

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE  
NATIONAL TECHNICAL UNIVERSITY OF UKRAINE  
“IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE”

Faculty of Electric Power Engineering and Automatics

**Yuliia Peretyatko**  
**Liudmyla Spinul**  
**Maksym Shcherba**

# **THEORETICAL FUNDAMENTALS OF ELECTRICAL ENGINEERING: DC Circuits**

## **Practice and Solutions**

*Recommended by the Methodological Board  
of Igor Sikorsky Kyiv Polytechnic Institute  
as a tutorial for students doing Bachelor's degree programmes in  
speciality 141 “Electric Power Engineering, Electrical Engineering, and  
Electromechanics”*

Kyiv  
Igor Sikorsky Kyiv Polytechnic Institute  
2022

Reviewer                      *Mykola Pushkar* Cand. Sc., Assoc. Prof.  
Editors                        *Galyna Mikhnenko*, Ph.D in Ed., Assoc.Prof.  
                                      *Anatolii Shcherba*, Doc. Sc., Prof.

*Recommended by the Methodological Board  
of Igor Sikorsky Kyiv Polytechnic Institute (Minutes No 3 of 27.01.2022 p.)  
Approved by the Supervisory Board of the Faculty of Electric Power Engineering and Automatics  
(Minutes No 3 of 01.11.2021 p.)*

Study e-book

*Yuliia Peretyatko*, Cand. Sc., Assoc. Prof.  
*Liudmyla Spinul*, Cand. Sc., Assoc. Prof.  
*Maksym Shcherba*, Doct. Sc., Assoc. Prof.

# **THEORETICAL FUNDAMENTALS of ELECTRICAL ENGINEERING: DC circuits**

## **Practice and Solutions**

Theoretical fundamentals of electrical engineering: DC circuits. Practice and Solutions [Electronic resource]: tutorial for students doing Bachelor's degree programmes in speciality "141 Electric Power Engineering, Electrical Engineering, and Electromechanics" / Yuliia Peretyatko, Liudmyla Spinul, Maksym Shcherba; Igor Sikorsky Kyiv Polytechnic Institute. – Electronic text data (1 file: 11.8 MB). – Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2022. – 81 p.

The tutorial "Theoretical fundamentals of electrical engineering: DC circuits. Practice and Solutions" is intended for undergraduate students of speciality 141 "Electric Power Engineering, Electrical Engineering, and Electromechanics".

This educational publication deals with one section of theoretical fundamentals of electrical engineering – single phase DC circuits. It includes a brief theoretical overview and provides some examples to make the explanation simple and clear.

Being written in English, the tutorial can be useful for both international and Ukrainian students.

© Yuliia Peretyatko, Liudmyla Spinul, Maksym Shcherba 2022  
© Igor Sikorsky Kyiv Polytechnic Institute, 2022

## **CONTENTS**

|                                                                           |    |
|---------------------------------------------------------------------------|----|
| Introduction                                                              | 4  |
| 1. Fundamentals of Linear Electrical DC Circuits                          | 5  |
| 2. Connections of Resistive Elements in Electrical Circuits               | 10 |
| 3. The Direct Application of Ohm's Law and Kirchhoff's Laws               | 14 |
| 4. The Mesh Method of Circuit calculation (Maxwell's Method)              | 35 |
| 5. Node-Analysis Method                                                   | 43 |
| 6. The Method of Equivalent Transformations and Star-Delta Transformation | 50 |
| 7. The Superposition Method                                               | 59 |
| 8. Thevenin's Theorem and Equivalent Generator Method                     | 68 |
| References                                                                | 81 |

## INTRODUCTION

The study of disciplines that can be combined under the concept of *Theoretical Fundamentals of Electrical Engineering* requires continuous independent work on educational literature, solving problems, and individual tasks. Thus, satisfactory mastering the discipline is impossible without solving problems.

The main purpose of this tutorial is to help the students who study Theoretical Fundamentals of Electrical Engineering at universities at the time when the teaching hours are decreasing and distance learning is of great importance.

Here we deal with one section of theoretical fundamentals of electrical engineering – single phase DC circuits, and split the material into eight chapters, each of which includes a set of the solution of typical problems. Some typical calculations are complex and time-consuming tasks, each of which consists of a number of subtasks or several independent tasks. The step-by-step process of doing standard calculations is shown to develop creative skills of students in the comparative analysis of the results, which is typical for engineering practice.

The tutorial takes into account the intensive development of modern computer technology: all calculations and graphing are adapted to the use of the computer mathematical system MathCAD.

# 1. FUNDAMENTALS OF LINEAR ELECTRICAL DC CIRCUITS

## *Basic definitions of electrical dc circuits*

When any person begins to explore the world of electricity, it is vital to start with understanding the basics of voltage, current and resistance.

At first, these concepts can be difficult to understand because we cannot “see” them. We must use measurement tools such as multimeters, spectrum analyzers, and oscilloscopes to visualize what is happening with the charge in a system or a device.

Secondly, we will study how each of the circuit elements behaves individually and how their interaction is governed by circuit laws when they are interconnected.

An **electrical network** is a set of devices connected together and intended for transmission, distribution and mutual transformation of electric and other kinds of energy if the processes proceeding in devices can be described by means of concepts of electromotive force, current and voltage.

An **electrical circuit** is a graphic representation of the network providing one or more closed loops for one or more currents to flow.

The network topology terms are as follows:

- a **branch** is formed of one or several circuit elements connected in series;
- a **node** is the point of connection between three or more branches; a node is represented by a dot sign;
- a **loop** is any closed path in a circuit. A loop is said to be independent if it contains at least one branch which is not a part of any other independent loop;
- a **mesh** is a loop that has no other loops inside.

**Voltage** is the potential difference between two points:  $U = \varphi_1 - \varphi_2$

**Current** is the rate of movement of electrical charges. The electric current is the time rate of charge passing through a specified area  $i = \frac{dq}{dt}$ . If the current flows

in one direction and does not change its value over time, then it is called **Direct Current (DC)**.

### ***The elements of a DC electrical circuit***

A **resistor** is a passive two-terminal electrical element in which an irreversible conversion of electric energy into heat energy takes place when the current flows through it.

A resistor implements electrical resistance as a circuit element. ***The resistor helps to control/ reduce the flow of current in a circuit.***

Resistive elements for which the volt-ampere characteristic is a straight line are called *linear*.

Resistance is the parameter of the linear resistive element which equals the ratio of voltage across its terminals to the current passing through it:

$$R = \frac{U}{I}$$

An **ideal source of electromotive force** is a source of electric energy which provides the unchanged value of the potential difference across its output terminals.

It is generally accepted that the internal resistance of an ideal source of electromotive force is equal to zero, namely the output voltage is independent of the current across the source.

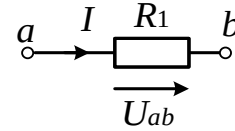
An **ideal source of current** is such an electric energy source that provides output current with constant value. Its internal resistance is equal to infinity; therefore, the output voltage is dependent on the load resistance value.

## Basic laws for electrical DC circuits

**Ohm's Law** for a linear circuit establishes the relation between the voltage across the terminals of a branch or one element and the current flowing through all the elements of this branch or through one element.

$$U_{ab} = \varphi_a - \varphi_b = IR_1$$

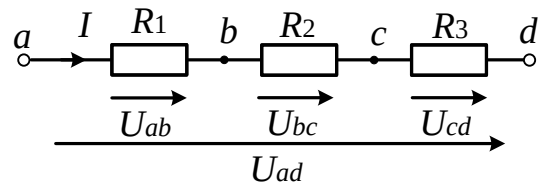
$$I = \frac{\varphi_a - \varphi_b}{R_1} = \frac{U_{ab}}{R_1}$$



$$U_{ad} = \varphi_a - \varphi_d = \\ = U_{ab} + U_{bc} + U_{cd} =$$

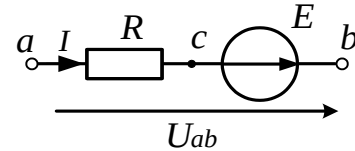
$$= IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

$$I = \frac{U_{ad}}{R_1 + R_2 + R_3} = \frac{\varphi_a - \varphi_d}{R_1 + R_2 + R_3}$$



$$U_{ab} = \varphi_a - \varphi_b = IR - E \Rightarrow$$

$$I = \frac{\varphi_a - \varphi_b + E}{R} = \frac{U_{ab} + E}{R}$$



**Kirchhoff's Current Law (KCL) for DC circuits.** KCL is the First Kirchhoff's Law. It states that arithmetical sum of the currents entering a node is exactly equal to the arithmetical sum of the currents leaving the same node:

$$\sum_{k=1}^n I_k = 0$$

**Kirchhoff's Voltage Law (KVL) for DC circuits.** KVL is the Second Kirchhoff's Law. It states that the algebraic sum of the voltages across each element individually within any closed loop is equal to zero:

$$\sum_{k=1}^m U_k = 0$$

An alternative form of Kirchhoff's Voltage Law states that the algebraic sum of the voltages across resistors within the closed loop equals the algebraic sum of the EMFs in the same closed loop:

$$\sum_{k=1}^n I_k R_k = \sum_{k=1}^m E_k$$

### Combinations in electrical circuits.

**Series connection** is such a connection of elements in the branch of electrical circuit between two nodes when the same current flows through all the elements.

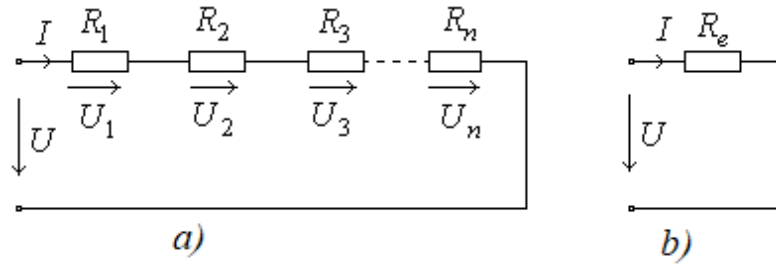


Fig. 1.1

Applying Kirchhoff's Voltage law, for the circuit in Fig. 1.1, *a* , we obtain

$$U = U_1 + U_2 + U_3 + \dots + U_n .$$

After applying Ohm's law for each voltage of the previous equation, we have

$$IR_{eq} = IR_1 + IR_2 + IR_3 + \dots + IR_n .$$

$$IR_{eq} = I(R_1 + R_2 + R_3 + \dots + R_n) ,$$

$$\text{thus } R_{eq} = R_1 + R_2 + R_3 + \dots + R_n = \sum_{k=1}^n R_k .$$

**Parallel connection** is such a connection of elements in the electrical circuit in which all the elements are connected between two nodes (Fig. 1.2, *a* and *b*).

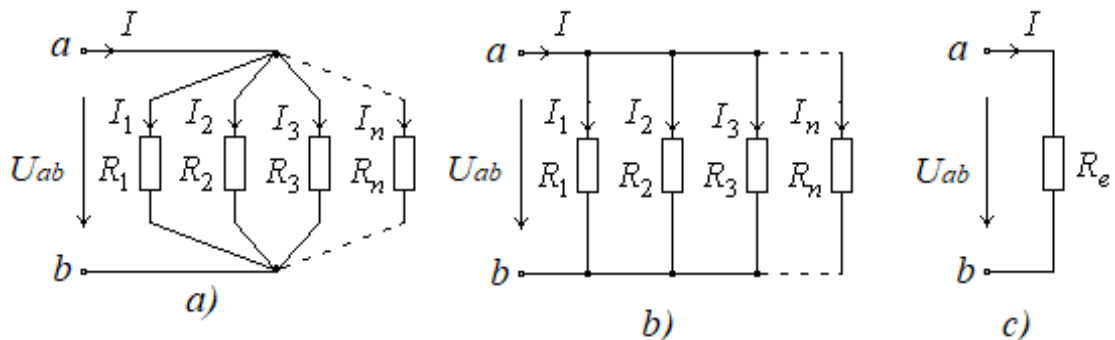


Fig. 1.2

According to Kirchhoff's Current Law, we can write the equation:

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

or

$$\frac{U}{R_e} = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3} + \dots + \frac{U}{R_n}.$$

Consequently, the resistors connected in parallel may be replaced with one equivalent resistor (Fig. 1.2, c), whose resistance is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

In the general case:  $\frac{1}{R_{eq}} = \sum_{k=1}^n \frac{1}{R_k}.$

### ***The method for analyzing the electrical circuits***

In the study of linear circuits, five methods are used to analyze the circuits. Every method of analysis is based on a particular principle or law. These four methods and their corresponding principles are given in Table 1.1 below.

Table 1.1

| <b><i>Methods of Analysis</i></b>                | <b><i>Principles of Analysis</i></b>                |
|--------------------------------------------------|-----------------------------------------------------|
| Method of Direct Application of Kirchhoff's Laws | Kirchhoff's Current Law;<br>Kirchhoff's Voltage Law |
| Mesh analysis method                             | Kirchhoff's Voltage Law                             |
| Node analysis method                             | Kirchhoff's Current Law;<br>Ohm's law               |
| Superposition method                             | Principle of superposition                          |
| Thevenin's method                                | Thevenin's Equivalent Circuit                       |

## 2. CONNECTIONS OF RESISTIVE ELEMENTS IN ELECTRICAL CIRCUITS

### Problem 2.1

In this series-parallel circuit, resistors  $R_1$  and  $R_2$  are in series with each other, but the resistor  $R_3$  is neither in series nor in parallel with  $R_1$  or  $R_2$ :

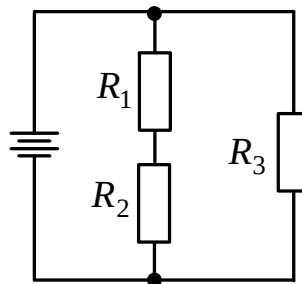


Fig. 2.1

Normally, the first step of a mathematical analysis of a circuit such as this is to determine the total circuit resistance or the equivalent circuit resistance. In other words, we need to calculate how much resistance the voltage source “sees” in the network formed by  $R_1$ ,  $R_2$  and  $R_3$ .

If the circuit were a simple series configuration, our task would be easy.

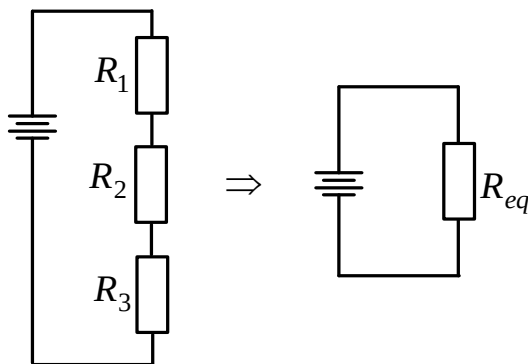


Fig. 2.2

The equivalent circuit resistance of the series connection is given by:

$$R_{eq} = R_1 + R_2 + R_3.$$

After the calculation, we can replace the series connection with one resistor whose resistance is equal to  $R_{eq}$ .

Similarly, if the circuit were a simple parallel configuration, we would have no difficulty in calculating a total resistance.

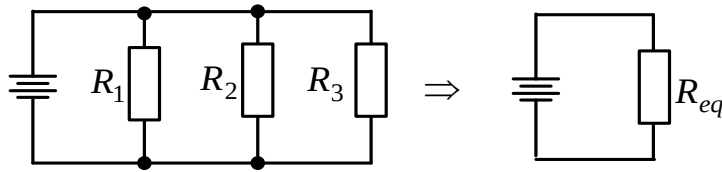


Fig. 2.3

The equivalent circuit resistance of the parallel connection is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

After calculating, we can also replace the parallel connection with one resistor whose resistance is equal to  $R_{eq}$ .

Notice that our circuit is neither purely series nor purely parallel, so the calculation of total resistance is not a simple one-step operation. However, there is a way to simplify the circuit to either a simple series or simple parallel circuit.

### Task:

Describe how that might be done and demonstrate it using the numerical values for resistors  $R_1$ ,  $R_2$  and  $R_3$ .

### Problem 2.2

Identify which of these components are connected directly in series with each other, and which are connected directly in parallel with each other. Assume that the open wire ends are connection points to a power source.

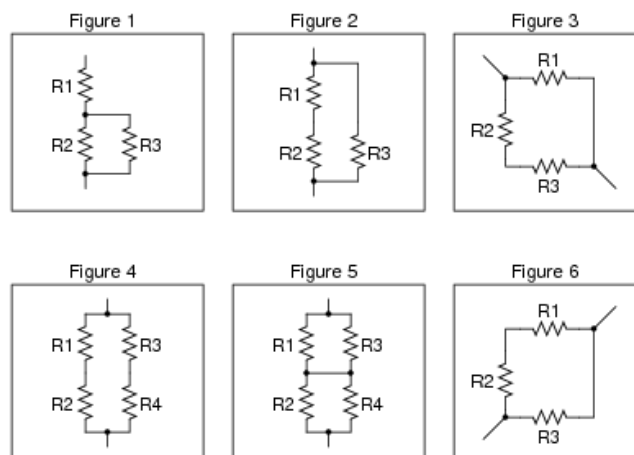


Fig. 2.4

### Problem 2.3

Identify which of these components are connected directly in series with each other, and which are connected directly in parallel with each other. Assume that the open wire ends are connection points to a power source. In the circuits where ground symbols appear consider the ground as the other side of the power source.

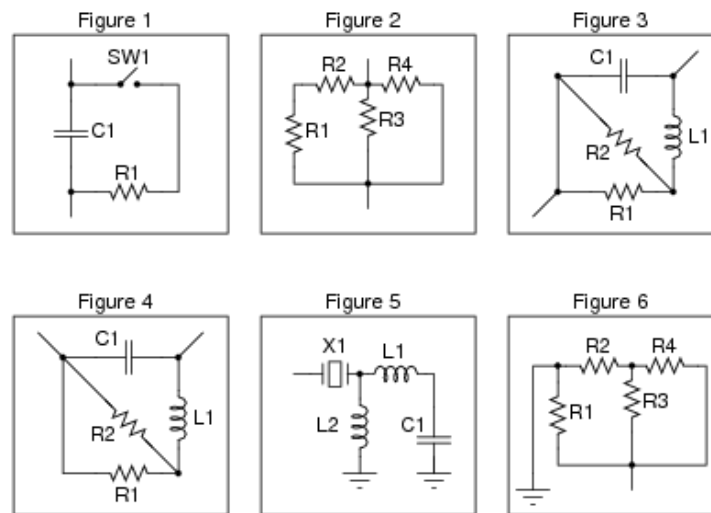


Fig. 2.5

### Problem 2.4

Calculate the resistance between points **A** and **B** ( $R_{AB}$ ) for the following resistor networks:

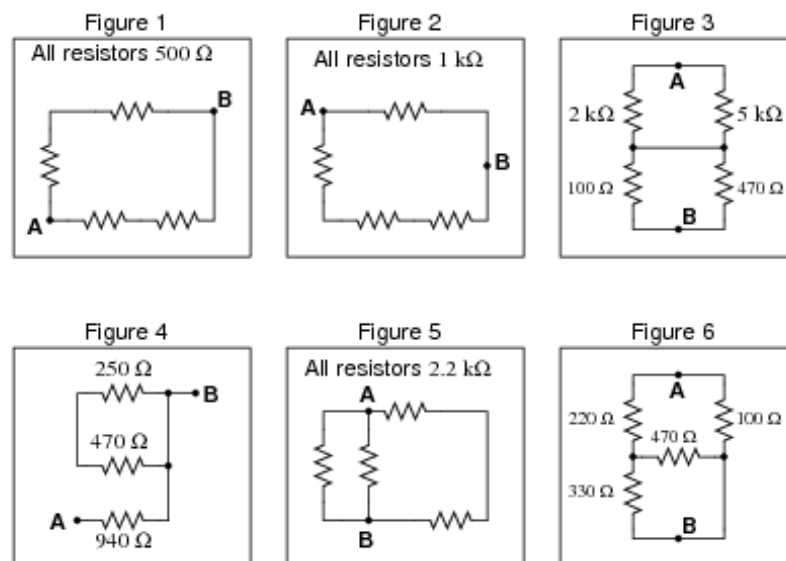


Fig. 2.6

### Problem 2.5

Determine the equivalent resistance of each circuit in Fig. 2.7 between terminals 1-1' for two cases:

1. when points 2 and 2' form an open circuit;
2. when points 2 and 2' are short-circuited.

Resistances in Ohms are given.

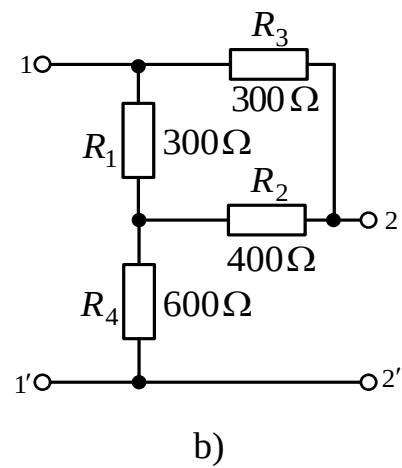
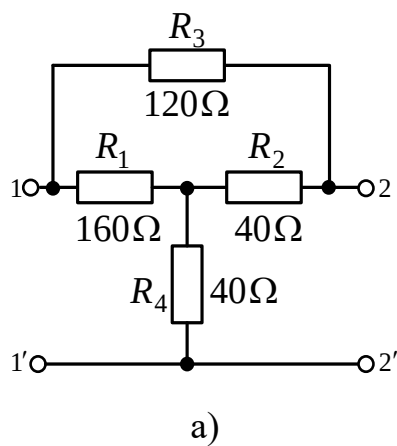


Fig. 2.7

### 3. THE DIRECT APPLICATION OF OHM'S LAW AND KIRCHHOFF'S LAWS

#### Problem 3.1

Determine the reading of the voltmeter for the specified parameters of the circuit:

$$R_1 = 10 \, \Omega;$$

$$R_2 = 20 \, \Omega;$$

$$E_1 = 60 \, \text{V};$$

$$E_2 = 20 \, \text{V};$$

$$E_3 = 10 \, \text{V};$$

$$U = 40 \, \text{V}.$$

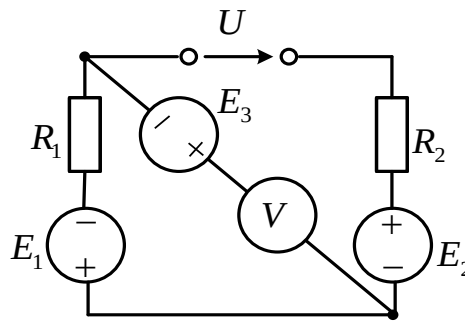


Fig. 3.1

#### Solution

An ideal voltmeter is an open circuit. An open circuit is a limiting approximation for a real voltmeter, which will have some large (but not infinite) resistance.

An ideal ammeter is a short circuit. A short circuit is a limiting approximation for a real ammeter, which will have some small (but not zero) resistance.

In a given circuit, one current flows through the elements connected in series. We arbitrarily set the positive direction of current flow, for example, counterclockwise and, by applying the second Kirchhoff's law, write the equation with respect to the terminals of the voltage source  $U$ .

$$U = I \cdot (R_1 + R_2) - E_1 - E_2$$

Then

$$I = \frac{U + E_1 + E_2}{R_1 + R_2} = \frac{40 + 60 + 20}{10 + 20} = 4 \, \text{A}.$$

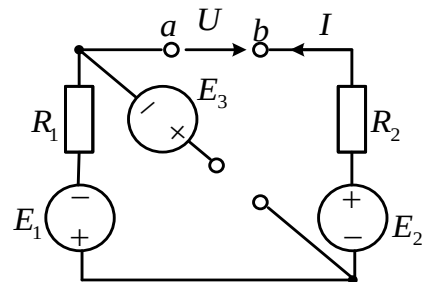


Fig. 3.2

The reading of the voltmeter is calculated as:

$$U_V = E_3 + IR_1 - E_1 = 10 + 4 \cdot 10 - 60 = -10 \text{ V}$$

or

$$U_V = E_3 + U + E_2 - IR_2 = 10 + 40 + 20 - 4 \cdot 20 = -10 \text{ V}.$$

Regardless of the direction of the circuit, the same values of the voltmeter are obtained.

### Problem 3.2

Determine the reading of the voltmeter for the specified parameters of the circuit:  $E = 100 \text{ V}$ ,  $R_1 = 50 \Omega$ ,  $R_2 = 30 \Omega$ ,  $R_3 = 20 \Omega$ .

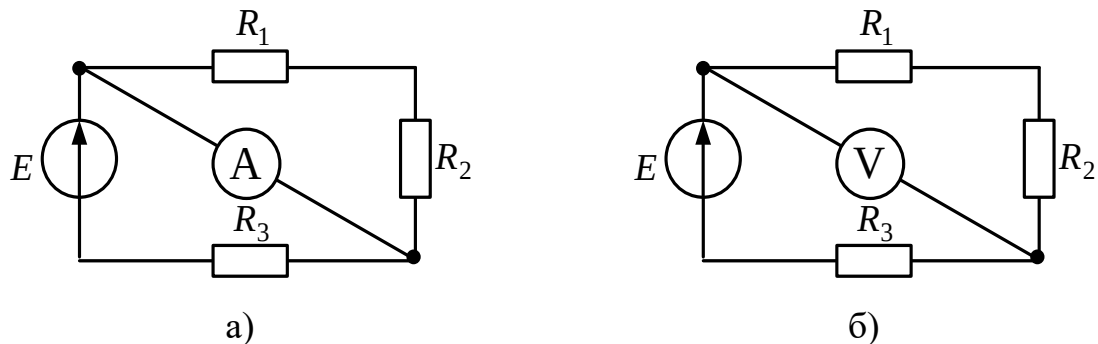


Fig. 3.3

### Problem 3.3

Look at the figure below and determine the electric current through  $R_1$  if

Resistor 1 ( $R_1$ ) =  $4 \Omega$ ,

Resistor 2 ( $R_2$ ) =  $4 \Omega$ ,

Resistor 3 ( $R_3$ ) =  $8 \Omega$ ,

Electric voltage ( $U$ ) =  $40 \text{ V}$ .

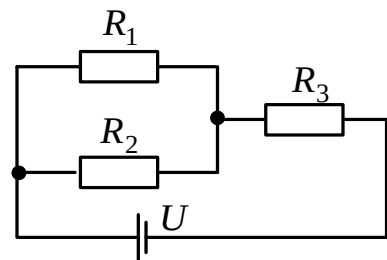
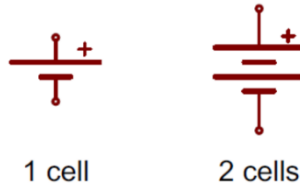


Fig. 3.4

### Solution.

It is known that the electric current flows from high potential to low potential. As a result, the direction of electric current in the circuit above is the same as a clockwise direction according to the polarity of a single-cell battery.



The typical single-celled and two-celled battery symbols are shown. In this schematic symbol, the longer line represents the positive terminal, and the shorter line is the negative terminal. You might also see batteries with more than two lines which indicate more than one cell in the battery.

At first, we calculate the equivalent resistance ( $R_{eq}$ ) by replacing all resistors with one. After that we calculate the electric current using Ohm's law:

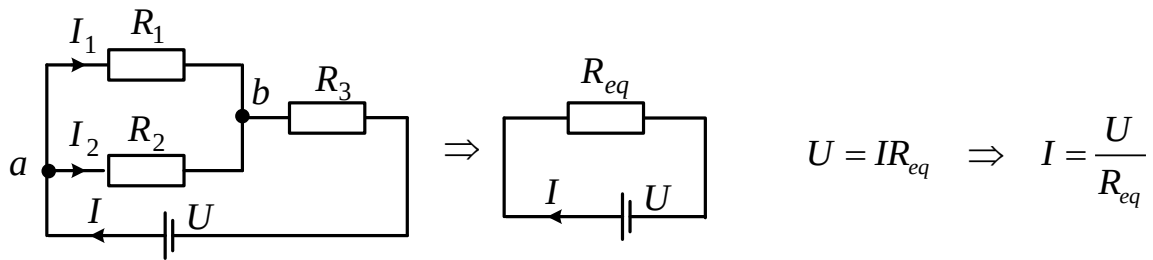


Fig. 3.5

where  $U$  = voltage,  $I$  = electric current,  $R_{eq}$  = the equivalent resistance.

**Calculation of the equivalent resistance:**

Resistor  $R_1$  and resistor  $R_2$  are connected in parallel. The equivalent resistance is calculated as:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{12} = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \cdot 4}{4 + 4} = 2 \Omega.$$

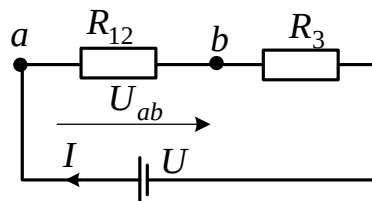


Fig. 3.5

Now we have resistor  $R_{12}$  and resistor  $R_3$  which are connected in series. The equivalent resistance is equal to:

$$R_{eq} = R_{12} + R_3 = 2 + 8 = 10 \Omega.$$

**Calculation of the electric current that flows through the branch with a battery (input current):**

$$I = \frac{U}{R_{eq}} = \frac{40}{10} = 4 \text{ A.}$$

The electric current that flows through the branch with the battery and  $R_3$  is equal to 4 Amperes.

**Calculation of the currents that flow through parallel branches:**

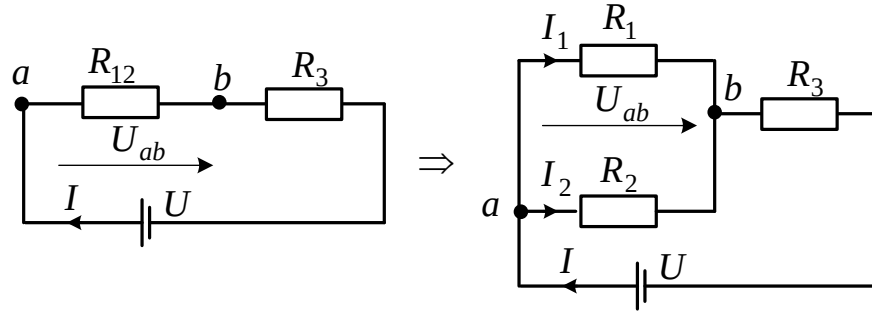


Fig. 3.6

If the battery, resistors  $R_3$  and  $R_{12}$  are connected in series and form mesh, then the electric current flowing through the battery flows through resistors  $R_3$  and  $R_{12}$  and is equal to 4 Amp.

Based on Ohm's law, we find the electric voltage  $U_{ab}$ :

$$U_{ab} = IR_{12} = 4 \cdot 2 = 8 \text{ V.}$$

Based on second Kirchhoff's law, it is equal to

$$U_{ab} + IR_3 = U \Rightarrow 8 \text{ V} + 4 \text{ A} \cdot 8 \Omega = 40 \text{ V.}$$

Then

$$I_1 = \frac{U_{ab}}{R_1} = \frac{8}{4} = 2 \text{ A,}$$

$$I_2 = \frac{U_{ab}}{R_2} = \frac{8}{4} = 2 \text{ A.}$$

Based on first Kirchhoff's law, we find:

$$I_1 + I_2 = I \Rightarrow 2 \text{ A} + 2 \text{ A} = 4 \text{ A}$$

**There is a rule allowing us to determine the currents flowing through the two branches connected in parallel if only the input current is known.**

$$I = I_1 + I_2 \Rightarrow \frac{U}{R_E} = \frac{U}{R_1} + \frac{U}{R_2} \Rightarrow$$

$$U = IR_E = I \frac{R_1 R_2}{R_1 + R_2} = I_1 R_1 = I_2 R_2$$

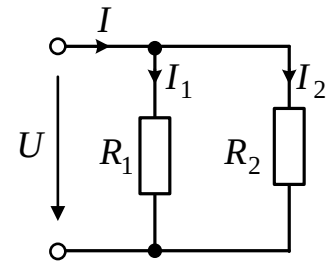


Fig. 3.7

$$I_1 R_1 = I \frac{R_1 R_2}{R_1 + R_2} \Rightarrow I_1 = I \frac{R_2}{R_1 + R_2}$$

$$I_2 R_2 = I \frac{R_1 R_2}{R_1 + R_2} \Rightarrow I_2 = I \frac{R_1}{R_1 + R_2}$$

### Problem 3.4

Look at Fig. 3.8 and determine all electric currents if

Resistor 1 ( $R_1$ ) = 4  $\Omega$ ,

Resistor 2 ( $R_2$ ) = 6  $\Omega$ ,

Resistor 3 ( $R_3$ ) = 1.6  $\Omega$ ,

Electric voltage ( $U$ ) = 16 V.

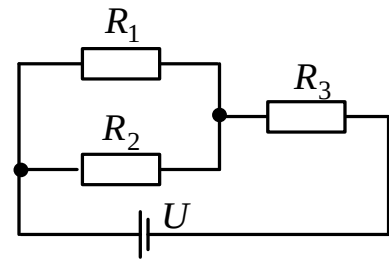


Fig. 3.8

### Problem 3.5

For electric circuit in Fig. 3.9

Resistor 1 ( $R_1$ ) = 100  $\Omega$ ,

Resistor 2 ( $R_2$ ) = 150  $\Omega$ ,

Resistor 3 ( $R_3$ ) = 50  $\Omega$ ,

Electric voltage ( $U$ ) = 150 V.

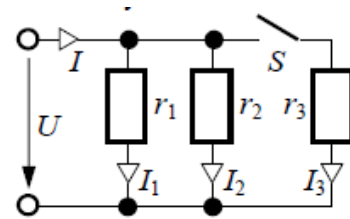


Fig. 3.8

By how many times do the values of all currents change after the switch is closed?

**Solution.**

To answer the problem task, we compare the current through each branch before connecting  $R_3$  with the corresponding value of the current after switching.

***Analyzing the circuit before switching (switch  $S$  is opened).***

The currents through two parallel branches are determined by Ohm's law:

$$I_1 = \frac{U}{R_1} = \frac{150}{100} = 1.5 \text{ A},$$

$$I_2 = \frac{U}{R_2} = \frac{150}{150} = 1 \text{ A}.$$

$$I_3 = 0 \text{ – as the branch is opened.}$$

Based on the Kirchhoff's current law, the input current of the parallel connection is calculated as

$$I = I_1 + I_2 = 1.5 + 1 = 2.5 \text{ A}$$

***Analyzing the circuit after switching (switch  $S$  is closed).***

We find the currents through three parallel branches using Ohm's law again:

$$I_1 = \frac{U}{R_1} = \frac{150}{100} = 1.5 \text{ A},$$

$$I_2 = \frac{U}{R_2} = \frac{150}{150} = 1 \text{ A},$$

$$I_3 = \frac{U}{R_3} = \frac{150}{50} = 3 \text{ A}.$$

Based on the Kirchhoff's current law, the input current of the parallel connection is calculated as:

$$I = I_1 + I_2 + I_3 = 1.5 + 1 + 3 = 5.5 \text{ A}.$$

As a result, we make a conclusion that the more resistors are connected in parallel, the less equivalent resistance and the more is the input current of the parallel connection.

In this problem,

- the input current after switching is risen by about twice;

- the currents through the parallel branches do not change except for the current through the  $R_3$ .

### Problem 3.6

Complete the table of values for this circuit:

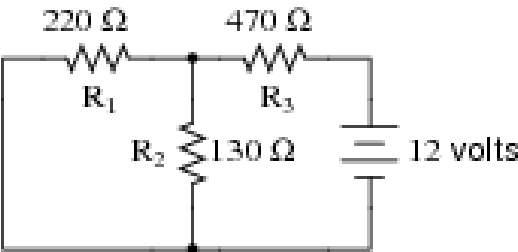


Fig. 3.9

|     | $R_1$        | $R_2$        | $R_3$        | Total |
|-----|--------------|--------------|--------------|-------|
| $U$ |              |              |              |       |
| $I$ |              |              |              |       |
| $R$ | 220 $\Omega$ | 130 $\Omega$ | 470 $\Omega$ |       |
| $P$ |              |              |              |       |

### Problem 3.7

Complete the table of values for this circuit:

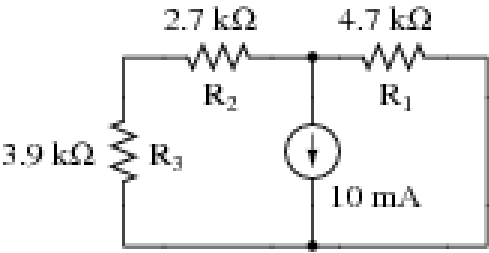


Fig. 3.10

|     | $R_1$          | $R_2$          | $R_3$          | Total |
|-----|----------------|----------------|----------------|-------|
| $U$ |                |                |                |       |
| $I$ |                |                |                | 10 mA |
| $R$ | 4.7 k $\Omega$ | 2.7 k $\Omega$ | 3.9 k $\Omega$ |       |
| $P$ |                |                |                |       |

**Problem 3.8**

Complete the table of values for this circuit:

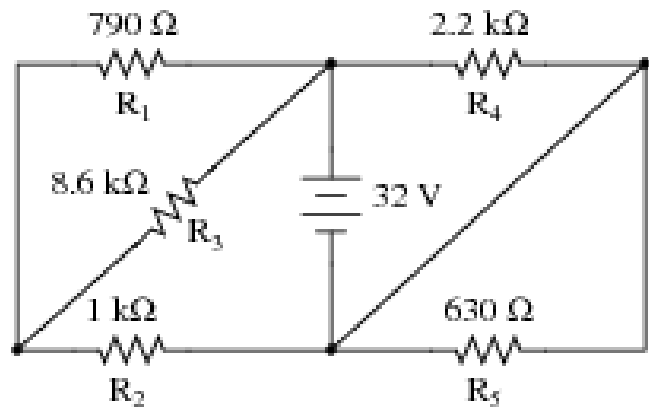


Fig. 3.11

|     | $R_1$        | $R_2$        | $R_3$          | $R_4$          | $R_5$        | Total |
|-----|--------------|--------------|----------------|----------------|--------------|-------|
| $U$ |              |              |                |                |              |       |
| $I$ |              |              |                |                |              |       |
| $R$ | 790 $\Omega$ | 1 k $\Omega$ | 8.6 k $\Omega$ | 2.2 k $\Omega$ | 630 $\Omega$ |       |
| $P$ |              |              |                |                |              |       |

**Problem 3.9**

Determine the resistance  $R$  and its power for the specified parameters of the circuit.

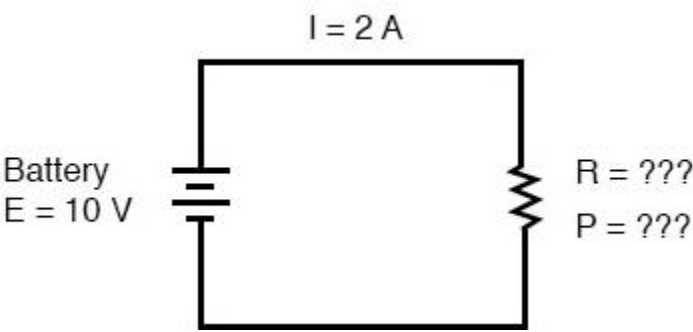


Fig. 3.12

**Solution.**

Here we have the battery voltage (10 volts) and the circuit current (2 amps). We do not know the resistance of the resistor in ohms and the power dissipated by

it in watts. According to Ohm's Law, we use the equation that helps to find the answers by means of the known quantities of voltage and current:

$$E = IR \Rightarrow R = \frac{E}{I}$$

$$P = EI = I^2 R$$

Inserting the known quantities of voltage ( $E$ ) and current ( $I$ ) into these two equations, we can determine circuit resistance ( $R$ ) and power dissipation ( $P$ ):

$$R = \frac{E}{I} = \frac{10 \text{ V}}{2 \text{ A}} = 5 \Omega,$$

$$P = EI = 10 \cdot 2 = 20 \text{ W} \quad \text{or} \quad P = I^2 R = 2^2 \cdot 5 = 20 \text{ W}.$$

For the circuit conditions of 10 volts and 2 amps, the resistor resistance must be  $5 \Omega$ . If we were designing a circuit to operate at these values, we would have to specify a resistor with a minimum power rating of 20 watts, or else it would overheat and fail.

### Problem 3.10

For the specified parameters of the circuit in Fig. 3.13, determine the currents and compare the power of batteries with the power of consumption.

$$R_1 = 20 \Omega$$

$$R_2 = 20 \Omega$$

$$R_3 = 80 \Omega$$

$$R_4 = 120 \Omega$$

$$R_5 = 80 \Omega$$

$$R_6 = 400 \Omega$$

$$R_7 = 36 \Omega$$

$$U = 100 \text{ V}$$

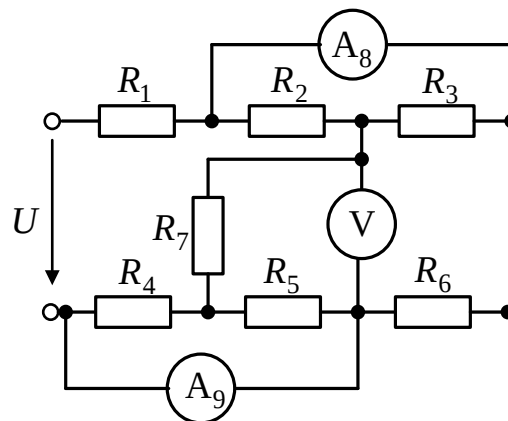


Fig. 3.13

### Solution

In order to determine the currents, we should:

- replace all measurement tools with their internal resistances;

- label the nodes. These labels serve as references and do not have another meaning. The usefulness of these labels will be understood soon;
- find out the equivalent resistance.

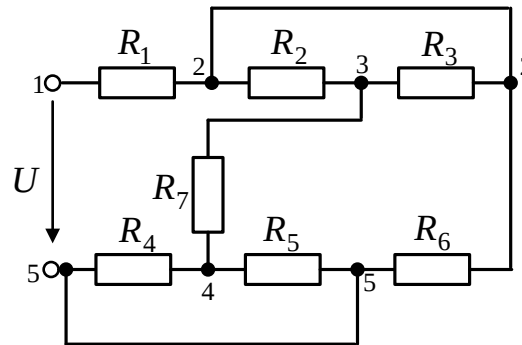


Fig. 3.14

### ***Calculation of the equivalent resistance:***

In order to simplify the calculation of equivalent resistance, let us represent the given circuit in a different way (see Fig. 3.15):

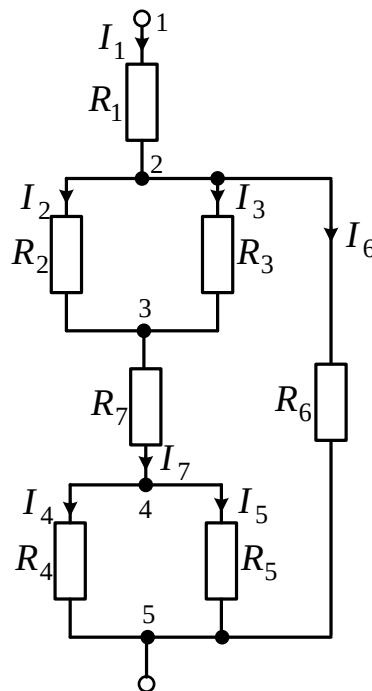


Fig. 3.15

Then we replace  $R_2$  and  $R_3$  with  $R_{23}$ ; and  $R_4$  and  $R_5$  with  $R_{45}$ .

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{20 \cdot 80}{20 + 80} = 16 \quad \Omega,$$

$$R_{45} = \frac{R_4 R_5}{R_4 + R_5} = \frac{120 \cdot 80}{120 + 80} = 48 \quad \Omega,$$

After that  $R_{23}$ ,  $R_{45}$  and  $R_7$  are in series which are in parallel with  $R_6$  (Fig. 3.16).

The equivalent resistance is equal to the sum of  $R_1$  and the parallel connection:

$$\begin{aligned} R_E &= R_1 + \frac{R_6 (R_{23} + R_7 + R_{45})}{R_6 + (R_{23} + R_7 + R_{45})} = \\ &= 20 + \frac{400 \cdot (16 + 36 + 48)}{400 + (16 + 36 + 48)} = 100 \quad \Omega \end{aligned}$$

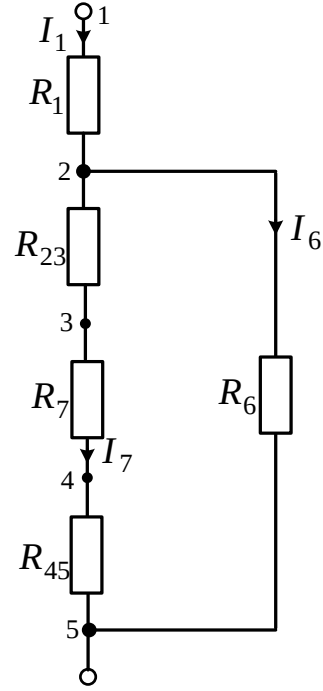


Fig. 3.16

By means of Ohm's law, the input current can be given as:

$$I_1 = \frac{U}{R_E} = \frac{100}{100} = 1 \text{ A.}$$

The currents through the parallel branches can be calculated either by Ohm's law or the rule for two parallel branches:

$$I_6 = \frac{U_{25}}{R_6} \quad \text{or} \quad I_6 = I_1 \frac{R_{23} + R_7 + R_{45}}{R_6 + (R_{23} + R_7 + R_{45})} = 1 \frac{16 + 36 + 48}{400 + (16 + 36 + 48)} = 0.2 \text{ A,}$$

$$I_7 = \frac{U_{25}}{R_{23} + R_7 + R_{45}} \quad \text{or} \quad I_7 = I_1 \frac{R_6}{R_6 + (R_{23} + R_7 + R_{45})} = 1 \frac{400}{400 + (16 + 36 + 48)} = 0.8 \text{ A,}$$

$$I_2 = I_7 \frac{R_3}{R_3 + R_2} = 0.8 \cdot \frac{80}{80 + 20} = 0.64 \text{ A,}$$

$$I_3 = I_7 \frac{R_3}{R_3 + R_2} = I_7 - I_2 = 0.8 \cdot \frac{20}{80 + 20} = 0.16 \text{ A,}$$

$$I_4 = I_7 \frac{R_5}{R_4 + R_5} = 0.8 \cdot \frac{80}{80 + 120} = 0.32 \text{ A,}$$

$$I_5 = I_7 \frac{R_4}{R_4 + R_5} = I_7 - I_4 = 0.8 \cdot \frac{120}{80 + 120} = 0.48 \text{ A.}$$

The currents through the branches with ammeters are found by using the first Kirchhoff's law:

$$I_8 = I_1 - I_2 = 1 - 0.64 = 0.36 \text{ A},$$

$$I_9 = I_5 + I_6 = I_1 - I_2 = 0.48 + 0.2 = 0.68 \text{ A}$$

or

$$I_9 = I_1 - I_4 = 1 - 0.32 = 0.68 \text{ A}$$

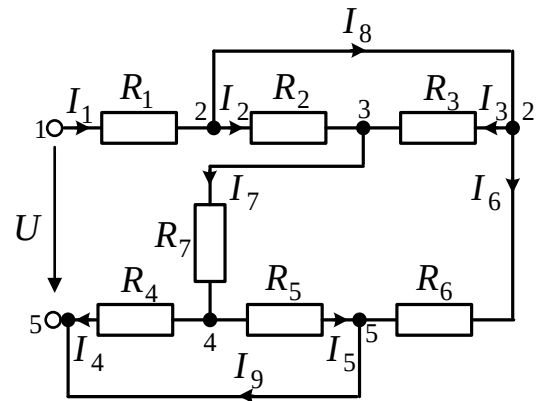


Fig. 3.17

### Problem 3.11

For the circuit in Fig. 3.18, the instruments reading are known: voltmeter  $V$  reads 120 V; wattmeter  $W$  reads 240 W. The resistances are given:  $r_1=16 \Omega$ ,  $r_2=40 \Omega$ . Determine the currents, resistance  $r_3$  and input voltage  $U$ . Check the power balance.

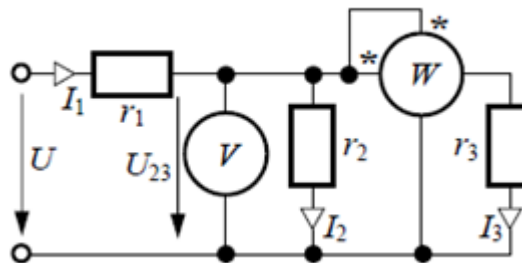


Fig. 3.18

### Solution.

Voltmeter  $V$  measures the voltage  $U_{23}$  across the section of the parallel connection  $r_2$  and  $r_3$ . According to Ohm's law, the currents flowing through each parallel branch can be determined as:

$$I_2 = \frac{U_{23}}{r_2} = \frac{120}{40} = 3 \text{ A},$$

$I_3 = \frac{U_{23}}{r_3}$  – and as  $r_3$  is unknown, the current  $I_3$  cannot be determined by this equation.

Wattmeter  $W$  measures the power consumed by  $r_3$  which can be represented by the equation:  $P_W = I_3^2 r_3 = \frac{U_{23}^2}{r_3} = U_{23} I_3$ . This equation allows us to find the resistance  $r_3$  and currents  $I_3$ :

$$r_3 = \frac{U_{23}^2}{P_W} = \frac{120^2}{240} = 60 \Omega$$

$$I_3 = \frac{P_W}{U_{23}} = \frac{240}{120} = 2 \text{ A.}$$

The input current is calculated by the first Kirchhoff's law:

$$I_1 = I_2 + I_3 = 3 + 2 = 5 \text{ A.}$$

In accordance with the second Kirchhoff's law for the loop  $r_1 - r_2 - U$ , the input voltage is:  $U = I_1 r_1 + U_{23} = 5 \cdot 16 + 120 = 200 \text{ V}$ .

The power of the generator is  $P_G = UI_1 = 200 \cdot 5 = 1000 \text{ W}$ .

The total consumption power is:  $P_C = I_1^2 r_1 + I_2^2 r_2 + I_3^2 r_3 = 1000 \text{ W}$ .

As the power balance  $P_G = P_C$  is true, the problem is solved perfectly.

### Problem 3.12

For the specified parameters of the circuit in Fig. 3.19, determine the currents and compare the power of batteries with the power of consumption.

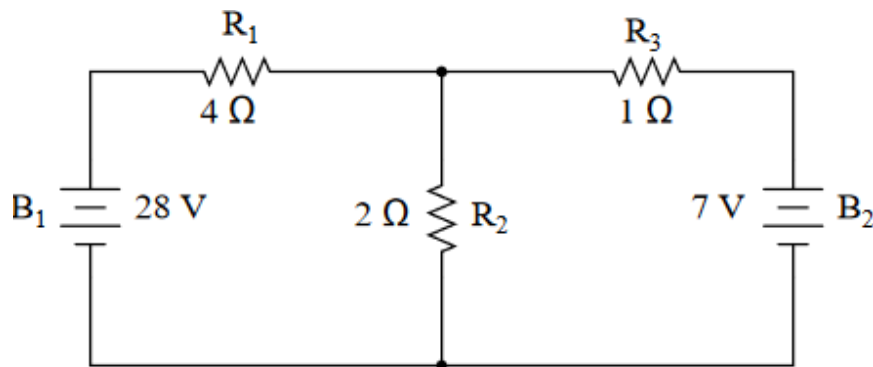


Fig. 3.19

### Solution.

It is known that when we solve any system of equations concerning multiple unknown values, we need the same number of equations.

The first and most straightforward network analysis technique is called the Direct Application of Kirchhoff's Laws. According to this method, we assume the directions of currents through the branches of the given network arbitrarily, then write the equations describing their relationships to each other by means of Kirchhoff's and Ohm's laws.

If the number of equations that is equal to the number of unknown currents, we can solve the simultaneous equations and determine all currents.

Let us use this circuit to illustrate the method of Direct Application of Kirchhoff's Laws.

The first step is to assume the directions of currents arbitrarily (Fig. 3.20). We will determine real directions of the currents after we mathematically solve the system of equations and find the currents (any "wrong" current direction will be shown up as negative numbers in our solution).

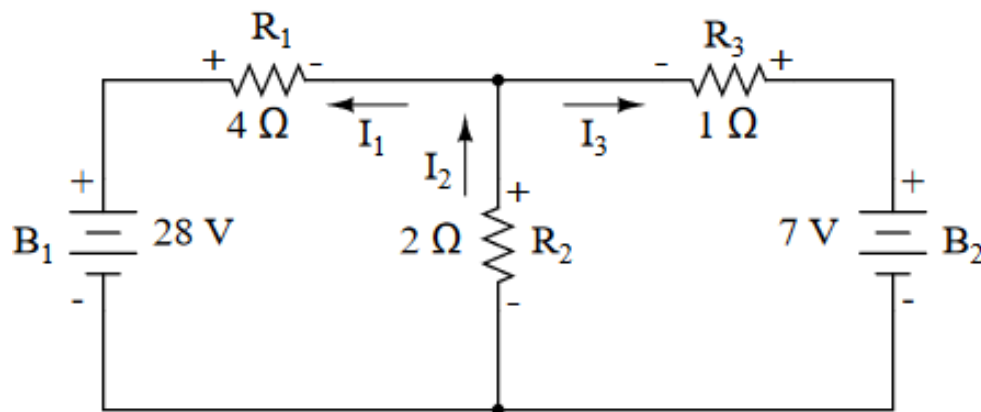


Fig. 3.20

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents entering and exiting a node must equal zero, so we can relate these three currents ( $I_1$ ,  $I_2$ , and  $I_3$ ) to each other in a single equation.

Note that any current entering the node has a positive sign, and any current exiting the node has a negative sign:

$$-I_1 + I_2 - I_3 = 0$$

The next step is to label all voltage drop polarities across resistors according to the assumed directions of the currents.

The battery polarities, of course, remain according to their symbols (the short end is negative, and the long end is positive). It is alright if the polarity of a resistor voltage drop does not match with the polarity of the nearest battery.

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of voltages across all elements in a loop must equal zero, so we can create more equations to find the currents ( $I_1$ ,  $I_2$ , and  $I_3$ ) for our simultaneous equations. To obtain a KVL equation, we

1) choose the moving direction for each mesh (the choice of starting points and directions is arbitrary);

2) apply KVL to each mesh summing up voltage drops in it, as though we were measuring voltages across each element of mesh with a real voltmeter.

First, we choose the moving direction for the left mesh, starting with the upper-left corner and moving anticlockwise. The result obtained is as follows:

$$U_{B1} + I_1 R_1 + I_2 R_2 = 0.$$

Then we choose the moving direction for the right mesh, starting with the upper-right corner and moving counterclockwise, and find the following result:

$$U_{B2} + I_2 R_2 + I_3 R_3 = 0.$$

After solving the system of equations,

$$\begin{cases} -I_1 + I_2 - I_3 = 0 \\ U_{B1} + I_1 R_1 + I_2 R_2 = 0 \\ U_{B2} + I_2 R_2 + I_3 R_3 = 0 \end{cases}$$

we get

$$\begin{cases} -I_1 + I_2 - I_3 = 0 \\ 28 + 4I_1 + 2I_2 = 0 \\ 7 + 2I_2 + 1I_3 = 0 \end{cases} \Rightarrow \begin{cases} I_1 = -5A, \\ I_2 = -4A, \\ I_3 = 1A. \end{cases}$$

Now we have to check our calculation. For this we should compare the power of batteries with the power of consumption. If the power balance  $P_G = P_C$  is true, the problem is solved perfectly.

The power of the generator is calculated as:

$$P_G = U_{B1}(-I_1) + U_{B2}(-I_3) = 28 \cdot (5) + 28 \cdot (-1) = 133 \text{ W}.$$

The total power consumption is calculated:

$$P_C = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 = (-5)^2 \cdot 4 + (-4)^2 \cdot 2 + 1^2 \cdot 1 = 133 \text{ W}.$$

The power balance  $P_G = P_C$  is right.

### Problem 3.13

For the given electrical circuit, we know the following parameters:

$$E_1 = 100 \text{ V}, E_3 = 150 \text{ V}, J = 5 \text{ A},$$

$$R_1 = 10 \Omega, R_2 = 20 \Omega, R_3 = 25 \Omega,$$

$$R_4 = 15 \Omega, R_5 = 15 \Omega.$$

Determine the currents.

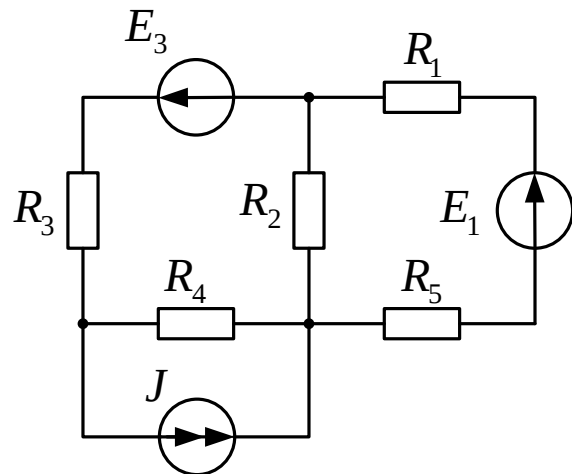


Fig.3.21

### Solution.

**Step 1.** Label the number of nodes  $q=3$  and branches  $k=5$ .

**Step 2.** Choose the current directions through the branches.

**Step 3.** Write  $q-1=2$  equations for the nodes by means of Kirchhoff's current law, for example:

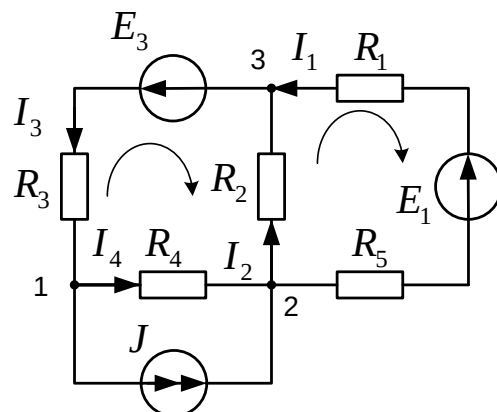


Fig.3.22

- for node 2:  $I_4 + J - I_2 - I_1 = 0$ ,

- for node 3:  $I_2 - I_3 + I_1 = 0$ .

**Step 4.** Determine the number of circuit meshes and choose arbitrarily positive directions of the traversal around these meshes.

**Step 5.** Write  $p-(q-1)$  equations for the meshes by means of Kirchhoff's voltage law. Notice that if the mesh includes a current source, then Kirchhoff's voltage law for this mesh is not applied.

$$-I_2 R_2 - I_4 R_4 - I_3 R_3 = -E_3,$$

$$I_2 R_2 - I_1 (R_1 + R_5) = -E_1.$$

**Step 6.** Solve the system of the following equations:

$$\begin{cases} I_4 + J - I_2 - I_1 = 0 \\ I_2 - I_3 + I_1 = 0 \\ -I_2 R_2 - I_4 R_4 - I_3 R_3 = -E_3 \\ I_2 R_2 - I_1 (R_1 + R_5) = -E_1 \end{cases} \Rightarrow \begin{cases} I_4 - I_2 - I_1 = -5 \\ I_2 - I_3 + I_1 = 0 \\ -I_2 \cdot 20 - I_4 \cdot 15 - I_3 \cdot 25 = -150 \\ I_2 \cdot 20 - I_1 \cdot 25 = -100 \end{cases}$$

The solution of the system gives the branch currents:

$$I_1 = 4.565 \text{ A}, \quad I_2 = 0.707 \text{ A}, \quad I_3 = 5.272 \text{ A}, \quad I_4 = 0.272 \text{ A}.$$

### Problem 3.14

For the given electrical circuit, it is known that

$$E_1 = 20 \text{ V}, \quad E_2 = 10 \text{ V},$$

$$E_6 = 20 \text{ V}, \quad J = 10 \text{ A},$$

$$R_1 = 5 \Omega, \quad R_4 = 5 \Omega,$$

$$R_5 = 15 \Omega, \quad R_6 = 10 \Omega.$$

Determine the currents.

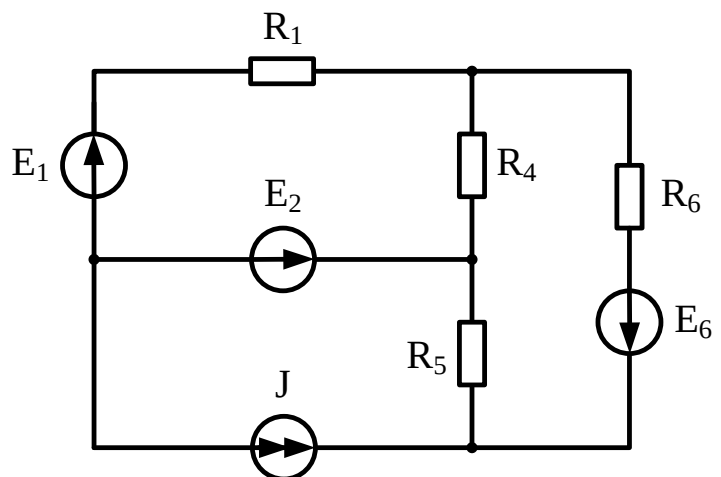


Fig. 3.23

**Solution.**

**Step 1.** Label the number of nodes  $q=4$  and branches  $k=6$ .

**Step 2.** Choose arbitrary current directions through branches.

**Step 3.** Draw up  $q-1=3$  equations for nodes by means of Kirchhoff's current law:

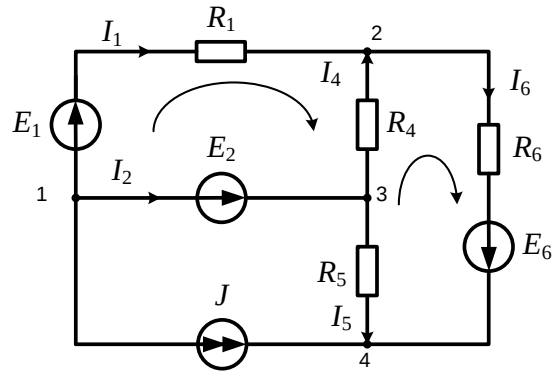


Fig.3.24

- for node 1:  $-J - I_2 - I_1 = 0$ ,

- for node 2:  $I_1 + I_4 - I_6 = 0$ ,

- for node 3:  $I_2 - I_4 - I_5 = 0$ ,

**Step 4.** Determine the number of circuit meshes and choose arbitrarily positive directions of the traversal around these meshes.

**Step 5.** Write  $p-(q-1)$  equations for the meshes by means of Kirchhoff's voltage law. One mesh includes a current source, so the equation for this mesh is not given by Kirchhoff's voltage law because we do not know the voltage across the current source.

$$I_1 R_1 - I_4 R_4 = E_1 - E_2,$$

$$I_6 R_6 - I_5 R_5 + I_4 R_4 = E_6.$$

**Step 6.** Solve the system of the following equations:

$$\begin{cases} -J - I_2 - I_1 = 0 \\ I_1 + I_4 - I_6 = 0 \\ I_2 - I_4 - I_5 = 0 \\ I_1 R_1 - I_4 R_4 = E_1 - E_2 \\ I_6 R_6 - I_5 R_5 + I_4 R_4 = E_6 \end{cases} \Rightarrow \begin{cases} -I_2 - I_1 = 10 \\ I_1 + I_4 - I_6 = 0 \\ I_2 - I_4 - I_5 = 0 \\ I_1 \cdot 5 - I_4 \cdot 5 = 10 \\ I_6 \cdot 10 - I_5 \cdot 15 + I_4 \cdot 5 = 20 \end{cases}$$

The solution of the system gives the branch currents:

$$I_1 = 1.273 \text{ A}, I_2 = 8.727 \text{ A}, I_4 = 3.273 \text{ A}, I_5 = 5.455 \text{ A}, I_6 = 4.545 \text{ A}.$$

### Problem 3.15

For the given electrical circuit, we know the following parameters:

$$I_2 = 1 \text{ A}, I_3 = 5 \text{ A}, I_4 = 2 \text{ A},$$

$$R_1 = 4 \Omega, R_2 = 2 \Omega, R_3 = 10 \Omega,$$

$$R_4 = 4 \Omega.$$

Determine the EMFs  $E_1, E_2$  and resistance  $R$ .

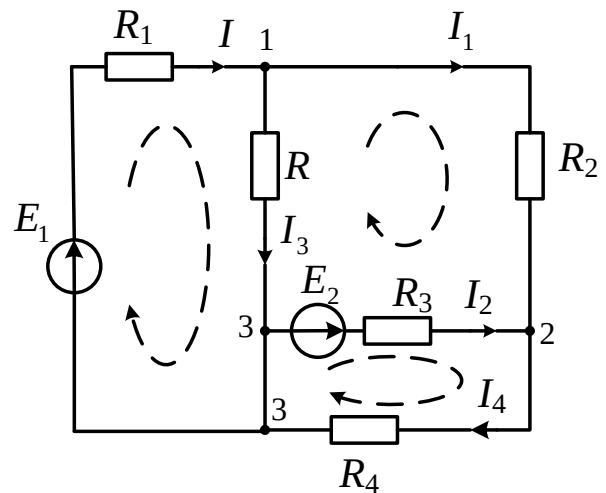


Fig. 3.25

### Solution.

**Step 1.** Label the number of nodes  $q=3$  and branches  $k=5$ .

**Step 2.** If there are five unknown quantities ( $E_1, E_2, R, I, I_1$ ), solve the system consisting of five equations.

**Step 3.** Draw up two equations for the nodes by means of Kirchhoff's current law:

- for node 1:  $I - I_1 - I_3 = 0$ ;

- for node 2:  $I_1 + I_2 - I_4 = 0$ .

**Step 4.** Choose arbitrarily the positive directions of the traversal around the meshes and write down 3 equations for these meshes by means of Kirchhoff's voltage law:

$$IR_1 + I_3R - E_1 = 0;$$

$$I_1R_2 - I_2 \cdot R_3 - I_3R = -E_2;$$

$$I_2 \cdot R_3 + I_4 \cdot R_4 = E_2.$$

**Step 5.** After solving the created system, the results are obtained:

$$\left\{ \begin{array}{l} 1. I - I_1 - I_3 = 0 \\ 2. I_1 + I_2 - I_4 = 0 \\ 3. IR_1 + I_3 R - E_1 = 0 \\ 4. I_1 R_2 - I_2 \cdot R_3 - I_3 R = -E_2 \\ 5. I_2 \cdot R_3 + I_4 \cdot R_4 = E_2 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} 2. I_1 = I_4 - I_2 = 1 \text{ A}; \\ 1. I = I_3 + I_1 = 5 + 1 = 6 \text{ A}; \\ 5. E_2 = 10 + 8 = 18 \text{ V}; \\ 4. R = \frac{E_2 + I_1 R_2 - I_2 \cdot R_3}{I_3} = \frac{18 + 1 \cdot 2 - 1 \cdot 10}{5} = 2 \Omega; \\ 3. E_1 = IR_1 + I_3 R = 6 \cdot 4 + 5 \cdot 2 = 34 \text{ V}. \end{array} \right.$$

### Problem 3.16

For the specified parameters of the circuit in Fig. 3.26, determine the currents, resistance  $r_1$  and compare the power of sources with the power of consumption.

$$E_1 = 120 \text{ V}, E_4 = 80 \text{ V},$$

$$E_5 = 6 \text{ V}, r_2 = r_4 = 6 \Omega,$$

$$r_3 = r_5 = 2 \Omega,$$

$$r_6 = 3 \Omega.$$

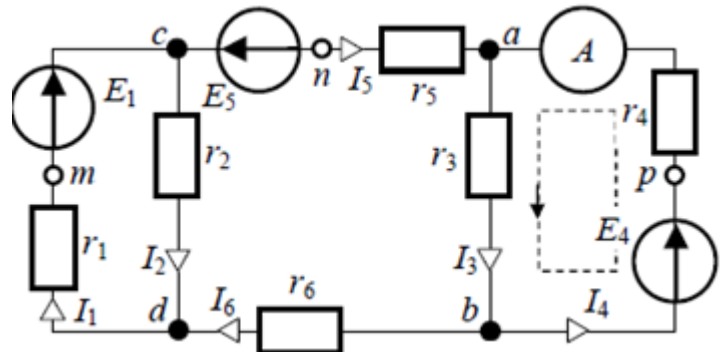


Fig. 3.26

### Problem 3.17

For the specified parameters of the circuit in Fig. 3.22, determine the currents, resistance  $r_6$ , EMF  $E_1$  and compare the power of sources with the power of consumption.

$$R_1 = R_2 = 1 \Omega,$$

$$R_3 = 4 \Omega, R_4 = 2 \Omega, R_5 = 1 \Omega,$$

$$E_2 = 3 \text{ V}, I_{A1} = 4 \text{ A}, I_{A4} = 3 \text{ A}.$$

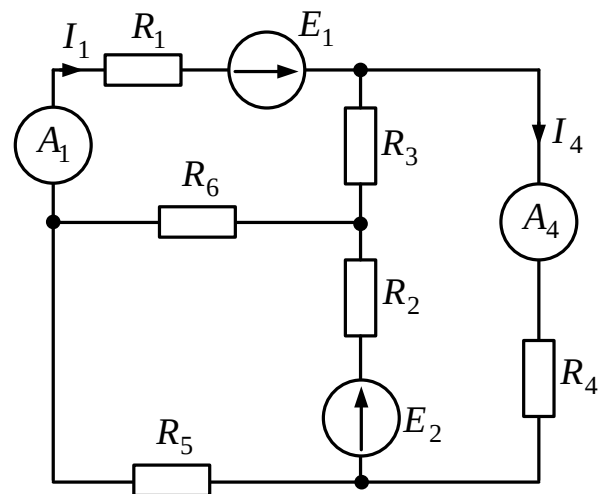


Fig. 3.27

### Problem 3.18

For the specified parameters of the circuit in Fig. 3.28, determine the currents, EMF  $E_1$  and compare the power of sources with the power of consumption.

$$R_1 = 2 \, \Omega;$$

$$R_2 = 6 \, \Omega;$$

$$R_3 = R_4 = 4 \, \Omega;$$

$$R_5 = 5 \, \Omega;$$

$$J = 1 \, \text{A},$$

$$I_5 = 2 \, \text{A}.$$

Determine  $E_1$

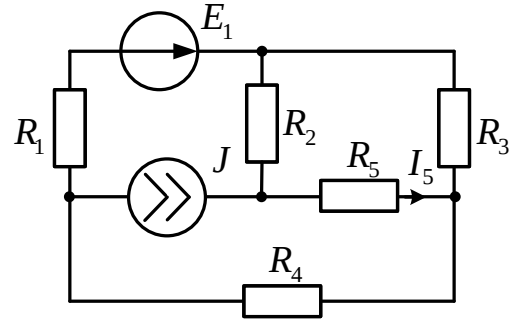


Fig. 3.28

### Problem 3.19

For the specified parameters of the circuit in Fig. 3.29, determine the currents, parameters of elements  $E_0, E_2, R_1, R_6$  and compare the power of sources with the power of consumption.

$P = 148 \, \text{W}$  – the generator power;

$P_{R_6} = 20 \, \text{W}$  – the power of consumption

by  $R_6$ ;

$$R_2 = 4 \, \Omega; \quad R_3 = 10 \, \Omega;$$

$$R_4 = 5 \, \Omega; \quad R_5 = 20 \, \Omega;$$

$$I_3 = 2 \, \text{A}; \quad I_5 = 1 \, \text{A}.$$

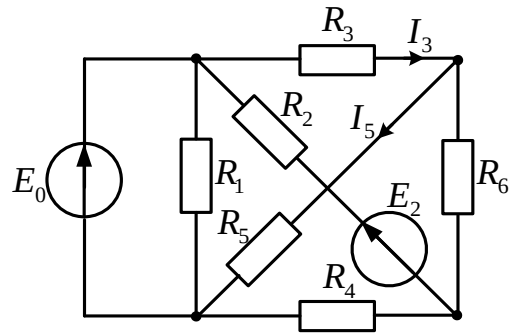


Fig. 3.29

#### 4. THE MESH METHOD OF CIRCUIT CALCULATION (MAXWELL'S METHOD)

##### Problem 4.1

Solve the DC circuit by applying the mesh analysis and find the currents  $I_1, I_2, I_3$ . The following parameters are given for the scheme in Fig. 4.1.

$$E_1 = 10V; \quad E_2 = 20V.$$

$$R_1 = 10 \Omega;$$

$$R_2 = 20 \Omega;$$

$$R_3 = 40 \Omega.$$

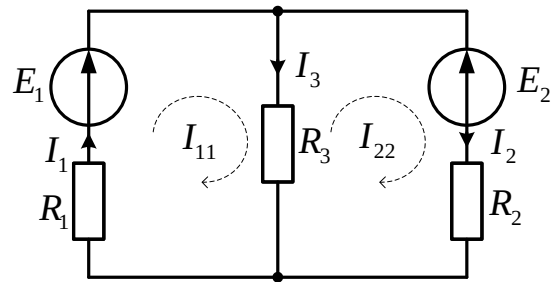


Fig. 4.1

The first step in the Mesh Current method is to identify the “loops” within the circuit encompassing all components. There are two meshes in the circuit. In our example circuit, the loop formed by  $E_1$ ,  $R_2$  and  $R_1$  will be the first while the loop formed by  $E_2$ ,  $R_3$  and  $R_2$  will be the second.

So, we need to assign two mesh currents. It is better to have all the mesh current loops flowing in the same direction (usually clockwise) to prevent errors when drawing up the equations.

A mesh current is the current passing through the elements which are not shared by other loops. For example, the current of  $R_1$  is  $I_1 = I_{11}$ , the current of  $R_2$  is  $I_2 = -I_{22}$ . But how about the elements shared between two meshes such as  $R_3$ ? The current of such an element is the algebraic sum of both meshes. Let us assume that the current of  $R_3$  is defined with the direction from top to bottom, algebraic sum means that its current would be  $I_{11} - I_{22}$ . If one assumes the inverse direction, i.e., from the bottom to the top, it would be  $I_{22} - I_{11}$  because  $I_{11}$  is passing through  $R_3$  with the same direction of its defined current, but  $I_{22}$  is passing with the reverse direction.

First, we define current directions for all elements and find them in terms of mesh currents. For this purpose, the equations for unknown mesh currents are:

$$\begin{cases} I_{11}(R_1 + R_3) - I_{22}R_3 = E_1, \\ -I_{11}R_3 + I_{22}(R_2 + R_3) = -E_2 \end{cases}$$

With the numerals:

$$\begin{cases} I_{11}(10 + 40) - I_{22}40 = 10, \\ -I_{11}40 + I_{22}(20 + 40) = -20 \end{cases}$$

Therefore,  $I_{11} = -0.143$ ,  $I_{22} = -0.429$ .

The branch currents are as follows:

$$I_1 = -I_{11} = 0.143 \text{ A},$$

$$I_2 = -I_{22} = 0.429 \text{ A},$$

$$I_3 = I_2 - I_1 = 0.429 - 0.143 = 0.286 \text{ A}.$$

They change the direction as mesh currents have “-” sign before the result values (see Fig. 4.2).

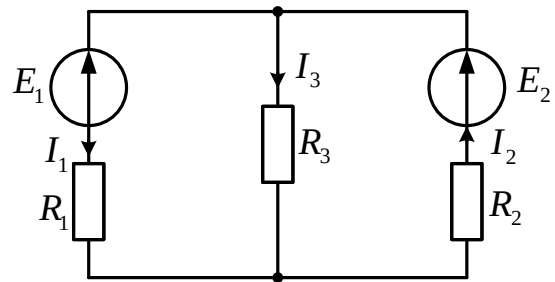


Fig. 4.2

The power balance equation is given by:

$$-E_1 I_1 + E_2 I_2 = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

The total power of sources is calculated as:

$$P_G = -E_1 I_1 + E_2 I_2 = -10 \cdot 0.143 + 20 \cdot 0.429 = 7.15 \text{ W}.$$

The total power consumption is equal to:

$$P_C = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 = 0.143^2 \cdot 10 + 0.429^2 \cdot 20 + 0.286^2 \cdot 40 = 7.157 \text{ W}.$$

If  $P_G = P_C$ , then problem is solved correctly.

### Problem 4.2

For the circuit in Fig. 4.3, we know the parameters and reading of the ammeter  $I_1 = 8 \text{ A}$ . Determine the other currents and compare the power of sources with the power of consumers.

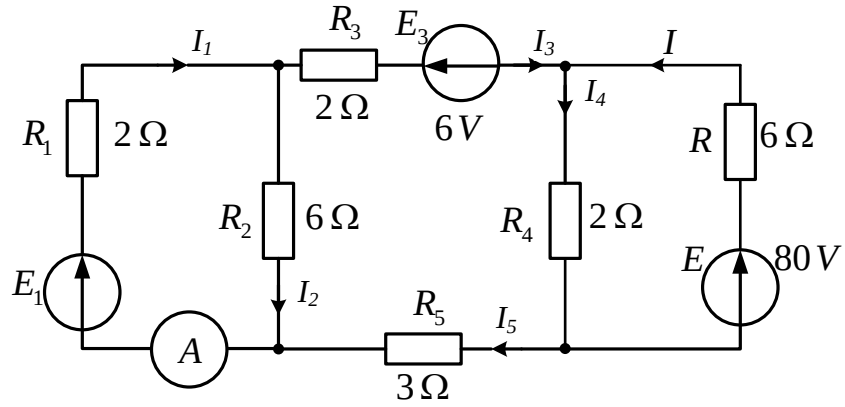


Fig. 4.3

**Solution.**

To analyze the given circuit, we apply the Mesh Method of Circuit Calculation. For this, we arbitrarily assume positive directions for real and mesh currents.

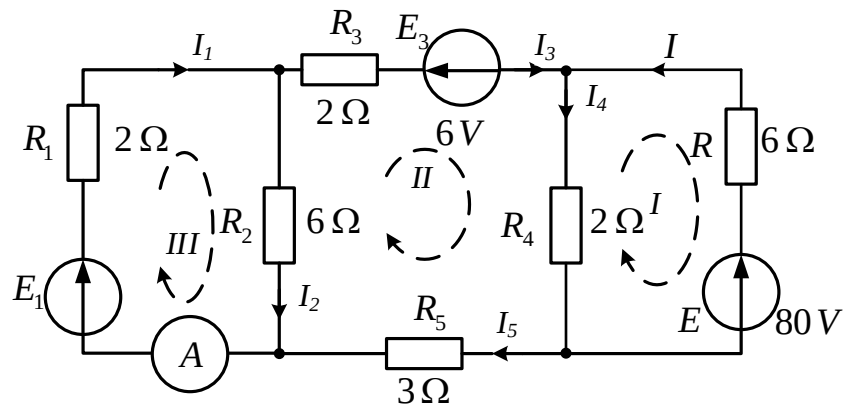


Fig. 4.4

As  $I_{III} = I_1$ ,  $I_{II} = I_3 = I_5$ ,  $I_I = -I$ , then

$$I_2 = I_1 - I_3 = I_{III} - I_{II},$$

$$I_4 = I_3 + I = I_{II} + I_I.$$

The system of equations is as follows:

$$\begin{cases} I_I (R + R_4) - I_{II} (R_4) = -E \\ I_{II} (R_3 + R_4 + R_2 + R_5) - I_I (R_4) - I_{III} (R_2) = -E_3 \\ I_{III} (R_1 + R_2) - I_{II} (R_2) = E_1 \end{cases}$$

Taking into consideration that  $I_{III} = 8 \text{ A}$ , we obtain:

$$\begin{cases} I_I(6+2) - I_{II}(2) = -80 \\ I_{II}(2+2+6+3) - I_I(2) - I_{III}(6) = -6 \\ I_{III}(2+6) - I_{II}(6) = E_1 \end{cases} \Rightarrow \begin{cases} I_I = -9.56A, \\ I_{II} = -1.76A, \\ E_1 = 53.44V. \end{cases}$$

We find the currents flowing through the circuit branches:

$$I_1 = I_{III} = 8A,$$

$$I_2 = I_{III} - I_{II} = 6.24A,$$

$$I_3 = I_{II} = 1.76A,$$

$$I_4 = I_{II} + I_I = 11.32A,$$

$$I_5 = I_{II} = -1.76A,$$

$$I = -I_I = 9.56A.$$

Then we compare the power of sources with the power of consumption to make sure the calculation is correct.

The power of consumption is calculated as:

$$P_C = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 + I_5^2 R_5 + I^2 R = 1182W$$

The power of sources is equal to:

$$P_G = E_1 \cdot I_1 - E_3 \cdot I_3 + E \cdot I = 1182W.$$

As the powers ( $P_G = P_C$ ) are the same, the currents are determined correctly.

**Problem 4.3.** For the electrical circuits in Fig. 4.5, use the given parameters to determine the currents by the mesh current method.

$$J = 1A; E_1 = 120V;$$

$$E_3 = 40V; E_4 = 20V;$$

$$R_1 = 20\Omega; R_2 = 40\Omega;$$

$$R_3 = 50\Omega; R_4 = 40\Omega;$$

$$R_5 = 10\Omega.$$

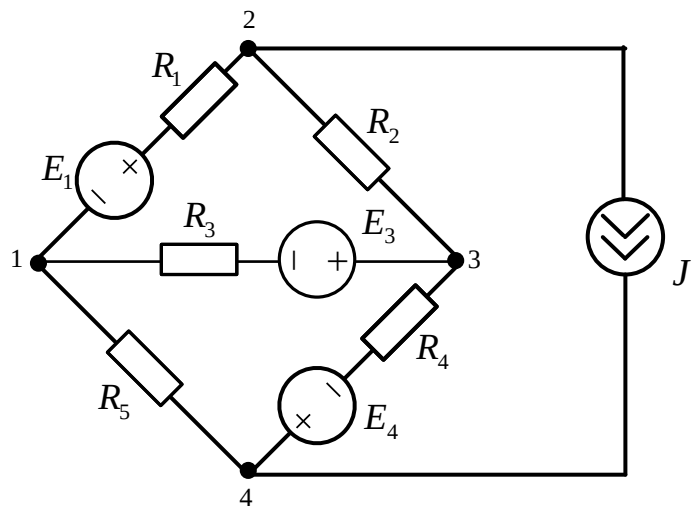


Fig. 4.5

**Solution.**

Firstly, identify the “loops” within the circuit encompassing all components as is shown in Fig. 4.6.

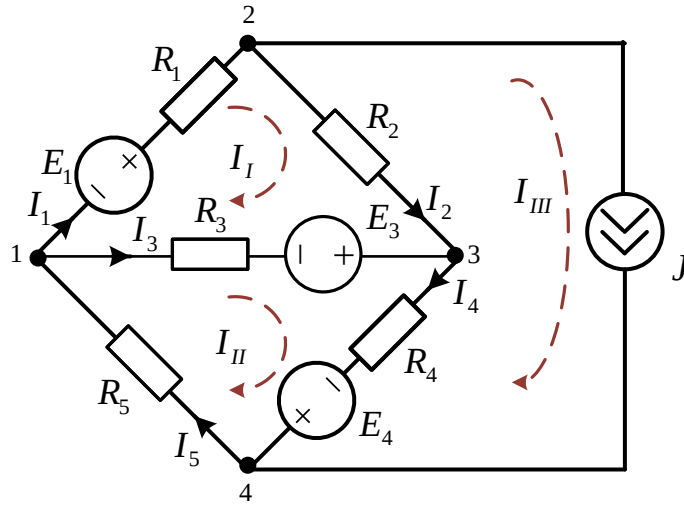


Fig. 4.6

After determining mesh current directions for each “loop”, the unknown mesh currents can be found by means of the system of equations:

$$\begin{cases} I_I (R_1 + R_2 + R_3) - I_{II} R_3 - I_{III} R_2 = E_1 - E_3, \\ -I_I R_3 + I_{II} (R_3 + R_4 + R_5) - I_{III} R_4 = E_3 + E_4 \end{cases}$$

With the numerals:

$$\begin{cases} I_I (20 + 40 + 50) - 50 I_{II} - 40 \cdot 1 = 120 - 40, \\ -50 I_I + I_{II} (50 + 40 + 10) - 40 \cdot 1 = 40 + 20 \end{cases}$$

Therefore,  $I_I = 2$ ,  $I_{II} = 2$ .

The branch currents are as follows:

$$I_1 = I_I = 2A,$$

$$I_2 = I_I - J = 1A,$$

$$I_3 = I_{II} - I_I = 0A,$$

$$I_4 = I_{II} - J = 1A,$$

$$I_5 = I_{II} = 2A.$$

The power balance equation is given by:

$$E_1 \cdot I_1 + E_3 \cdot I_3 + E_4 \cdot I_4 + J \cdot (E_4 - I_4 R_4 - I_2 R_2) = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 + I_5^2 R_5$$

The total power of sources is calculated as:

$$\begin{aligned}
 P_G &= E_1 \cdot I_1 + E_3 \cdot I_3 + E_4 \cdot I_4 + J \cdot U_{42} = \\
 &= E_1 \cdot I_1 + E_4 \cdot I_4 + J \cdot (E_4 - I_4 R_4 - I_2 R_2) = \\
 &= 120 \cdot 2 + 40 \cdot 0 + 20 \cdot 1 + 1 \cdot (20 - 1 \cdot 40 - 1 \cdot 40) = 200 \text{ W}
 \end{aligned}$$

The total power consumption is equal to:

$$\begin{aligned}
 P_C &= I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 + I_5^2 R_5 = \\
 &= 2^2 \cdot 20 + 1^2 \cdot 40 + 0^2 \cdot 50 + 1^2 \cdot 40 + 2^2 \cdot 10 = 200 \text{ W}.
 \end{aligned}$$

If  $P_G = P_C$ , then problem is solved correctly.

**Problem 4.4.** For the electrical circuits in Fig. 4.7, use the given parameters to determine the currents by the mesh current method.

$$\begin{aligned}
 E_1 &= 40 \text{ V}, E_2 = 80 \text{ V}, \\
 E_3 &= 20 \text{ V}, E_4 = 120 \text{ V}, \\
 R_1 &= 20 \Omega, R_2 = 10 \Omega, \\
 R_3 &= 10 \Omega, R_4 = 10 \Omega, \\
 R_5 &= 40 \Omega, R_6 = 60 \Omega.
 \end{aligned}$$

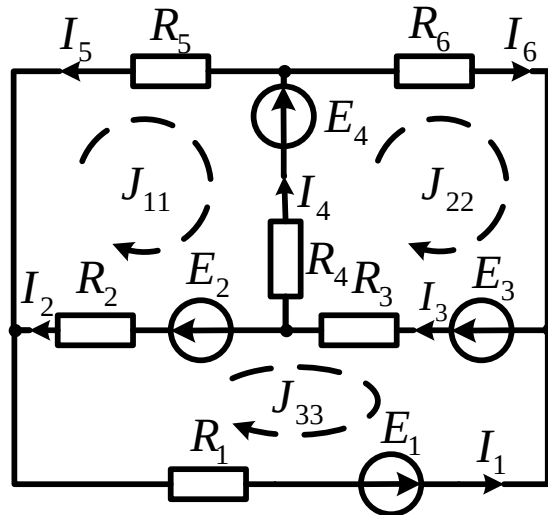


Fig. 4.7

### Solution.

Step 1. Determine the number of independent meshes.

Step 2. Choose arbitrarily the positive directions of the unknown mesh currents  $J_{11}$ ,  $J_{22}$ ,  $J_{33}$ , for example in a clockwise direction.

Step 3. Write the equations for independent meshes by means of Kirchhoff's voltage law,

$$\left. \begin{aligned} R_{11}J_{11} - R_{12}J_{22} - R_{13}J_{33} &= E_2 - E_4; \\ -R_{21}J_{11} + R_{22}J_{22} - R_{23}J_{33} &= E_3 + E_4; \\ -R_{31}J_{11} - R_{32}J_{22} + R_{33}J_{33} &= -E_2 - E_3 - E_1. \end{aligned} \right\}$$

where

$R_{11} = R_5 + R_4 + R_2$  – is the self-resistance of the first mesh;

$R_{22} = R_6 + R_3 + R_4$  – is the self-resistance of the second mesh;

$R_{33} = R_2 + R_3 + R_1$  – is the self-resistance of the third mesh;

$R_{12} = R_{21} = -R_4$  – is the mutual resistance of the first and the second meshes;

$R_{13} = R_{31} = -R_2$  – is the mutual resistance of the first and the third meshes;

$R_{23} = R_{32} = -R_3$  – is the mutual resistance of the second and the third meshes;

$E_{11} = E_1 + E_2$  – is the EMF across the first mesh;

$E_{22} = -E_2 - E_3$  – is the EMF across the second mesh.

The system of equations according to the mesh current method can be written as:

$$\left. \begin{aligned} (R_5 + R_4 + R_2)J_{11} - R_4J_{22} - R_2J_{33} &= E_2 - E_4; \\ -R_4J_{11} + (R_6 + R_3 + R_4)J_{22} - R_3J_{33} &= E_3 + E_4; \\ -R_2J_{11} - R_3J_{22} + (R_2 + R_3 + R_1)J_{33} &= -E_2 - E_3 - E_1. \end{aligned} \right\}$$

With the numerical data:

$$\left. \begin{aligned} 60J_{11} - 10J_{22} - 10J_{33} &= -40; \\ -10J_{11} + 80J_{22} - 10J_{33} &= 140; \\ -10J_{11} - 10J_{22} + 40J_{33} &= -140; \end{aligned} \right\} \text{ or } \left. \begin{aligned} 6J_{11} - 1J_{22} - 1J_{33} &= -4; \\ -1J_{11} + 8J_{22} - 1J_{33} &= 14; \\ -1J_{11} - 1J_{22} + 4J_{33} &= -14. \end{aligned} \right\}$$

Step 4. The solution of the system of equations will give:

$$J_{11} = -1,05\text{A}; \quad J_{22} = 1,19\text{A}; \quad J_{33} = -3,47\text{A}.$$

Step 5. Calculate the real currents in the common branch of the two circuits as the algebraic sum of the corresponding mesh currents:

$$I_1 = -J_{33} = -(-3,47) = 3,47 \text{ A};$$

$$I_2 = J_{11} - J_{33} = -1,05 - (-3,47) = 2,42 \text{ A};$$

$$I_3 = J_{22} - J_{33} = 1,19 + 3,47 = 4,66 \text{ A};$$

$$I_4 = J_{22} - J_{11} = 1,19 + 1,05 = 2,24 \text{ A};$$

$$I_5 = -J_{11} = 1,05 \text{ A};$$

$$I_6 = J_{22} = 1,19 \text{ A}.$$

If the direction of the mesh current and actual current coincide, then the mesh current is taken with plus. If the direction of the mesh current and actual current is opposite, then the mesh current is taken with minus.

## 5. NODE-ANALYSIS METHOD

### Problem 5.1

For the given circuit, determine the branch currents using the Node-analysis Method.

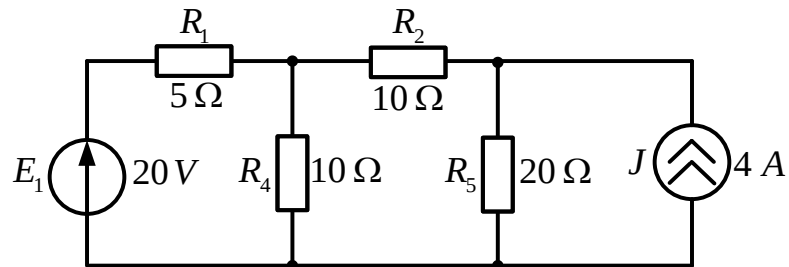


Fig. 5.1

### Solution

**Step 1.** There are **three principal nodes** in the circuit above. They are labelled as 1, 2, and 3 in Fig.5.2.

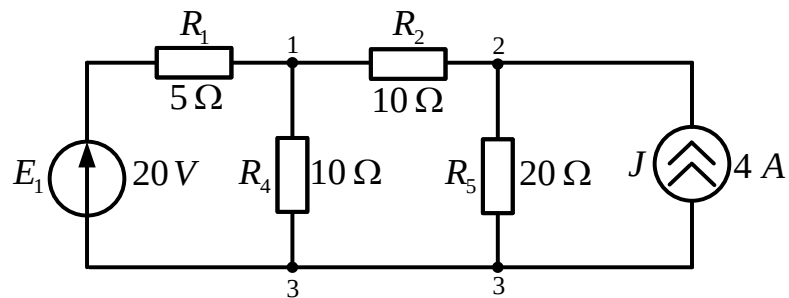


Fig. 5.2

**Node 3** is a reference node (Ground).

**Step 2.** The unknown potentials  $\phi_1$ ,  $\phi_2$  as the node voltages,  $U_1$  and  $U_2$ , are labelled in Fig. 5.3.

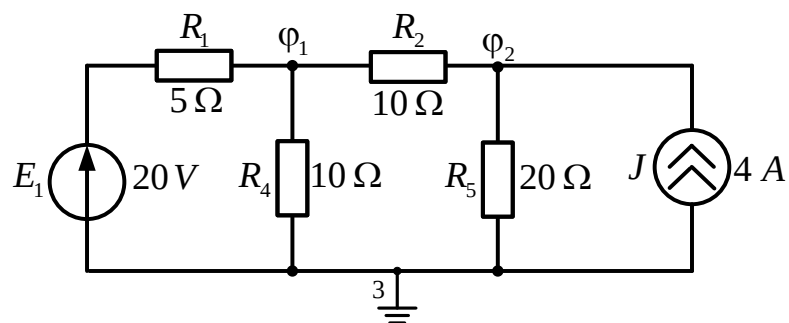


Fig. 5.3

In Fig. 5.3,  $U_{13} = \varphi_1 - \varphi_3 = \varphi_1 - 0 = \varphi_1$  is the voltage from node 1 with respect to ground and  $U_{23} = \varphi_2 - \varphi_3 = \varphi_2 - 0 = \varphi_2$  is the voltage from node 2 with respect to ground.

**Step 3.** In this case, we will get **two nodal equations**, since there are two principal nodes, 1 and 2, other than Ground.

The **nodal equation** at node 1 is

$$\varphi_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) - \varphi_2 \left( \frac{1}{R_2} \right) = E_1 \cdot \frac{1}{R_1}$$

The **nodal equation** at node 2 is

$$\varphi_2 \left( \frac{1}{R_2} + \frac{1}{R_5} \right) - \varphi_1 \left( \frac{1}{R_2} \right) = J$$

**Step 4.** We find the unknown potentials  $\varphi_1, \varphi_2$  by solving Equation 1 and Equation 2.

$$\begin{cases} \varphi_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) - \varphi_2 \left( \frac{1}{R_2} \right) = E_1 \cdot \frac{1}{R_1}; \\ \varphi_2 \left( \frac{1}{R_2} + \frac{1}{R_5} \right) - \varphi_1 \left( \frac{1}{R_2} \right) = J. \end{cases} \Rightarrow$$

$$\begin{cases} \varphi_1 \left( \frac{1}{5} + \frac{1}{10} + \frac{1}{10} \right) - \varphi_2 \left( \frac{1}{10} \right) = 20 \cdot \frac{1}{5}; \\ \varphi_2 \left( \frac{1}{10} + \frac{1}{20} \right) - \varphi_1 \left( \frac{1}{10} \right) = 4. \end{cases} \Rightarrow$$

$$\begin{cases} \varphi_1 \cdot 0.4 - \varphi_2 \cdot 0.1 = 4; \\ \varphi_2 \cdot 0.15 - \varphi_1 \cdot 0.1 = 4. \end{cases} \Rightarrow$$

We get  $\varphi_1 = 20 \text{ V}$  and  $\varphi_2 = 40 \text{ V}$ .

**Step 5.** Now, we can find the current flowing through each resistor by using Ohm's law. For this, we assume the movement directions of the currents flowing through the branches (Fig. 5.4).

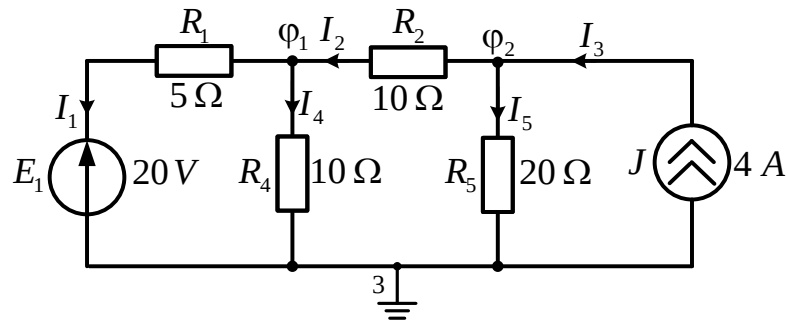


Fig. 5.4

$$I_1 = \frac{\varphi_1 - \varphi_3 - E_1}{R_1} = \frac{20 - 0 - 20}{5} = 0;$$

$$I_2 = \frac{\varphi_2 - \varphi_1}{R_2} = \frac{40 - 20}{10} = 2 \text{ A};$$

$$I_3 = J;$$

$$I_4 = \frac{\varphi_1 - \varphi_3}{R_4} = \frac{20 - 0}{10} = 2 \text{ A};$$

$$I_5 = \frac{\varphi_2 - \varphi_3}{R_5} = \frac{40 - 0}{20} = 2 \text{ A}.$$

The power balance equation is as follows:

$$-E_1 I_1 + J U_{23} = I_1^2 R_1 + I_2^2 R_2 + I_4^2 R_4 + I_5^2 R_5$$

The total power of sources is calculated as:

$$P_G = J U_{23} = J (\varphi_2 - \varphi_3) = 4 \cdot (40 - 0) = 160 \text{ W}$$

The total power consumption is equal to:

$$P_C = I_1^2 R_1 + I_2^2 R_2 + I_4^2 R_4 + I_5^2 R_5 = 0^2 \cdot 5 + 2^2 \cdot 10 + 2^2 \cdot 10 + 2^2 \cdot 20 = 160 \text{ W}.$$

If  $P_G = P_C$ , then problem is solved correctly.

### Problem 5.2

For the electrical circuit, we know:

$$J = 4 \text{ A}, E_1 = 120 \text{ V}, E_2 = 60 \text{ V},$$

$$R_1 = R_2 = 20 \Omega, R_3 = 5 \Omega, R_4 = 15 \Omega$$

Determine the currents by the node potential method.

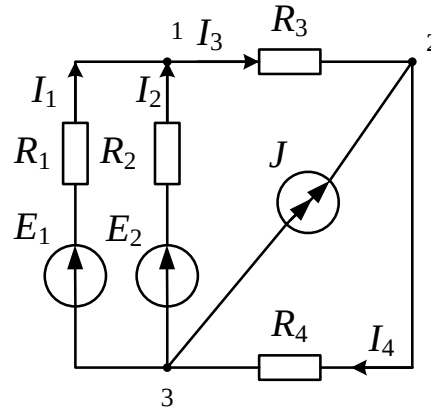


Fig. 5.5

### Solution.

There are **three principal nodes** in the circuit above. They are labelled as 1, 2, and 3 in Fig. 5.5. **Node 3** is the reference node ( $\varphi_3 = 0$ ).

For the nodes with unknown potentials, the system of the node equations is as follows:

$$\begin{cases} G_{11}\varphi_1 - G_{12}\varphi_2 = \sum_1 EG + \sum_1 J; \\ -G_{21}\varphi_1 + G_{22}\varphi_2 = \sum_2 EG + \sum_2 J. \end{cases} \quad (5.1)$$

Here  $G_{11}, G_{22}$  are self-conductances of the nodes (the total conductance of branches which are joined to the appropriate node):

$$G_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \quad G_{22} = \frac{1}{R_3} + \frac{1}{R_4}.$$

$G_{12}, G_{21}$  are the mutual conductances of the nodes (the total conductance of branches which join nodes 1 and 2):

$$G_{12} = G_{21} = \frac{1}{R_3}.$$

$\sum EG$  is the algebraic sum of multiplications of the branch EMFs, joined to the node  $k$ , by the conductance of this branches;

$\sum J$  is an algebraic sum of the currents of current sources in branches which are joined to the node  $k$ .

For node 1:  $\sum_1 EG + \sum_1 J = \frac{E_1}{R_1} + \frac{E_2}{R_2},$

for node 2:  $\sum_2 EG + \sum_2 J = J.$

Then the system (5.1) may be rewritten:

$$\begin{cases} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \varphi_1 - \frac{1}{R_3} \varphi_2 = \frac{E_1}{R_1} + \frac{E_2}{R_2}; \\ -\frac{1}{R_3} \varphi_1 + \left( \frac{1}{R_2} + \frac{1}{R_3} \right) \varphi_2 = J. \end{cases}$$

With the numerals and after simplification, we obtain:

$$\begin{cases} \left( \frac{1}{20} + \frac{1}{20} + \frac{1}{5} \right) \varphi_1 - \frac{1}{5} \varphi_2 = \frac{120}{20} + \frac{60}{20}; \\ -\frac{1}{5} \varphi_1 + \left( \frac{1}{20} + \frac{1}{5} \right) \varphi_2 = 4. \end{cases} \quad \text{or} \quad \begin{cases} 0,3\varphi_1 - 0,2\varphi_2 = 9; \\ -0,2\varphi_1 + 0,267\varphi_2 = 4. \end{cases}$$

From here  $\varphi_1 = 80 \text{ V}$ ,  $\varphi_2 = 75 \text{ V}$ .

The branch currents are calculated by Ohm's law:

$$I_1 = \frac{\varphi_3 - \varphi_1 + E_1}{R_1} = \frac{0 - 80 + 120}{20} = 2 \text{ A},$$

$$I_2 = \frac{\varphi_3 - \varphi_1 + E_2}{R_2} = \frac{0 - 80 + 60}{20} = -1,$$

$$I_3 = \frac{\varphi_1 - \varphi_2}{R_3} = \frac{80 - 75}{5} = 1 \text{ A},$$

$$I_4 = \frac{\varphi_2 - \varphi_3}{R_4} = \frac{75 - 0}{15} = 5 \text{ A}.$$

The power balance is  $E_1 I_1 + E_2 I_2 + (I_4 R_4) J = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4$

$$120 \cdot 2 + 60 \cdot (-1) + (5 \cdot 15) 4 = 2^2 \cdot 20 + 1^2 \cdot 20 + 1^2 \cdot 5 + 5^2 \cdot 15$$

$$480 \text{ W} = 480 \text{ W}.$$

The power balance is right. The problem is solved correctly.

### Problem 5.3.

Determine the currents by node potential method for the circuit shown in Fig. 5.6.

$$J = 10 \text{ A}, E_2 = 90 \text{ V},$$

$$E_3 = 5 \text{ V}, E_7 = 12 \text{ V},$$

$$R_1 = 8 \Omega, R_6 = 4 \Omega,$$

$$R_2 = R_3 = R_4 = R_5 = 10 \Omega.$$

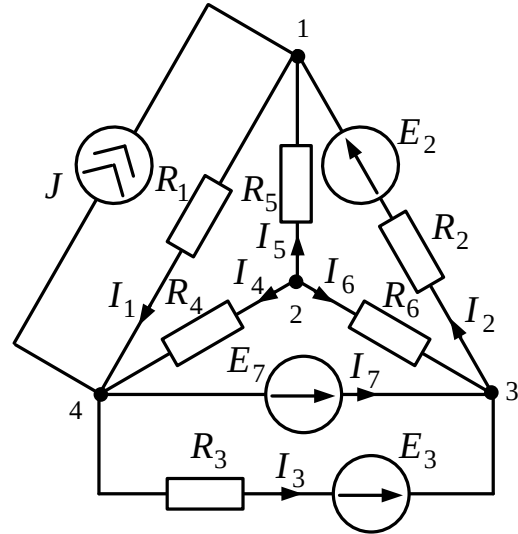


Fig. 5.6

### Solution.

Assume  $\varphi_4 = 0$ , then  $\varphi_3 = E_7 = 12 \text{ V}$ . For the nodes with unknown potentials  $\varphi_1, \varphi_2$  we write a system of the node equations:

$$\begin{cases} \varphi_1 \left( \frac{1}{R_1} + \frac{1}{R_5} + \frac{1}{R_2} \right) - \varphi_2 \left( \frac{1}{R_5} \right) - \varphi_3 \left( \frac{1}{R_2} \right) = J + \frac{E_2}{R_2} \\ -\varphi_1 \left( \frac{1}{R_5} \right) + \varphi_2 \left( \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6} \right) - \varphi_3 \left( \frac{1}{R_6} \right) = 0 \end{cases} \quad (5.2)$$

Where

$$\text{the self-conductance of node 1 is } G_{11} = \frac{1}{R_1} + \frac{1}{R_5} + \frac{1}{R_2} \text{ Sm},$$

$$\text{the self-conductance of node 2 is } G_{22} = \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6} \text{ Sm},$$

$$\text{the mutual conductance of nodes 1 and 2 is } G_{12} = G_{21} = -\frac{1}{R_5} \text{ Sm},$$

$$\text{the mutual conductance of nodes 1 and 3 is } G_{13} = G_{31} = -\frac{1}{R_2} \text{ Sm},$$

$$\text{the nodal current of node 1 is } J_{11} = J + \frac{E_2}{R_2} \text{ A},$$

$$\text{the nodal current of node 2 is } J_{22} = 0.$$

Then we can replace the equations (5.2) with (5.3)

$$\begin{cases} \varphi_1 G_{11} + \varphi_2 G_{12} + \varphi_3 G_{13} = J_{11} \\ \varphi_1 G_{21} + \varphi_2 G_{22} + \varphi_3 G_{23} = 0 \end{cases} \quad (5.3)$$

After solving this system of equations (5.3), we find the unknown potentials

$\varphi_1, \varphi_2$ :

$$\begin{cases} 0.325\varphi_1 - 0.1\varphi_2 = 20.2 \\ -0.1\varphi_1 + 0.45\varphi_2 = 3 \end{cases} \Rightarrow \begin{cases} \varphi_1 \approx 69V \\ \varphi_2 \approx 22V \end{cases}$$

The branch currents are calculated by using Ohm's law

$$I_1 = \frac{\varphi_1 - \varphi_4}{R_1} = \frac{69}{8} = 8.61 \text{ A},$$

$$I_2 = \frac{\varphi_3 - \varphi_1 + E_2}{R_2} = \frac{12 - 69 + 90}{10} = 3.3 \text{ A},$$

$$I_3 = \frac{\varphi_4 - \varphi_3 + E_3}{R_3} = \frac{-12 + 5}{10} = -0.7 \text{ A},$$

$$I_4 = \frac{\varphi_2 - \varphi_4}{R_4} = \frac{22}{10} = 2.2 \text{ A},$$

$$I_5 = \frac{\varphi_1 - \varphi_2}{R_5} = \frac{69 - 22}{10} = 4.7 \text{ A},$$

$$I_6 = \frac{\varphi_2 - \varphi_3}{R_6} = \frac{22 - 12}{4} = 2.5 \text{ A},$$

Using first Kirchhoff's law, we can find  $I_7$

$$I_7 = I_2 - I_6 - I_3 - J = 3.3 - 2.5 - (-0.7) = 1.5 \text{ A}$$

The power balance is calculated as:

$$E_2 I_2 + E_3 I_3 + E_7 I_7 + (\varphi_1 - \varphi_4) J = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 + I_5^2 R_5 + I_6^2 R_6$$

$$90 \cdot 3.3 + 5 \cdot (-0.7) + 12 \cdot 1.5 + (69 - 0) \cdot 10 =$$

$$= 8.61^2 \cdot 8 + 3.3^2 \cdot 10 + (-0.7)^2 \cdot 10 + 2.2^2 \cdot 10 + 4.7^2 \cdot 10 + 2.5^2 \cdot 4$$

$$1001.59 \text{ W} \approx 1001.66 \text{ W}.$$

## 6. THE METHOD OF EQUIVALENT TRANSFORMATIONS AND STAR-DELTA TRANSFORMATION

### Problem 6.1.

For the electrical circuit known as Wheatstone bridge and presented in Fig. 6.1, we know the following parameters:

$$R_1 = 20 \, \Omega, \, R_2 = 40 \, \Omega, \, R_3 = 60 \, \Omega,$$

$$R_4 = 30 \, \Omega, \, R_5 = 30 \, \Omega.$$

Determine the equivalent resistance  $R_{eq}$ .

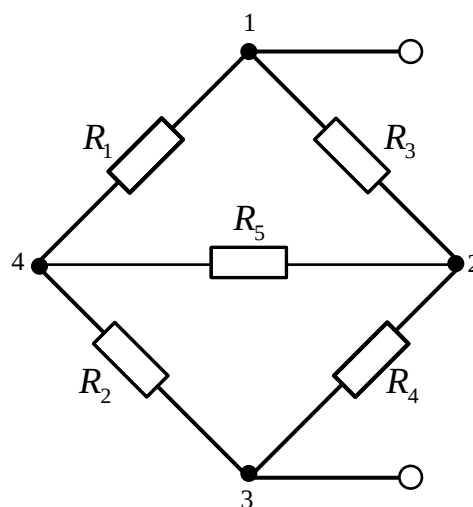


Fig. 6.1

### Solution.

There are neither series nor parallel connections of resistive elements in the given electric circuit. But there are two star-connections  $R_1 - R_5 - R_2$ ,  $R_3 - R_5 - R_4$  and two delta-connections  $R_1 - R_5 - R_3$  and  $R_4 - R_5 - R_2$  here. As a result of transformation of any of these four last connections, the scheme becomes simpler and has the mixed connection.

To determine the equivalent resistance of the given circuit, the star-delta transformation is required.

**Note!** If  $\frac{R_1}{R_4} = \frac{R_3}{R_2}$ , then  $I_5=0$ , and, as a result, the branch containing  $R_5$  can be replaced with the open circuit.

For the given circuit  $\frac{R_1}{R_4} \neq \frac{R_3}{R_2}$ .

We need both delta-star and star-delta transformations to find the equivalent resistance and then compare the results.

Firstly, we convert a delta  $R_1 - R_5 - R_3$  into equivalent star (see Fig. 6.2).

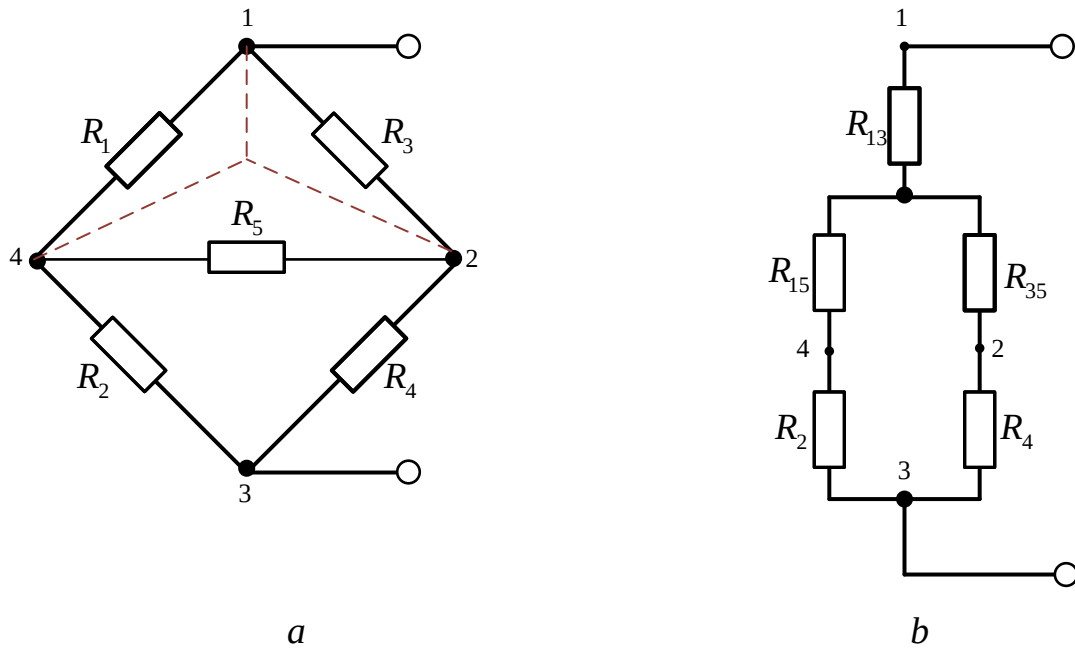


Fig. 6.2

In order to transform delta  $R_1 - R_5 - R_3$  (Fig. 6.2 a) into equivalent star  $R_{13} - R_{15} - R_{35}$  (fig. 6.2 b), the formulas of a delta-star transformation are applied:

$$R_{13} = \frac{R_1 \cdot R_3}{R_1 + R_5 + R_3} = \frac{20 \cdot 60}{20 + 60 + 30} = 10,909 \Omega$$

$$R_{15} = \frac{R_1 \cdot R_5}{R_1 + R_5 + R_3} = \frac{20 \cdot 30}{20 + 60 + 30} = 5,45 \Omega$$

$$R_{35} = \frac{R_3 \cdot R_5}{R_1 + R_5 + R_3} = \frac{30 \cdot 60}{20 + 60 + 30} = 16,36 \Omega$$

After the transformation (Fig. 6.2 b), the resistive elements  $R_{15}$ ,  $R_2$  and  $R_{35}$ ,  $R_4$  are connected in series consistently. Their equivalent resistances are as follows:

$$R_{215} = R_2 + R_{15} = 40 + 5,45 = 45,45 \Omega$$

$$R_{435} = R_4 + R_{35} = 30 + 16,36 = 46,36 \Omega$$

The resistors  $R_{215}$  and  $R_{435}$  are joined in parallel. Their equivalent resistances are

$$R_6 = \frac{R_{215} \cdot R_{435}}{R_{215} + R_{435}} = \frac{45,45 \cdot 46,36}{45,45 + 46,36} = 22,95 \Omega$$

Now the resistors  $R_6$  and  $R_{13}$  are connected in series. The equivalent resistance of the circuit is equal to:

$$R_{eq\Delta-\gamma} = R_{13} + R_6 = 10,909 + 22,95 = 33,86 \Omega.$$

Then we convert a star  $R_3 - R_5 - R_4$  into an equivalent delta connection (see Fig. 6.3).

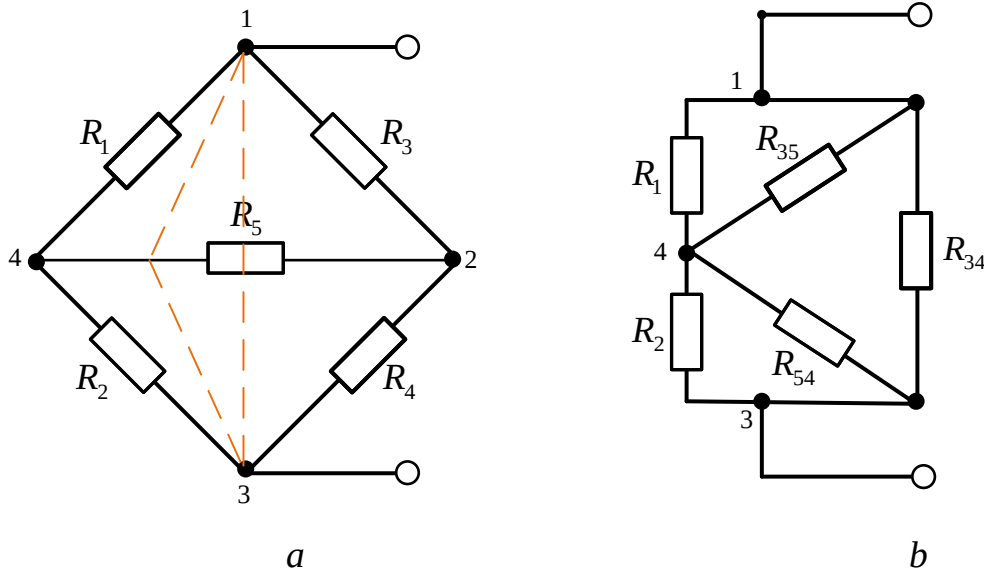


Fig. 6.3

To transform delta  $R_3 - R_5 - R_4$  (fig. 6.3 a) into equivalent star  $R_{35} - R_{54} - R_{34}$  (fig. 6.2 b), the formulas of a star-delta transformation are applied:

$$R_{35} = R_3 + R_5 + \frac{R_3 \cdot R_5}{R_4} = 60 + 30 + \frac{60 \cdot 30}{30} = 150 \Omega$$

$$R_{54} = R_5 + R_4 + \frac{R_5 \cdot R_4}{R_3} = 30 + 30 + \frac{30 \cdot 30}{60} = 75 \Omega$$

$$R_{34} = R_3 + R_4 + \frac{R_3 \cdot R_4}{R_5} = 60 + 30 + \frac{60 \cdot 30}{30} = 150 \Omega$$

After transformation (Fig. 6.3 b), the resistors  $(R_1, R_{35})$  and  $(R_2, R_{54})$  are joined in parallel. Their equivalent resistances are as follows:

$$R_6 = \frac{R_1 \cdot R_{35}}{R_1 + R_{35}} = \frac{20 \cdot 150}{20 + 150} = 17,65 \Omega$$

$$R_7 = \frac{R_2 \cdot R_{54}}{R_2 + R_{54}} = \frac{40 \cdot 75}{40 + 75} = 26,08 \Omega$$

After that the resistors  $R_6$  and  $R_7$  are connected in series. Their equivalent resistance is  $R_8 = R_6 + R_7 = 17,65 + 26,08 = 43,73\Omega$ .

The resistors  $R_8$  and  $R_{34}$  are joined in parallel. The equivalent resistance of the circuit is equal to:

$$R_{eqY-\Delta} = \frac{R_8 \cdot R_{34}}{R_8 + R_{34}} = \frac{43,73 \cdot 150}{43,73 + 150} = 33,86 \Omega$$

We make sure that applying both delta-star and star-delta transformations to find the equivalent resistance gives the same result:

$$R_{eq\Delta-Y} = R_{eqY-\Delta}$$

### Problem 6.2.

We have the direct current electric circuit (Fig. 6.4) with the parameters of energy sources (V) and resistances ( $\Omega$ ). The task is to calculate the currents in all branches of the electric circuit.

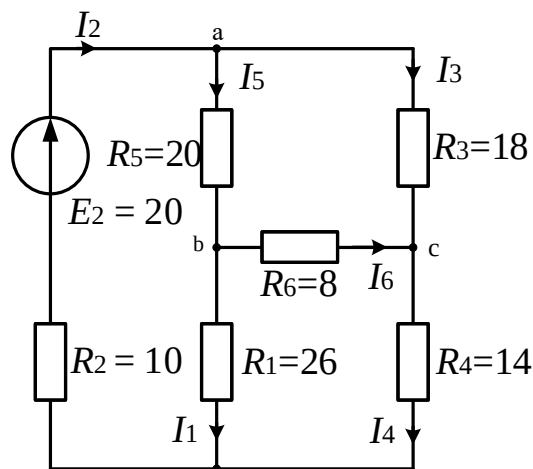


Fig. 6.4

### Solution.

The given circuit is a simple one which can be analyzed by the Method of Direct Application of Ohm's law, but first we should define equivalent resistance of the circuit relative to the terminals of the EMF source.

There are neither series connections nor parallel ones here. The connections  $R_3, R_5, R_6$  and  $R_1, R_6, R_4$  form a Delta-connection and the connections  $R_5, R_6, R_1$  and  $R_3, R_6, R_4$  form a Star-connection.

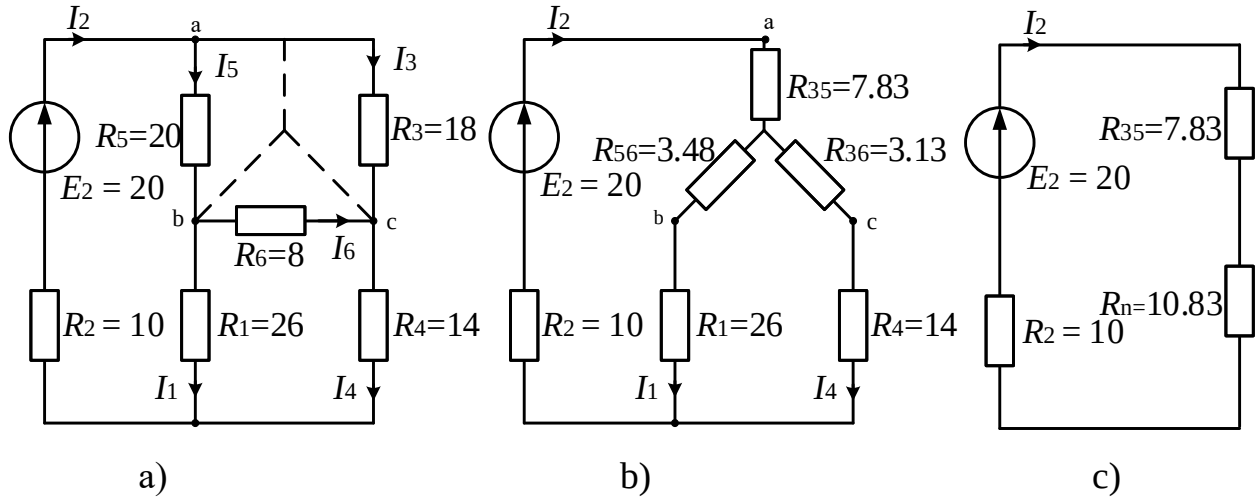


Fig. 6.5

By replacing the  $\Delta$ -connection of resistances  $R_3, R_5, R_6$  that forms the bridge (Fig. 6.5, a) with equivalent Y-connection, we obtain the new configuration of the given circuit (Fig. 6.5, b).

$$R_{35} = \frac{R_3 R_5}{R_3 + R_5 + R_6} = \frac{18 \cdot 20}{18 + 20 + 8} = 7.83 \Omega$$

$$R_{36} = \frac{R_3 R_6}{R_3 + R_5 + R_6} = \frac{18 \cdot 8}{18 + 20 + 8} = 3.13 \Omega$$

$$R_{56} = \frac{R_5 R_6}{R_3 + R_5 + R_6} = \frac{20 \cdot 8}{18 + 20 + 8} = 3.48 \Omega$$

The input circuit resistance (Fig. 6.5, c) with regard to the terminals of the EMF source is calculated as:

$$R_n = \frac{(R_1 + R_{56}) \cdot (R_4 + R_{36})}{R_1 + R_{36} + R_4 + R_{56}} = \frac{(26 + 3.48) \cdot (14 + 3.13)}{26 + 3.48 + 14 + 3.13} = 10.83 \Omega,$$

$$R_{\text{inp}} = R_2 + R_{35} + R_n = 10 + 7.83 + 10.83 = 28.66 \Omega.$$

The input current of the bridge scheme is

$$I_2 = \frac{E_2}{R_{inp}} = \frac{20}{28.66} = 0.698 \text{ A}.$$

The currents of the parallel branches (Fig. 1, b) are calculated as:

$$I_1 = \frac{I_2 \cdot (R + R_{36})}{R_1 + R_{36} + R_4 + R_{56}} = \frac{0.698 \cdot (14 + 3.13)}{26 + 3.48 + 14 + 3.13} = 0.256 \text{ A};$$

$$I_4 = I_2 - I_1 = 0.698 - 0.256 = 0.44 \text{ A}$$

To calculate the currents  $I_3, I_5, I_6$  through the branches of the bridge we can use Ohm's law. First we find the voltages  $U_{ab}, U_{bc}$ :

$$U_{ab} = I_2 \cdot R_{35} + I_4 \cdot R_{36} = 0.698 \cdot 7.89 + 0.44 \cdot 3.13 = 6.89 \text{ V},$$

$$U_{bc} = -I_1 \cdot R_{56} + I_4 \cdot R_{36} = -0.256 \cdot 3.48 + 0.44 \cdot 3.13 = 0.486 \text{ V}.$$

According to Ohm's law and first Kirchhoff's law, we obtain:

$$I_3 = \frac{U_{ab}}{R_3} = \frac{6.89}{18} = 0.382 \text{ A}, \quad I_6 = \frac{U_{bc}}{R_6} = \frac{0.486}{8} = 0.061 \text{ A}$$

$$I_5 = I_2 - I_3 = 0.698 - 0.382 = 0.316 \text{ A}.$$

To sum up:

$$I_1 = 0.256 \text{ A}, \quad I_2 = 0.698 \text{ A}, \quad I_3 = 0.382 \text{ A}, \quad I_4 = 0.44 \text{ A}$$

$$I_5 = 0.316 \text{ A}, \quad I_6 = 0.061 \text{ A}$$

The power balance equation is as follows:

$$E_2 \cdot I_2 = I_1^2 \cdot R_1 + I_2^2 \cdot R_2 + I_3^2 \cdot R_3 + I_4^2 \cdot R_4 + I_5^2 \cdot R_5 + I_6^2 \cdot R_6$$

$$20 \cdot 0.698 = 0.256 \cdot 2.26 + 0.698 \cdot 2.1 + 0.382 \cdot 2.18 + 0.44 \cdot 2.14 + 0.315 \cdot 2.2 + 0.061 \cdot 2.8$$

$$13.96 \text{ Wt} \cong 13.94 \text{ Wt}.$$

### Problem 6.3.

Find all currents while substituting the resistance triangle  $R_4 - R_5 - R_6$  with an equivalent star.

$$E_1 = 50 \text{ V}, E_2 = 30 \text{ V}, E_3 = 100 \text{ V},$$

$$R_1 = 3,5 \Omega, R_2 = 2 \Omega, R_3 = 7 \Omega,$$

$$R_4 = 6 \Omega, R_5 = 6 \Omega, R_6 = 12 \Omega.$$

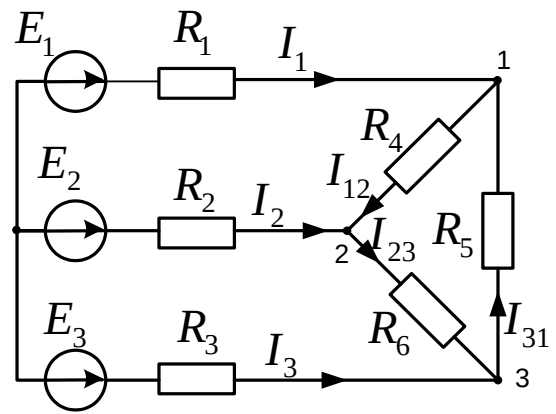


Fig. 6.5

**Solution.**

It is known that any mechanical structure can operate by an electric motor. Supposing an electrical circuit is a simplified example where a three-phase generator through a three-phase power line supplies a three-phase motor.

Firstly, we transform the delta  $R_4 - R_5 - R_6$  into star  $R_{45} - R_{46} - R_{65}$  and, by using the formulas of transformation, obtain the resistance of equivalent star:

$$R_{45} = \frac{R_4 \cdot R_5}{R_4 + R_5 + R_6} = \frac{6 \cdot 6}{6 + 6 + 12} = 1,5 \Omega$$

$$R_{46} = \frac{R_4 \cdot R_6}{R_4 + R_5 + R_6} = \frac{6 \cdot 12}{6 + 6 + 12} = 3 \Omega$$

$$R_{65} = \frac{R_6 \cdot R_5}{R_4 + R_5 + R_6} = \frac{12 \cdot 6}{6 + 6 + 12} = 3 \Omega$$

The result of a delta-star transformation is shown in Fig. 6.5.

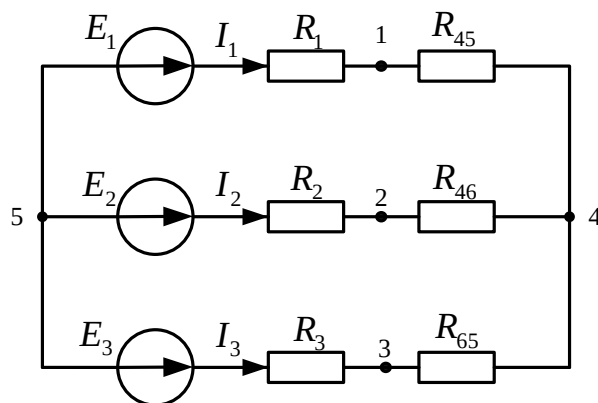


Fig. 6.5

Three branches are connected in parallel, and we can apply the node potential method to find the currents (fig. 6.5).

Let us assume  $\varphi_5 = 0$ , then the nodal equation at node  $\varphi_4$  is as follows:

$$\varphi_4 \left( \frac{1}{R_1 + R_{45}} + \frac{1}{R_2 + R_{46}} + \frac{1}{R_3 + R_{65}} \right) = \frac{E_1}{R_1 + R_{45}} + \frac{E_2}{R_2 + R_{46}} + \frac{E_3}{R_3 + R_{65}} \Rightarrow$$

$$\varphi_4 = \varphi_4 - \varphi_5 = U_{45} = \frac{\frac{E_1}{R_1 + R_{45}} + \frac{E_2}{R_2 + R_{46}} + \frac{E_3}{R_3 + R_{65}}}{\frac{1}{R_1 + R_{45}} + \frac{1}{R_2 + R_{46}} + \frac{1}{R_3 + R_{65}}} = \frac{J_{44}}{G_{44}},$$

$$\text{where } G_{44} = \frac{1}{R_1 + R_{45}} + \frac{1}{R_2 + R_{46}} + \frac{1}{R_3 + R_{65}} = \frac{1}{3,5 + 1,5} + \frac{1}{2 + 3} + \frac{1}{7 + 3} = 0,5$$

$$J_{44} = \frac{E_1}{R_1 + R_{45}} + \frac{E_2}{R_2 + R_{46}} + \frac{E_3}{R_3 + R_{65}} = \frac{50}{3,5 + 1,5} + \frac{30}{2 + 3} + \frac{100}{7 + 3} = 26$$

$$\varphi_4 = \frac{J_{44}}{G_{44}} = \frac{26}{0,5} = 52 \text{ V}.$$

The branch currents are calculated by Ohm's law:

$$I_1 = \frac{\varphi_5 - \varphi_4 + E_1}{R_1 + R_{45}} = \frac{0 - 52 + 50}{5} = -0,4 \text{ A},$$

$$I_2 = \frac{\varphi_5 - \varphi_4 + E_2}{R_2 + R_{46}} = \frac{0 - 52 + 30}{5} = -4,4 \text{ A},$$

$$I_3 = \frac{\varphi_5 - \varphi_4 + E_3}{R_3 + R_{65}} = \frac{0 - 52 + 100}{10} = 4,8 \text{ A}.$$

$$I_{12} = \frac{I_1 R_{45} - I_2 R_{46}}{R_4} = \frac{-0,4 \cdot 1,5 + 4,4 \cdot 3}{6} = 2,1 \text{ A}$$

$$I_{23} = \frac{I_2 R_{46} - I_3 R_{65}}{R_6} = \frac{-4,4 \cdot 3 - 4,8 \cdot 3}{12} = -2,3 \text{ A}$$

$$I_{31} = \frac{I_3 R_{65} - I_1 R_{45}}{R_5} = \frac{4,8 \cdot 3 + 0,4 \cdot 1,5}{6} = 2,5 \text{ A}$$

The power balance equation is:

$$E_1 I_1 + E_2 I_2 + E_3 I_3 = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_{12}^2 R_4 + I_{23}^2 R_6 + I_{31}^2 R_5$$

The total power of sources is calculated as:

$$P_G = E_1 I_1 + E_2 I_2 + E_3 I_3 = 50 \cdot (-0,4) + 30 \cdot (-4,4) + 100 \cdot 4,8 = 328 \text{ W}$$

The total power consumption is equal to:

$$\begin{aligned} P_C &= I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_{12}^2 R_4 + I_{23}^2 R_6 + I_{31}^2 R_5 = \\ &= 0,4^2 \cdot 3,5 + 4,4^2 \cdot 2 + 4,8^2 \cdot 7 + 2,1^2 \cdot 6 + 2,3^2 \cdot 12 + 2,5^2 \cdot 6 = 328 \text{ W}. \end{aligned}$$

If  $P_G = P_C$  , the problem is solved correctly.

## 7. THE SUPERPOSITION METHOD

### Problem 7.1

Let us consider the familiar two-source, two-resistor circuit one more time (fig. 7.1, a). First of all, we try superposition to find  $I_1$ ,  $I_2$ ,  $I_3$ ,  $U_{R1}$ ,  $U_{R2}$ .

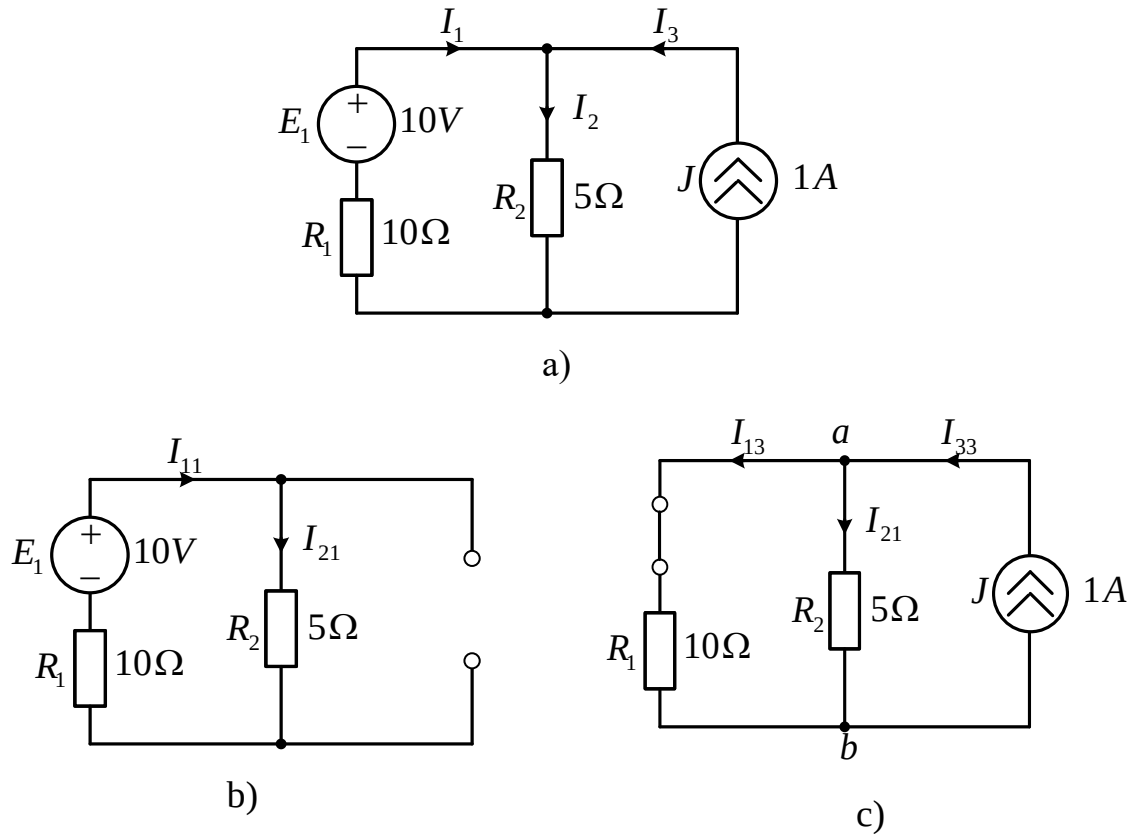


Fig. 7.1

**Step 1.** We set the positive directions of actual currents (see Fig. 7.1, a).

**Step 2.** Now we are to calculate currents in all branches of the given circuit from each energy source separately.

For this purpose, we should deactivate (turn off) one of the sources – the order does not matter.

**Firstly**, we deactivate the current source – set the value of  $J$  to zero, which has the same effect as replacing  $J$  with an open circuit (Fig. 7.1, b). The result is a simple voltage divider, and we can easily calculate the partial results using Ohm's law.

We indicate a partial current in the 1st branch as  $I_{11}$  and a particular current in the 2nd branch as  $I_{21}$ , when EMF acts only in the 1st branch. So, we obtain:

$$I_{11} = I_{21} = \frac{E_1}{R_1 + R_2} = \frac{10}{10 + 5} = 0,667 \text{ A},$$

$$I_{31} = 0$$

Then we indicate a particular voltage across  $R_1$  as  $U_{11}$  and a particular voltage across  $R_2$  as  $U_{21}$ , when EMF acts only in the 1st branch. Thus, we obtain:

$$U_{11} = I_{11} \cdot R_1 = 0,667 \cdot 10 = 6.67 \text{ V},$$

$$U_{21} = I_{21} \cdot R_2 = 0,667 \cdot 5 = 3.33 \text{ V}.$$

**Secondly**, we go back to the original circuit and turn off the other source – set  $E_1$  to zero, which is the same as replacing it with a short circuit (Fig. 8.1, c). To calculate these partial currents, it is convenient to use Ohm's law and equivalent transformations of circuit. Shorting  $E$  causes  $R_1$  to be in parallel with  $R_2$ .

We indicate a partial current in the 1st branch as  $I_{13}$  and a partial current in the 2nd branch as  $I_{23}$ , when current source acts only in the 3rd branch. So, we have:

$$I_{13} = \frac{U_{ab}}{R_1} = \left( J \cdot \frac{R_1 R_2}{R_1 + R_2} \right) \frac{1}{R_1} = J \cdot \frac{R_2}{R_2 + R_1} = 1 \cdot \frac{5}{10 + 5} = 0.333 \text{ A},$$

$$I_{23} = \frac{U_{ab}}{R_2} = \left( J \cdot \frac{R_1 R_2}{R_1 + R_2} \right) \frac{1}{R_2} = J \cdot \frac{R_1}{R_2 + R_1} = 1 \cdot \frac{10}{10 + 5} = 0.667 \text{ A},$$

$$I_{33} = J.$$

Next we indicate particular voltages across  $R_1$  as  $U_{13}$  and partial voltages across  $R_2$  as  $U_{23}$  when the current source acts only in the 3rd branch, and calculate:

$$U_{13} = I_{13} \cdot R_1 = 0,333 \cdot 10 = 3.33 \text{ V},$$

$$U_{23} = I_{23} \cdot R_2 = 0,667 \cdot 5 = 3.33 \text{ V}.$$

**Step 3.** The complete answer is the algebraic sum (superposition) of the two partial answers.

$$I_1 = I_{11} - I_{13} = 0.667 - 0.333 = 0.334 \text{ A},$$

$$I_2 = I_{31} + I_{32} = 0.667 + 0.667 = 1.334 \text{ A},$$

$$I_3 = J = 1 \text{ A},$$

$$U_{R1} = U_{11} - U_{13} = 6.67 - 3.33 = 3.34 \text{ V},$$

$$U_{R2} = U_{21} + U_{23} = 3.33 + 3.33 = 6.66 \text{ V}.$$

### Problem 7.2

For the circuit shown in Fig. 8.2, use superposition to find the power which is being dissipated in  $R_3$ .

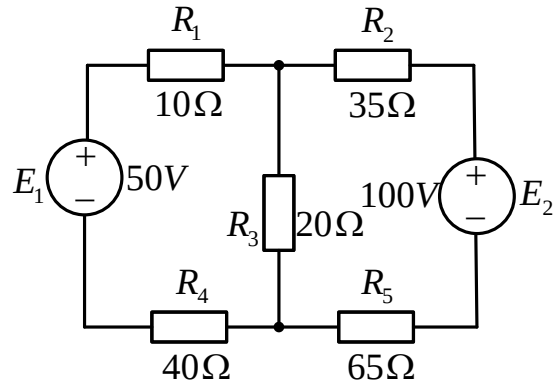


Fig. 7.2

By using superposition, we cannot find “partial powers”: we should find either a total voltage or a total current and then calculate the power. So, we find  $U_3$ .

**Step 1.** Start by deactivating  $E_2$  (Fig. 7.3, a).

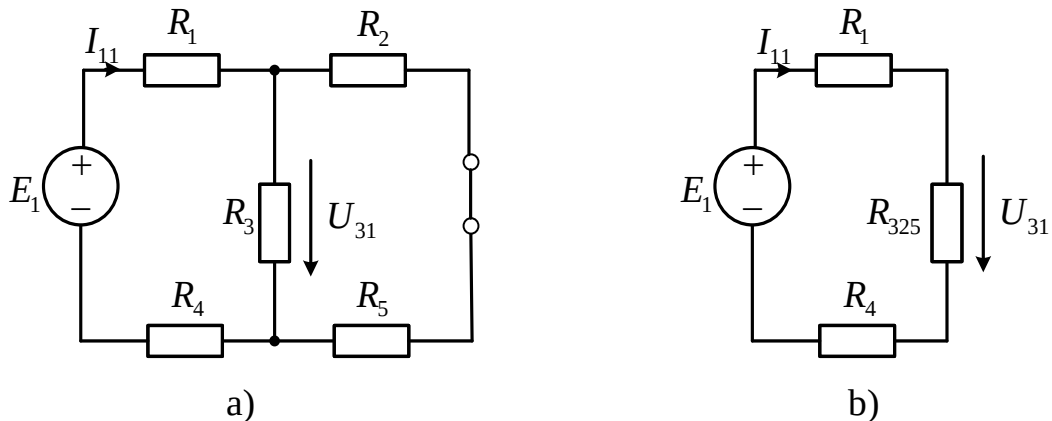


Fig. 7.3

We note that the series combination of  $R_2$  and  $R_5$  is in parallel with  $R_3$ .

$$R_{325} = \frac{R_3(R_2 + R_5)}{R_3 + R_2 + R_5} = \frac{20(35 + 60)}{20 + 35 + 60} = 16.67\Omega.$$

According to Ohm's law, the partial voltage across  $R_3$  when  $E_1$  acts is equal to:

$$U_{31} = I_{11}R_{325} = \left( \frac{E_1}{R_{325} + R_1 + R_4} \right) R_{325} = \frac{50}{16.67 + 10 + 40} 16.67 = 12.5V.$$

**Step 2.** Return to the original circuit and deactivate  $E_1$ . Note that with  $E_1$  is shorted, and the series combination of  $R_1$  and  $R_4$  is in parallel with  $R_3$ .

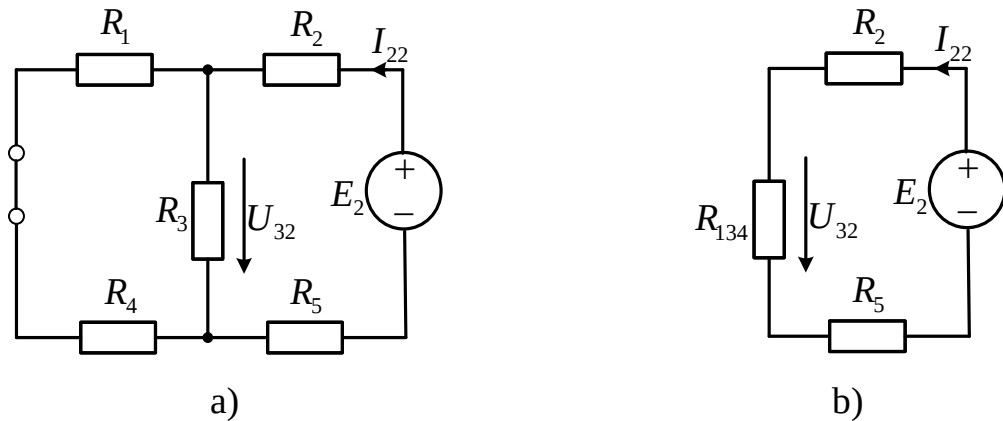


Fig. 7.4

$$R_{134} = \frac{R_3(R_1 + R_4)}{R_3 + R_1 + R_4} = \frac{20(10 + 40)}{20 + 10 + 40} = 14.29\Omega .$$

According to Ohm's law, the particular voltage across  $R_3$  when  $E_2$  acts is equal to:

$$U_{32} = I_{22}R_{134} = \left( \frac{E_2}{R_{134} + R_2 + R_5} \right) R_{134} = \left( \frac{100}{14.29 + 35 + 65} \right) 14.29 = 12.5V .$$

**Step 3.** The complete solution is the sum of the two partial answers:

$$U_3 = U_{31} + U_{32} = 12.5 + 12.5 = 25V$$

And finally,

$$P_{R_3} = \frac{U_3^2}{R_3} = \frac{25^2}{20} = 31.25W .$$

### Problem 7.3

Find the branch currents of the following circuit using **the** superposition method if the following parameters are known:

$$E_1 = 120 \text{ V}, E_4 = 45 \text{ V},$$

$$R_1 = 20 \Omega, R_2 = 30 \Omega,$$

$$R_3 = 15 \Omega, R_5 = 60 \Omega.$$

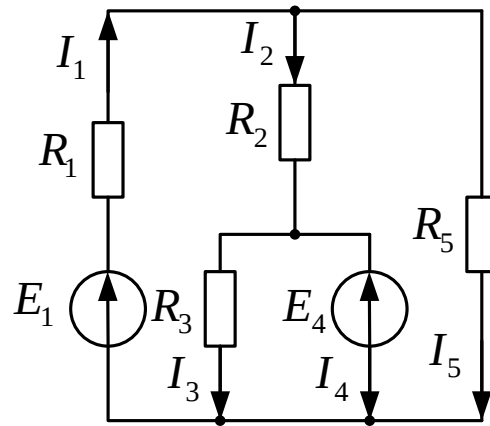


Fig. 7.5

### Solution.

First, we assume the positive direction for each branch current. The circuit has two energy sources, so according to the superposition principle, we analyze each energy source separately.

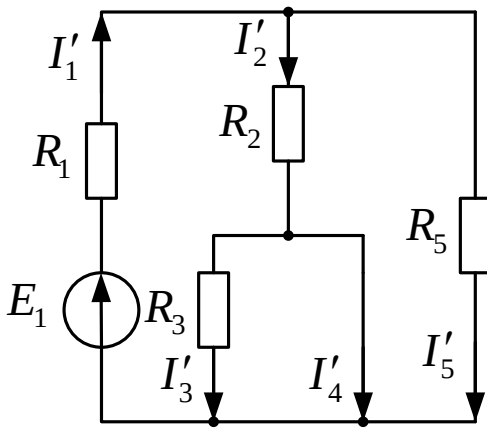


Fig. 7.6, a

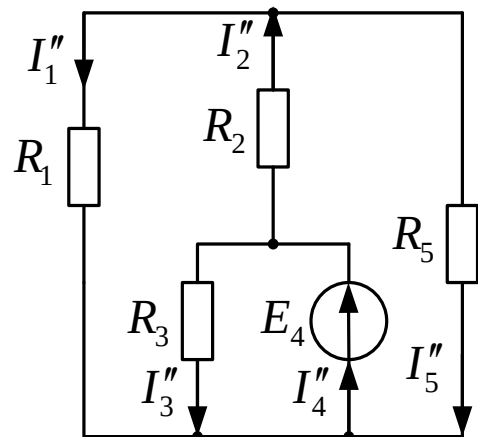


Fig. 7.6, b

We deactivate the  $E_4$  - set the value of  $E_4$  to zero, which has the same effect as replacing  $E_4$  with the short circuit (Fig. 7.6, a). The result is a simple voltage divider, and we can easily calculate the partial results using Ohm's law.

Then we indicate and calculate a partial current in each branch, when EMF acts only in the 1st branch.

After deactivating,  $E_4$  forms the short circuit, so  $R_{34} = 0$ ,  $I'_3 = 0$  and  $I'_4 = I'_2$ .

The equivalent resistance  $R_{eq}'$  is calculated as:

$$R_{eq} = R_1 + \frac{R_2 \cdot R_5}{R_2 + R_5} = 20 + \frac{30 \cdot 60}{30 + 60} = 40 \Omega.$$

The partial current  $I'_1$  is calculated by Ohm's law:

$$I'_1 = \frac{E_1}{R_{eq}} = \frac{120}{40} = 3 \text{ A}.$$

The current  $I'_2$  is determined by means of the current dispersion rule:

$$I'_2 = I'_1 \frac{R_5}{R_2 + R_5} = 3 \frac{60}{30 + 60} = 2 \text{ A}.$$

The current  $I'_5$  is determined by means of Kirchhoff's current law:

$$I'_5 = I'_1 - I'_2 = 3 - 2 = 1 \text{ A}.$$

Then we return to the original circuit and turn off the other source: we set  $E_1$  to zero, which is the same as replacing it with a short circuit (Fig. 7.6, b). To calculate these partial currents, it is convenient to use Ohm's law and equivalent transformations of a circuit.

The resistors  $R_1, R_5$  and  $(R_2, R_{54})$  are joined in parallel. Their equivalent resistance is equal to:

$$R_{15} = \frac{R_1 \cdot R_5}{R_1 + R_5} = \frac{20 \cdot 60}{20 + 60} = 15 \Omega.$$

Then the resistors  $R_6$  and  $R_2$  are connected in series and this connection can be replaced with one:

$$R_{15,2} = R_{15} + R_2 = 15 + 30 = 45 \Omega.$$

The equivalent resistance of the partial circuit is equal to:

$$R_{eq} = \frac{R_3 \cdot R_{15,2}}{R_3 + R_{15,2}} = \frac{15 \cdot 45}{15 + 45} = 7,5 \Omega.$$

The partial current  $I''_4$  is calculated by Ohm's law:  $I''_4 = \frac{E_4}{R_{eq}} = \frac{45}{7,5} = 4 \text{ A}.$

The partial current  $I_3''$  is calculated by Ohm's law, too:  $I_3'' = \frac{E_4}{R_3} = \frac{45}{15} = 3 \text{ A}$ .

The current  $I_2''$  is determined by means of Kirchhoff's current law:

$$I_2'' = I_4'' - I_3'' = 3 - 2 = 1 \text{ A}.$$

The currents  $I_1''$ ,  $I_5''$  are determined by means of the current division rule:

$$I_1'' = I_2'' \frac{R_5}{R_1 + R_5} = 1 \frac{60}{20 + 60} = 0,75 \text{ A}, \quad I_5'' = I_2'' \frac{R_1}{R_1 + R_5} = 1 \frac{20}{20 + 60} = 0,25 \text{ A}.$$

In accordance with the superposition principle, we define the currents in the branches as the appropriate partial currents produced by a separate energy source, while taking into account their directions:

$$I_1 = I_1' - I_1'' = 3 - 0,75 = 2,25 \text{ A},$$

$$I_2 = I_2' - I_2'' = 2 - 1 = 1 \text{ A},$$

$$I_3 = I_3' + I_3'' = 0 + 3 = 3 \text{ A},$$

$$I_4 = I_4' - I_4'' = 2 - 4 = -2 \text{ A},$$

$$I_5 = I_5' + I_5'' = 1 + 0,25 = 1,25 \text{ A}.$$

#### Problem 7.4

Determine the branch current in the given circuit (Fig. 7.7), if

$$E = 16 \text{ V}, \quad J = 1 \text{ A},$$

$$R_1 = R_2 = R_3 = R_4 = R_5 = 6 \Omega.$$

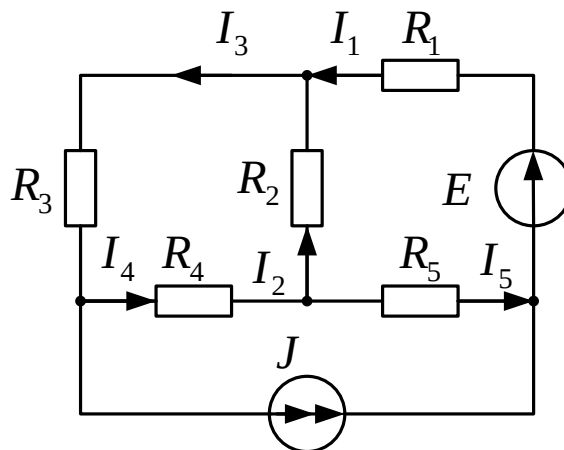


Fig. 7.7

### Solution.

First of all, we assume the positive direction of the branch currents in the given circuit (see Fig. 7.7) and deactivate the current source: we set the value of  $J$  to zero, which has the same effect as replacing  $J$  with an open circuit (fig. 7.8).

We determine the equivalent resistance  $R_{eq}$  as:

$$R_{eq} = R_1 + R_5 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4} =$$

$$= 6 + 6 + \frac{6(6 + 6)}{6 + 6 + 6} = 16\Omega$$

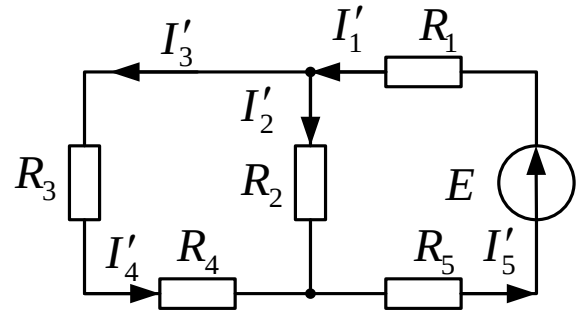


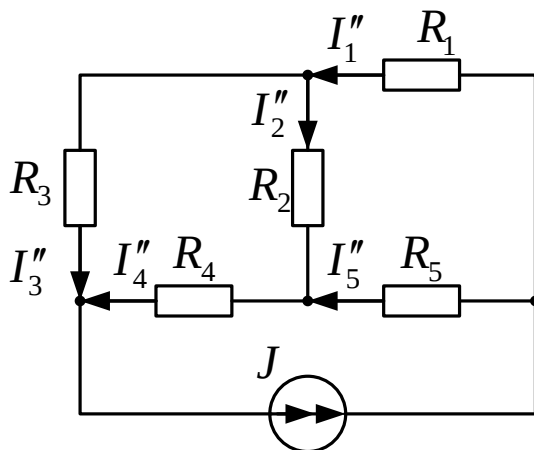
Fig. 7.8

Next we determine the partial branch currents:

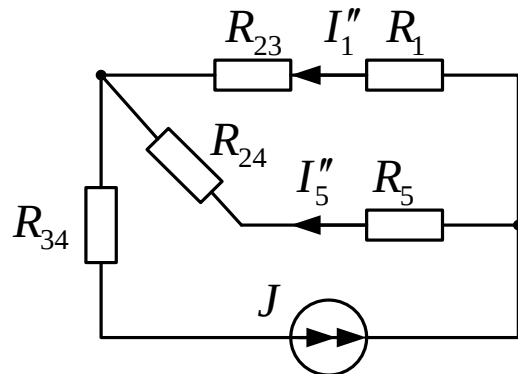
$$I'_1 = I'_5 = \frac{E}{R_{eq}} = \frac{16}{16} = 1\text{A}, \quad I'_2 = I'_1 \frac{R_3 + R_4}{R_2 + R_3 + R_4} = 1 \frac{6 + 6}{6 + 6 + 6} = 0,667\text{A},$$

$$I'_3 = I'_4 = I'_1 - I'_2 = 1 - 0,667 = 0,333\text{A}.$$

Secondly, we deactivate the other source: set  $E$  to zero, which is the same as replacing it with a short circuit (Fig. 8.1, c).



a



b

Fig.7.9

To determine the currents in the partial circuit (Fig. 7.9 b), we transform the resistance triangle  $R_2 - R_3 - R_4$  into the equivalent star. Taking into account that the resistances of triangle  $R_2 - R_3 - R_4$  are the same, we obtain

$$R_{23} = R_{24} = R_{34} = \frac{R_2 \cdot R_3}{R_2 + R_3 + R_4} = 2 \Omega .$$

As a result of transformation, there is a circuit in Fig. 7.9 b, from where we find the currents  $I_1''$ ,  $I_5''$ :

$$I_1'' = J \frac{R_5 + R_{24}}{R_1 + R_{23} + R_5 + R_{24}} = 1 \frac{2 + 6}{2 + 6 + 2 + 6} = 0,5 \text{ A} ,$$

$$I_5'' = J - I_1'' = 1 - 0,5 = 0,5 \text{ A}$$

According to Kirchhoff's voltage law for a loop  $R_1 - R_2 - R_5$ , we determine the current  $I_2''$ :

$$I_1'' R_1 + I_2'' R_2 - I_5'' R_5 = 0 , \text{ from here } I_2'' = \frac{I_5'' R_5 - I_1'' R_1}{R_2} = \frac{0,5 \cdot 6 - 0,5 \cdot 6}{6} = 0 .$$

Using Kirchhoff's current law, we determine the other currents:

$$I_4'' = I_2'' + I_5'' = 0,5 \text{ A} , \quad I_3'' = I_1'' - I_2'' = 0,5 \text{ A} .$$

In accordance with the superposition principle, we determine the actual branch currents:

$$I_1 = I_1' + I_1'' = 1 + 0,5 = 1,5 \text{ A} ,$$

$$I_2 = -I_2' - I_2'' = -0,667 \text{ A} ,$$

$$I_3 = I_3' + I_3'' = 0,333 + 0,5 = 0,833 \text{ A} ,$$

$$I_4 = I_4' - I_4'' = 0,333 - 0,5 = -0,167 \text{ A} ,$$

$$I_5 = I_5' - I_5'' = 1 - 0,5 = 0,5 \text{ A} .$$

## 8. THEVENIN'S THEOREM AND EQUIVALENT GENERATOR METHOD

### Problem 8.1

Consider the circuit in Fig. 8.1. The task is to replace the given circuit with the Thevenin equivalent circuit.

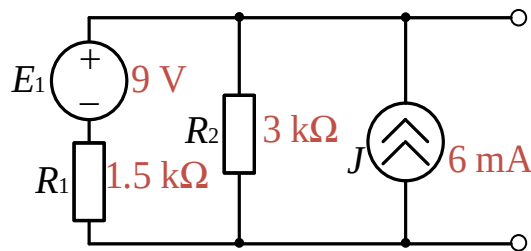


Fig. 8.1

**Step 1.** Determine the open-circuit voltage using whatever method you prefer. Use the node voltage in this case.

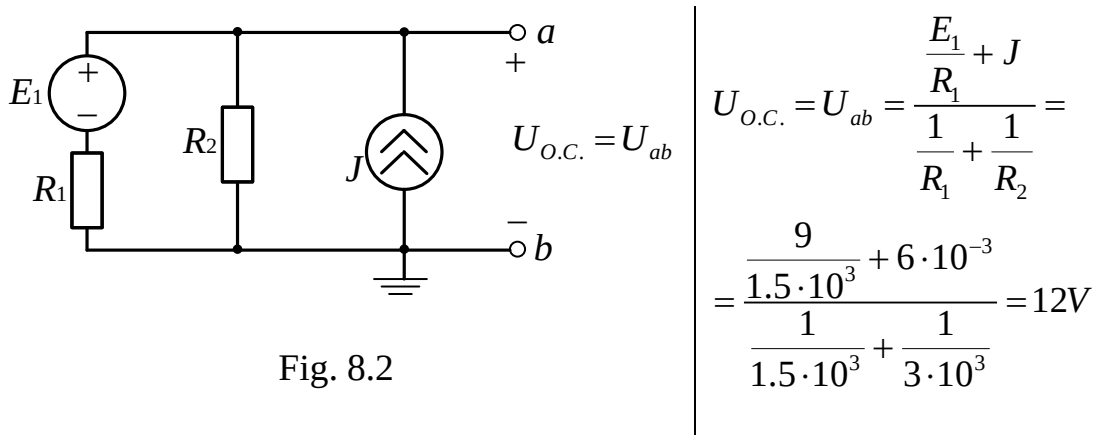


Fig. 8.2

$$E_{Th} = U_{O.C.} = 12V$$

**Step 2.** Determine the short-circuit current using whatever method you prefer. We decide to use Kirchhoff's first law, but carefully as the short circuit introduces some undesirable effects into the circuit analysis.

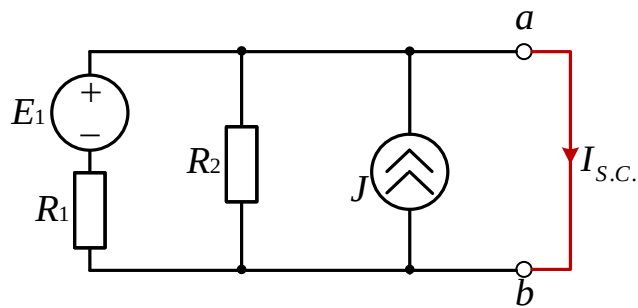


Fig. 8.3

a: because of the short circuit,

$$U_{ab} = 0;$$

b: because of the short circuit,

$$U_{R_2} = U_{ab} = 0 \text{ and } I_{R_2} = \frac{U_{R_2}}{R_2} = 0.$$

So  $R_2$  plays no role and can be removed.

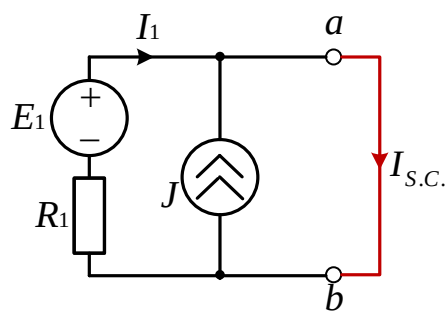


Fig. 8.4

$$I_{s.c.} = I_{R_1} + J = \frac{E_1}{R_1} + J =$$

$$= \frac{9}{1.5 \cdot 10^3} + 6 \cdot 10^{-3} = 12 \text{ mA}$$

As a result, we can calculate as:

$$R_{Th} = \frac{U_{O.C.}}{I_{s.c.}} = \frac{12 \text{ V}}{12 \cdot 10^{-3} \text{ A}} = 1 \text{ k}\Omega$$

And the given circuit can be replaced with Thevenin equivalent circuit:

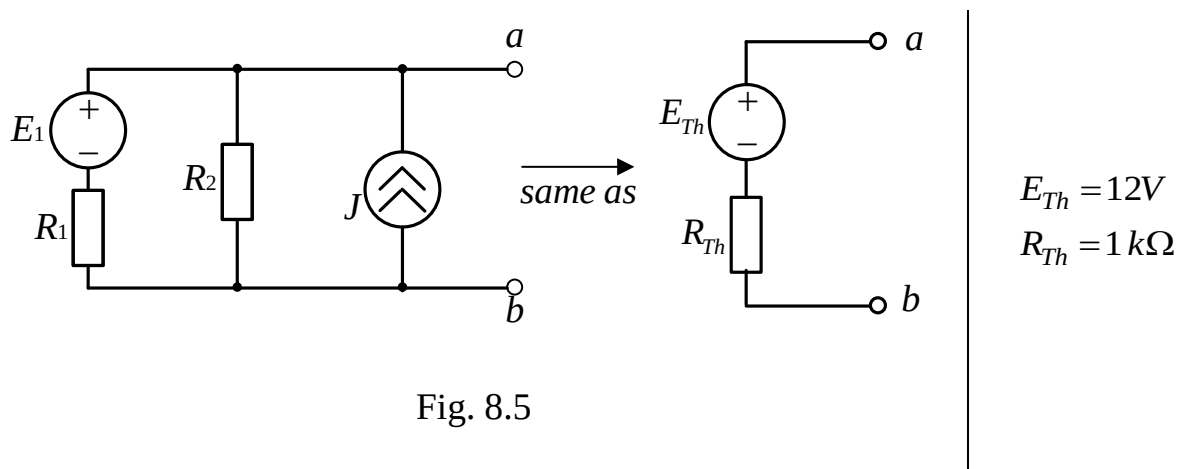


Fig. 8.5

*The alternative method for  $R_{Th}$ .* If the circuit consists of independent sources and resistors only, then the Thevenin resistance can also be found by de-activating the independent sources and finding the equivalent resistance regarding the output terminals.

De-activating the sources:

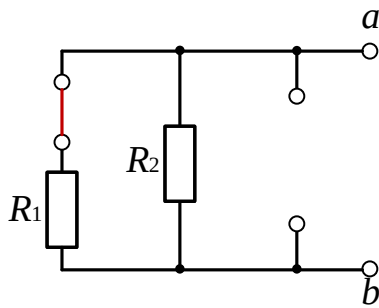


Fig. 8.6

$$R_{Th} = R_{ab} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{1.5 \cdot 10^3 \cdot 3 \cdot 10^3}{1.5 \cdot 10^3 + 3 \cdot 10^3} = 1 \text{ k}\Omega$$

## Problem 8.2

Attach different load resistors varying from 1  $\Omega$  to 100  $\text{k}\Omega$  to the output terminals of the circuit in Fig. 8.1 (problem 8.1). For each load resistance, calculate the load voltage (as well as current and power) for each of the circuits (Fig. 8.7).

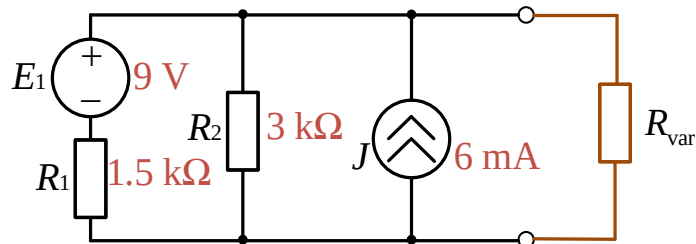


Fig. 8.7

We have already learned that the given circuit (Fig. 8.7) can be replaced with the Thevenin equivalent circuit and how to determine  $E_{Th}$  and  $R_{Th}$  which were calculated in problem 8.1. As a result, we have a circuit shown in Fig. 8.8, *b*.

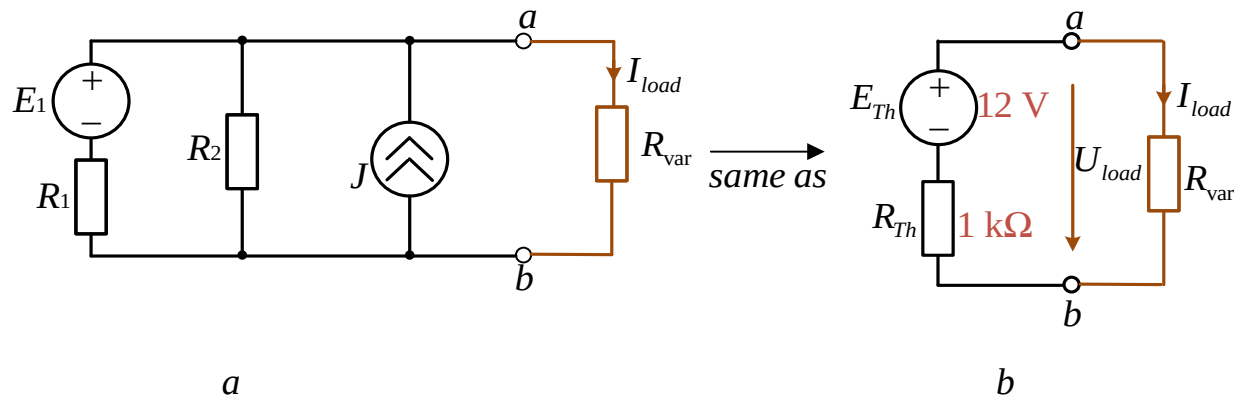


Fig. 8.8

Applying Ohm's law for the Thevenin equivalent circuit (Fig. 8.8, b), we can calculate the required values as:

- current through the load resistor  $I_{load} = \frac{E_{Th}}{R_{Th} + R_{var}};$
- load voltage across output terminals  $a-b$   $U_{load} = U_{ab} = I_{load} R_{var}$
- load power  $P_{load} = U_{load} I_{load} = I_{load}^2 \cdot R_{var}$

Table 8.1

| $R_{var} = R_{load}$ | $U_{load}$ | $I_{load}$ | $P_{load}$ |
|----------------------|------------|------------|------------|
| 1 $\Omega$           | 11.99 mV   | 11.99 mA   | 0.144 mW   |
| 10 $\Omega$          | 118.8 mV   | 11.88 mA   | 1.41 mW    |
| 100 $\Omega$         | 1.091 V    | 10.91 mA   | 11.90 mW   |
| 1 k $\Omega$         | 6.0 V      | 6.0 mA     | 36 mW      |
| 10 k $\Omega$        | 10.91 V    | 1.09 mA    | 11.9 mW    |
| 100 k $\Omega$       | 11.88 V    | 0.119 mA   | 1.41 mW    |

Now we can plot the graph of three functions  $U_{load}(R_{load})$ ,  $I_{load}(R_{load})$ ,  $P_{load}(R_{load})$ , using the table above, and based on these points, sketch the graph of these functions.

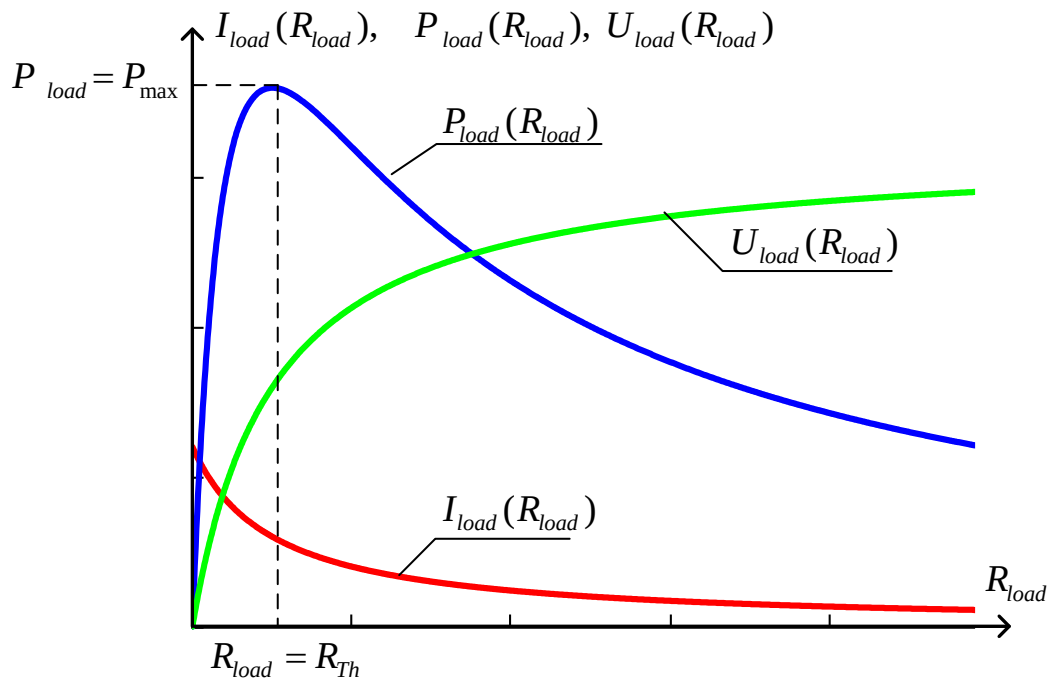


Fig. 8.9

The plot presents the data showing dependence of the voltage across the load, currents through the load, and the load power on the change of the resistance of this load.

As is seen from this graph, the increase in the load resistance leads to the load power increase, then it reaches the peak and slowly goes down.

The table and the graph show that the **Maximum Power Transfer** occurs in the load when the load resistance  $R_{load}$  is equal to the source resistance  $R_{Th}$ , i.e.  $R_{load} = R_{Th} = 1\text{ k}\Omega$ . This is called a “matched condition” and, generally, maximum power is transferred from an active device such as a power supply or battery to an external device when the impedance of the external device exactly matches the impedance of the source.

### Problem 8.3

Consider DC electric circuit (Fig. 8.10, a) with the following parameters of energy sources:  $E_1 = 240\text{ V}$ ,  $E_6 = 600\text{ V}$ , and the resistances:  $R_1 = 18\ \Omega$ ,  $R_2 = 120\ \Omega$ ,  $R_3 = 200\ \Omega$ ,  $R_4 = 200\ \Omega$ ,  $R_5 = 80\ \Omega$ . Determine the current  $I_1$  flowing through the branch with resistor  $R_1$ .

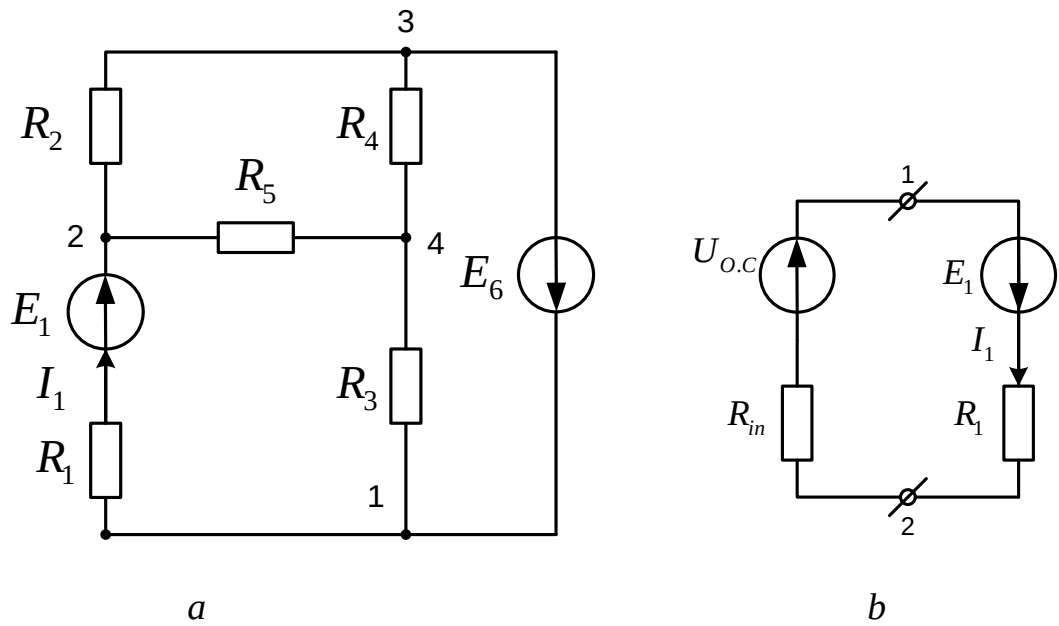


Fig. 8.10

**Solution.**

According to Thevenin's Theorem the current through  $R_1$  does not change if the circuit relative to nodes 1 and 2 is replaced with a simple equivalent circuit (named the Thevenin Equivalent Circuit), consisting of a single resistance in series with a voltage source (Fig. 8.10, *b*).

**Step 1.** Calculate  $E_{Th}$  and  $R_{Th}$ .

In order to determine  $E_{Th}$ , it is necessary to design an open circuit relative to nodes 1 and 2, assuming that  $R_1 \rightarrow \infty$  and  $I_1 = 0$  (Fig. 8.11).

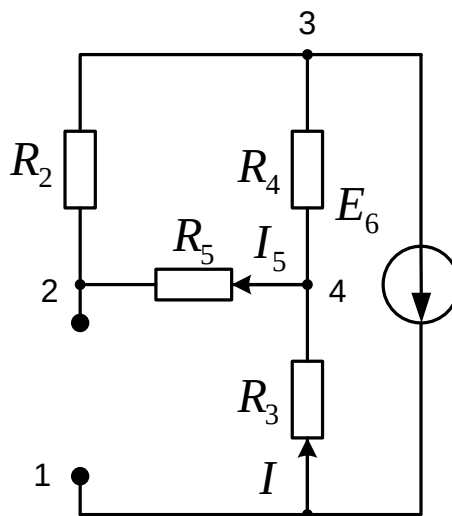


Fig. 8.11

Using any method, we determine the voltage across points 1 and 2  $U_{12}$ .

**Note:** After forming an open circuit, the points 1 and 2 are not the nodes. To find the voltage  $V_{12}$ , we calculate the currents  $I$  flowing through  $E_6$  and  $R_3$ , and  $I_5$  flowing through the  $R_2$  and  $R_3$ .

The equivalent resistance  $R_{eq}$  for the circuit shown in Fig. 8.11 regarding terminals 3 and 1 of a source EMF is given by:

$$R_{eq} = \frac{(R_2 + R_5)R_4}{R_2 + R_5 + R_4} + R_3 = \frac{(120 + 80) \cdot 200}{120 + 80 + 200} + 200 = 300 \Omega.$$

According to Ohm's law:

$$I = \frac{E_6}{R_{eq}} = \frac{600}{300} = 2 \text{ A}.$$

Using the rule for determining currents of a voltage divider, we find:

$$I_5 = I \frac{R_4}{R_2 + R_4 + R_5} = 2 \frac{200}{120 + 200 + 80} = 1 \text{ A}$$

The voltage  $U_{12}$  using Kirchhoff's voltage law is equal to:

$$U_{12} = R_5 I_5 + R_3 I = 80 \cdot 1 + 200 \cdot 2 = 480 \text{ V}.$$

The voltage  $U_{12}$  is an open-circuit voltage  $U_{O.C.}$  appearing across the terminals of the removed branch.

In order to determine  $R_{Th}$ , we deactivate the sources replacing EMF by a short circuit (Fig. 8.12) and calculate the equivalent resistance regarding the output terminals 1 and 2.

$$R_6 = \frac{R_3 \cdot R_4}{R_3 + R_4} + R_5 = \frac{200 \cdot 200}{200 + 200} + 80 = 180 \Omega,$$

$$R_{Th} = R_{in} = \frac{R_6 \cdot R_2}{R_6 + R_2} = \frac{180 \cdot 120}{180 + 120} = 72 \Omega.$$

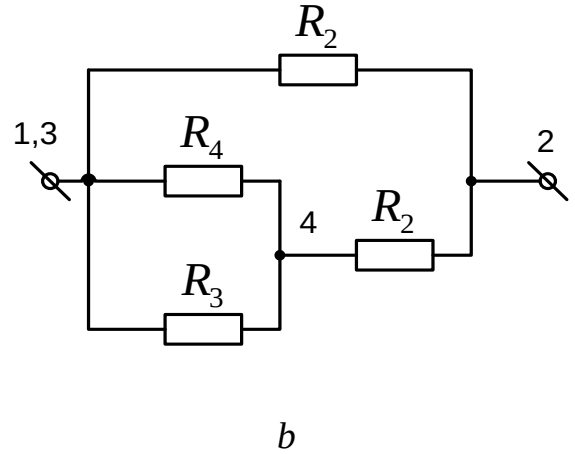
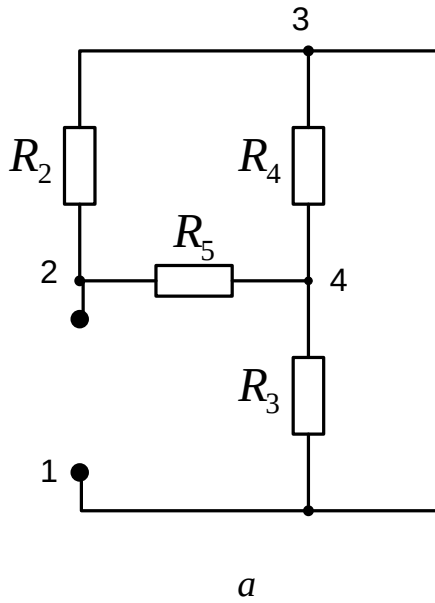


Fig. 8.12

**Step 2.** After replacing the given circuit by Thevenin Equivalent Circuit and determining its parameters (Fig. 8.13), we can find the current  $I_1$  using Ohm's law:

$$I_1 = \frac{U_{o.c.} + E_1}{R_{in} + R_1} = \frac{480 + 240}{72 + 18} = 8 \text{ A}$$

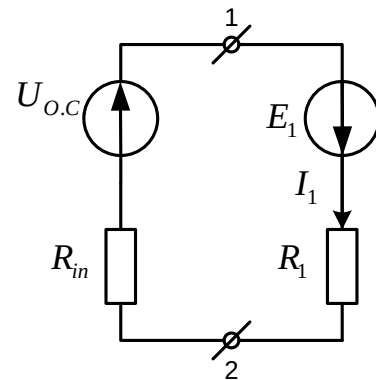


Fig. 8.13

#### Problem 8.4

The parameters of sources are:  $J = 5 \text{ A}$ ,  $E_1 = 25 \text{ V}$ ,  $E_2 = 120 \text{ V}$ ,  $E_5 = 60 \text{ V}$  and the resistances are  $R_1 = 10 \Omega$ ,  $R_3 = 25 \Omega$ ,  $R_4 = 15 \Omega$ ,  $R_5 = 30 \Omega$ . Define the current  $I_3$  in the branch with resistor  $R_3$ .

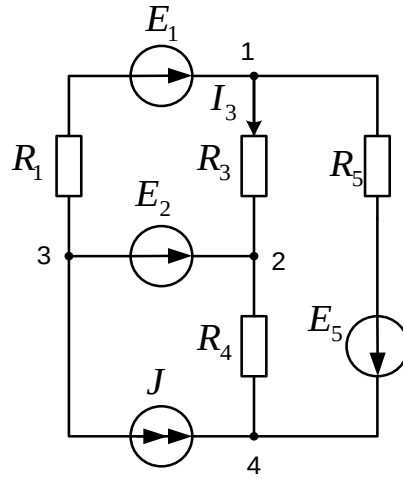


Fig. 8.14

**Solution.**

**Step 1.** Calculate the Thevenin Equivalent Circuit parameters  $-E_{Th}$  and  $R_{Th}$ .

In order to determine  $E_{Th}$ , we remove the branch with resistor  $R_3$  (Fig. 8.15).

The open circuit voltage across points 1 and 2 is equal to  $E_{Th}$

$$E_{Th} = U_{O.C.} = U_{12}$$

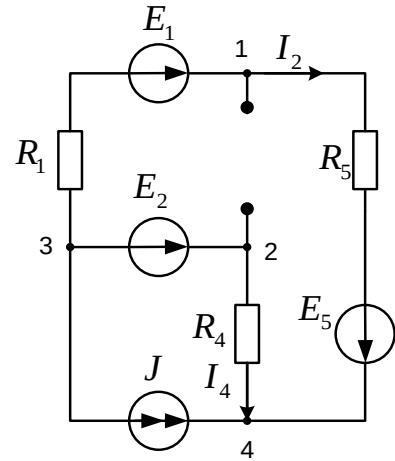


Fig. 8.15

To find the voltage  $U_{12}$  between points 1 and 2, we analyze the circuit in Fig. 8.15 using the nodal method:

$$\varphi_3 = 0$$

$$\varphi_4 \left( \frac{1}{R_4} + \frac{1}{R_1 + R_5} \right) = \frac{E_2}{R_4} + \frac{E_1 + E_5}{R_1 + R_5} + J \Rightarrow \varphi_4 = 165V$$

The branch currents are calculated as:

$$I_2 = \frac{\varphi_3 - \varphi_4 + E_1 + E_5}{R_1 + R_5} = -2A,$$

$$I_4 = \frac{\varphi_4 - \varphi_3 - E_2}{R_4} = -J - I_2 = -3\text{ A}.$$

The voltage  $U_{12}$  is found by Kirchhoff's voltage law:

$$U_{12} = R_5 I_2 - E_5 + R_4 I_4 = -75\text{ V}$$

or

$$U_{12} = E_1 - R_1 I_2 = -75\text{ V}$$

The voltage  $U_{12}$  is an open-circuit voltage  $U_{o.c.}$  appearing across the terminals of the removed branch with  $R_3$ .

In order to determine the internal resistance of the Thevenin Equivalent Circuit, we replace each source with its internal resistance ( $R_E = 0$ ,  $R_J \rightarrow \infty$ ), and the removed branch regarding points 1 and 2 remains as an open circuit (Fig. 8.16).

The internal resistance of the Thevenin Equivalent Circuit equals the equivalent resistance regarding the points 1 and 2 (Fig. 8.16).

$$\begin{aligned} R_{Th} = R_{in} &= \frac{R_1 \cdot (R_4 + R_5)}{R_1 + R_4 + R_5} = \\ &= \frac{10 \cdot (15 + 30)}{10 + 15 + 30} = \frac{450}{55} = 8,1818\Omega \end{aligned}$$

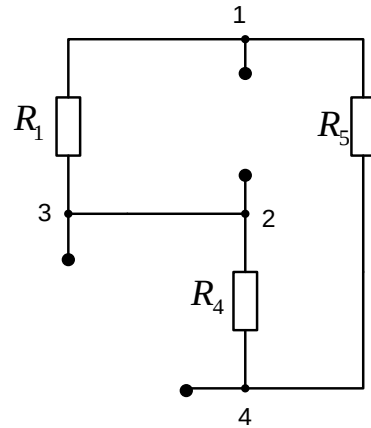


Fig. 8.16

**Step 2.** After replacing the given circuit with the Thevenin Equivalent Circuit and determining its parameters (Fig. 8.17), we find the current  $I_3$  using Ohm's law:

$$I_3 = \frac{U_{o.c.}}{R_{in} + R_3} = \frac{-75}{8,1818 + 25} = -2.26 \text{ A}$$

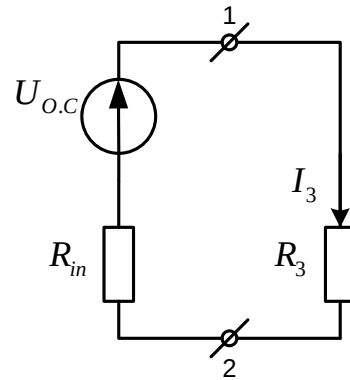


Fig. 8.17

#### Problem 8.4

The parameters of energy sources are:  $J = 10 \text{ A}$ ,  $E_2 = 20 \text{ V}$ ,  $E_3 = 10 \text{ V}$ ,  $E_6 = 10 \text{ V}$ , and the resistances are:  $R_1 = 20 \Omega$ ,  $R_2 = 10 \Omega$ ,  $R_4 = 5 \Omega$ ,  $R_5 = 5 \Omega$ ,  $R_6 = 15 \Omega$ . Determine the current  $I_5$  in the branch with the resistor  $R_5$ .

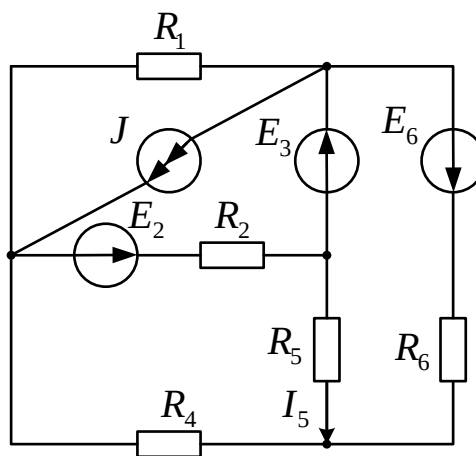


Fig. 8.18

**Solution.**

**Step 1.** Calculate the Thevenin Equivalent Circuit parameters  $-E_{Th}$  and  $R_{Th}$ .

To determine  $E_{Th}$ , we remove the branch with the resistor  $R_5$  (Fig. 8.19).

The open circuit voltage across points 1 and 2 is equal to  $E_{Th}$

$$E_{Th} = U_{O.C.} = U_{12}$$

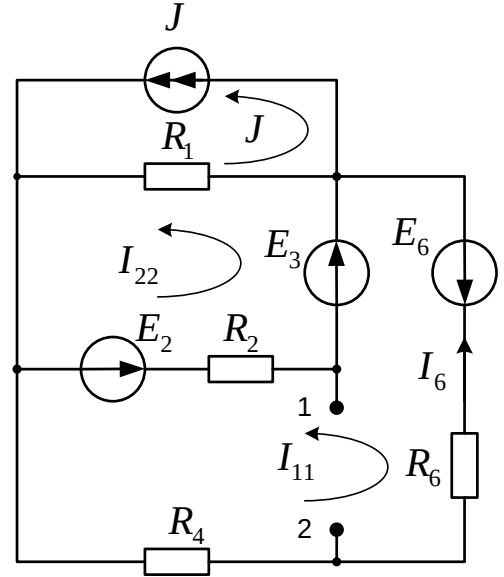


Fig. 8.19

To find the voltage  $U_{12}$  between points 1 and 2, we analyze the circuit in Fig. 8.19 and calculate the current  $I_6$  by applying the currents method.

$$\begin{cases} I_{11}(R_2 + R_4 + R_6) - I_{22}R_2 = -E_3 - E_2 - E_6, \\ -I_{11}R_2 + I_{22}(R_1 + R_2) - JR_1 = E_2 + E_3. \end{cases}$$

With the numerals:

$$\begin{cases} 30I_{11} - 10I_{22} = -40, \\ -10I_{11} + 30I_{22} = 30. \end{cases}$$

The mesh currents equal  $I_{11} = -1,125 \text{ A}$ ,  $I_{22} = 0,625 \text{ A}$

The branch current is  $I_6 = I_{11} = -1,125 \text{ A}$ ,  $I_2 = I_{11} - 5 = -5,1818 \text{ A}$ .

To find the voltage  $U_{12}$ , we use Kirchhoff's voltage law:

$$U_{12} = -E_3 - E_6 - R_6 I_6 = -10 - 10 + 15 \cdot 1,125 = -3,125 \text{ V}.$$

Voltage  $U_{12}$  is an open-circuit voltage  $U_{O.C.}$  appearing across the terminals of the removed branch.

In order to determine  $R_{Th}$ , we deactivate the sources replacing EMF with a short circuit and the current source with an open circuit (Fig. 8.20). Then we calculate the equivalent resistance regarding the output terminals 1 and 2.

$$R_7 = \frac{R_1 \cdot R_2}{R_1 + R_2} + R_4 =$$

$$= \frac{20 \cdot 10}{30} + 5 = 11,667 \Omega$$

$$R_{Th} = R_{in} = \frac{R_7 \cdot R_6}{R_7 + R_6} = 6,5626 \Omega .$$

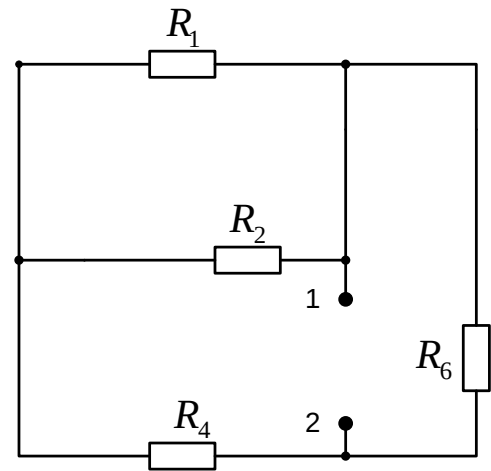


Fig. 8.20

**Step 2.** After replacing the given circuit with the Thevenin Equivalent Circuit and determining its parameters (Fig. 8.21), we can find the current  $I_3$  using Ohm's law:

$$I_5 = \frac{U_{O.C.}}{R_{in} + R_5} = \frac{-3,125}{6,5626 + 5} = -0,2703 \text{ A}$$

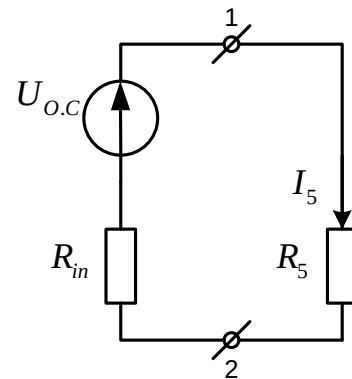


Fig. 8.21

## REFERENCES

1. Балла М.І. *Англо-український словник*. Київ: Освіта, 1996.
2. Бессонов Л.А. *Теоретические основы электротехники*. Москва: Высшая школа, 1988. 528 с.
3. Зевеке Г.В., Ионкин П.А., Нетушил А.В. и др. *Основы теории цепей*. Москва: Энергоатомиздат, 1987. 528 с.
4. Набокова О. В. *Теоретична електротехніка: Навчальний посібник* Запоріжжя: ЗНТУ, 2015. 477 с.
5. Atabekov G. I. *Linear Network Theory*. London, Great Britan: Pergamon Press, 1964. 635p.
6. Khilov V. S. *Theoretical Fundamentals of Electrical Engineering*. Dnipro, Ukraine: National Mining University. 2018. 467 p.
7. *Electronics Tutorials*: веб-сайт. URL: <https://www.electronics-tutorials.ws/>
8. Lessons-In-DC-Electrical-Circuits. Retrieved from <https://electrical-engineering-portal.com/res/Lessons-In-DC-Electrical-Circuits.pdf>