lambda + 2G} H_g^*, \quad (13)$$

where $\varepsilon_{\rho\rho}^* = \frac{\partial u_\rho^*}{\partial \rho}$, $\varepsilon_{g,g}^* = \frac{u_\rho^*}{\rho}$. The circular component of the vector H_g^* vector of the intensity of the variable magnetic field will be determined by the radius R_0 of the middle line of the heart, i.e $H_g^* = \frac{NI^*}{2\pi R_0}$. At the same time, normal stresses $\sigma_{\rho\rho}^*$ i $\sigma_{g,g}^*$ are determined by ratios:

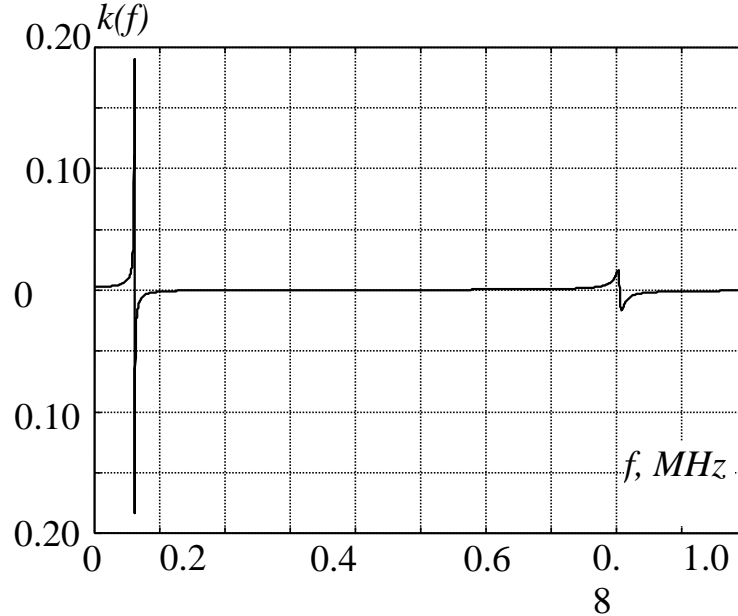
$$\begin{aligned} \sigma_{\rho\rho}^* &= \frac{4G(\lambda + G)}{\lambda + 2G} \left[\frac{\partial u_\rho^*}{\partial \rho} + \nu \frac{u_\rho^*}{\rho} - \frac{m_{21}^0 H_g^*}{2(\lambda + G)} \right], \\ \sigma_{g,g}^* &= \frac{4G(\lambda + G)}{\lambda + 2G} \left[\nu \frac{\partial u_\rho^*}{\partial \rho} + \frac{u_\rho^*}{\rho} + \frac{m_{21}^0 \lambda - m_{22}^0 (\lambda + 2G)}{4G(\lambda + G)} H_g^* \right], \end{aligned} \quad (14)$$

where $\nu = \lambda / [2(\lambda + G)]$ – Poisson's ratio of demagnetized FM.

To take into account the finite Q factor of the oscillating system and to simplify mathematical calculations, an effective shift modulus was used $G_c = G \left(1 + \frac{i}{Q} \right)$, where

$i = \sqrt{-1}$. This leads to the fact that all functions that depend on the wavenumber γ are complex.

Figure 2 shows the graph of changes in the real part of the dynamic coupling coefficient $k(f)$, which was calculated for the following set of geometrical and physicomachanical parameters of the coil core: $R_1 = 10,5 \text{ mm}$; $R_2 = 13,5 \text{ mm}$;



$\rho_0 = 7,4 \cdot 10^3 \text{ kg} / \text{m}^3$; $G = 60 \text{ GPa}$; $\nu = 0,3$; $m_{21} = 250 \text{ T}$; $m_{22} = 500 \text{ T}$; $\mu_2^e = 350 \mu_0$;
 $\mu_0 = 4\pi \cdot 10^{-7} \text{ G} / \text{m}$; $Q = 150$.

Figure 2. Dependence of the real part of the dynamic coupling coefficient $k(f)$ from the frequency f .

Calculation of the coefficient $k(f)$ started with frequency $f = 1 \text{ Hz}$. It should be emphasized that at $f = 0 \text{ Hz}$, that is, in the static solution, the radial displacement $U_p(0)$ and, accordingly, the coupling coefficient $k(0)$ has relatively small but non-zero numerical values. This is quite natural, because when added to a permanent magnetization field H_g^0 of a small static field with tension H_g^* there is an inevitable change in the physical state of the heart.

Practical use of ratios for calculating the material constants of the FM core

The input values are the dimensions of the core, its density and the number of turns. Two frequencies of magnetomechanical resonances are experimentally measured f_1 i f_2 , for $k(f) > 0$ and two frequencies of magnetomechanical antiresonances f_1^a and f_2^a , for $k(f) < 0$. At an intermediate frequency $f_1 < f_0 < f_2$ inductance is measured $L_k = L_0$, as a result of which the magnetic permeability is determined μ_2^* . By relation f_2/f_1 using a table of function roots D, which is prepared in advance, the numerical value of Poisson's ratio is determined ν and the corresponding value of the first root:

$x_1 = \gamma_1 R_0 = \frac{2\pi f_1 R_0}{V}$, де $V = \sqrt{\frac{2G}{\rho_0(1-\nu)}}$. By size x_1 the shear modulus is determined G and modulus of elasticity $\lambda = \frac{2G\nu}{(1-\nu)}$.

At antiresonance frequencies f_1^a і f_2^a (at $Q \rightarrow \infty$) equalities must be fulfilled $k(f_1^a) = -1$, $k(f_2^a) = -1$, where. Piezomagnetic constants are determined from these equalities m_{21}^0 і m_{22}^0 . Based on the found piezomagnetic constants, the corresponding radial field magnetizations are determined H_g^0 constants $m_1 = \frac{m_{22}^0}{H_g^0}$ and $m_2 = \frac{m_{21}^0}{H_g^0}$, which will completely determine the components of the fourth rank tensor m_{pqij} magnetostrictive constants.

According to the known piezomagnetic constant m_{21}^0 , modulus of elasticity λ and G and magnetic permeability μ_2^* magnetic permeability is calculated $\mu_2^\varepsilon = \mu_2^* - \frac{(m_{21}^0)^2}{\lambda + 2G}$, which is not determined in a physical experiment, but is used when writing the equations of the physical state [4,75 and is called the magnetic permeability in the regime of invariance (equal to zero) of the deformations of the ferromagnet.

Thus, according to the five quantities measured in the experiment, namely frequencies f_1 , f_2 , f_1^a , f_2^a and inductance L_0 , five material constants are determined by calculation, namely: Poisson's ratio ν , shear modulus G , magnetostrictive constants m_1 and m_2 , magnetic permeability μ_2^ε .

The results of a theoretical study of the dependence of the inductance of the coil on the frequency of the core, which is capable of mechanical oscillations, are given. It is shown that the frequencies of magnetomechanical resonances are determined both by the shape and dimensions of the oscillating core and by the material constants of the substance from which this core is made. An algorithm for determining Poisson's ratio, shear modulus, magnetostrictive constants, and magnetic permeability of the core material is formulated. The developed mathematical model and the proposed algorithm for determining the material constants can become the basis for further experimental studies of magnetostrictive materials.

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