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THREE-MODEL METHODS FOR EXTREMUM EXTIMATION IN NON-LINEAR DYNAMIC SYSTEM IDENTIFICATION

Annotation: In this paper the properties and behaviour of the three-model approaches to determine the quality function maximum in the adaptive-searching identification methods are under research. The simple model of the relation between criterion and parameters is used and its correlation with real chaotic system is investigated. Advantages and drawbacks of the two extremum point estimation methods are considered. Approximate calculations to determine right identification system parameters is given.

Keywords: adaptive-searching identification, dynamic system identification, chaotic system, searching agents ensemble, extremum point.

Introduction

Onr of the most challenging task in the identification theory is identification of the non-linear dynamic systems, especially, systems with chaotic dynamics. There are many methods and approaches to solve this task. One of the most known ancestor of working approaches is adaptive-searching identification methods. However, sequential development of these methods, namely approaches, which uses multi-model techniques [1,2], allows to simultaneously improve both speed and precision of the identification.

Numerious researches shows the suitability and effectivnes of the identification methods, which based on ensemble of the searching agents. But, some probles require inversigation. First of all, searching tactics is quite obvious far from target point, but good behaviour near extremum is not achieved. Moreover, the good values of the parameters searching agent itself must be determined.

Problem definition

Every identification method with parralell models require adequate identification criterion [2]. In this paper, we assume, that such criterion (q) is received by previous reserch. It seems to be possible evven for chaotic dinamic systems. The precise form of such criterion may by very different. To receive identification simulation results, which is independent of particular dynamic system properties, the model of identification error, and the identification criterion is required.

In this paper the properties of three agents near quality function maximum is considered. The overoall behaviour of the identification system is not under consideration. Moreover, we will neglect the agents dinamic, and will consider only the possibility to determine extremum point near these three agents.

The main reason of the accuracy loss, is case of the good criterion, low noise and unlimited idetification time, is criterion non-symmetrical form. So, the model must have a uniformly controlled part, which describes such phenomena. In this paper simple representation will be used:

$$\Delta q = a_l \Delta p + a_a |\Delta p| + w(t), \tag{1}$$

where $\Delta p = p_m - p_o$ – parameter difference, p_o – object parameter, p_m – model (one of) parameter, Δq – criteria difference, a_l – model sensitivity to parameter change coefficient, a_a – coefficient, which describes unsymmetrical properties, $w(t)$ – measurement error (in this paper – Gaussian with σ_w).

Quality function F will be defined as usual in adaptive-searching identification:

$$F(\Delta q) = \exp\left(-\frac{\Delta q^2}{q_\gamma^2}\right), \tag{2}$$

where q_γ – sensitivity scale.

To explore properties of the extremum estimation methods near the extremum, we assume, that exist 3 models, each designated by indexes “l” (left), “r” (right) and “c” (central). There coefficients is given by equation:

$$p_l = p_c - A; \; p_r = p_c + A.$$

We assume, that $p_o \in [p_l; p_r]$, so this additinal restriction will be used. If any of the appreaches will give us other result, we artificially will limit the p_o value.

There are three approaches to extremum estimation, that was considered in [3]: global COG – “center of gravity”, local COG – the same, but with only 3 points near extremum, and QAE – “quadratic approximation near extremum”. As in this task we consider only 3 models, first and second approaches will be the same. In this case, estimated extremum point p_{ge} defined as:

$$p_{ge} = \frac{F_l p_l + F_c p_c + F_r p_r}{F_l + F_c + F_r}. \tag{3}$$

The “QAE” approach uses parabolic approximation of the function $F(p)$ near the extremum. The axis origin will be moved to the point (p_c, F_c). In this definitions:

$$\tilde{p}_c = 0, \; \tilde{p}_l = p_l - p_c, \; \tilde{p}_r = p_r - p_c.$$

$$\tilde{F}_c = 0, \; \tilde{F}_l = F_l - F_c, \; \tilde{F}_r = F_r - F_c.$$

$$\begin{cases} a_2\tilde{p}_l^2 + a_1\tilde{p}_l = \tilde{F}_l \\ a_2\tilde{p}_r^2 + a_1\tilde{p}_r = \tilde{F}_r \end{cases}.$$

$$a_1 = \frac{\tilde{F}_r\tilde{p}_l^2 - \tilde{F}_l\tilde{p}_r^2}{\tilde{p}_l^2\tilde{p}_r + \tilde{p}_l\tilde{p}_r^2}.$$

$$a_2 = \frac{\tilde{F}_r\tilde{p}_l - \tilde{F}_l\tilde{p}_r}{\tilde{p}_l^2\tilde{p}_r + \tilde{p}_l\tilde{p}_r^2}.$$

$$\tilde{p}_e = -\frac{a_1}{2a_2};\; p_{ee} = p_c - -\frac{a_1}{2a_2}. \tag{4}$$

Due to different disagreements between identification error model and real object properties, the value of p_{ee} may be not in range of $[p_l, p_r]$, or even $a_2 \geq 0$. In this situation the p_{ee} will be artificially limited by this range. Identification error we define as:

$$e_{ge} = p_{ge} - p_o,\; e_{ee} = p_{ee} - p_o. \tag{5}$$

Simulation results

To investigate properties of different approaches to extremum (identification error minimum) estimation, a series of simulations was carried out with the aid of the "qontrol" program. During each simulation, the p_o, p_c, A, q_γ values was set to fixed values, and RMS error values was calculated for all simulation time T:

$$\overline{e_{eg}} = \frac{1}{T}\sqrt{\int_0^T e_{eg}^2(t)dt}.$$

The main task is to determine the possible ranges of A and q_γ values. As a secondary task, the overall shape of functions $\overline{e_{ge}}(A, q_\gamma)$ and $\overline{e_{ee}}(A, q_\gamma)$ are under interest.

In fig. the results of simulation with $\overline{e_{eg}}$ calculation are represented. The model parameters was set by this way: $a_a = 0.2$, $p_o = -50$, $p_c = 0$, $A \in [50; 450]$, $q_\gamma \in [40; 4000]$. Three subplots corresponds different a_l values: 2.0, 5.0, 8.0.

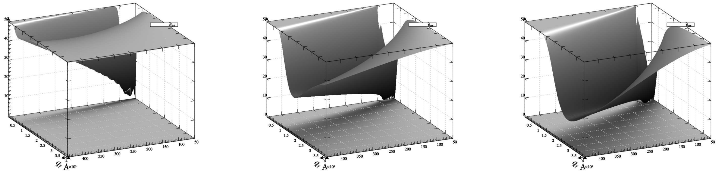

Figure 1 – Dependencies $\overline{e_{ge}}(A, q_\gamma)$ with different a_l values

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As we can see, the plot of the $\overline{e_{ge}}(A, q_\gamma)$ function forms a valley-like shape. As predicted, minimal errors is observed near $A \approx |p_o|$, and bottom part of the "valley" forms near a straight line in (A, q_γ) coordinates.

In fig. the results of simulation with the same conditions, but with $\overline{e_{ee}}$ calculation are represented.

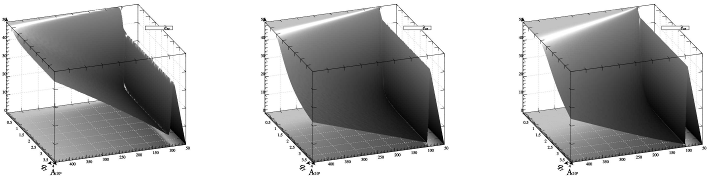

Figure 2 – Dependencies $\overline{e_{ee}}(A, q_\gamma)$ with different a_l values

Suddenly, these relationships have much more complex shape, than previous. First of all, we can observe flat linear rising part in the area, where $A \in [|p_o|, 2|p_o|]$. This is due to p_{ee} limitation – in such conditions $p_{ee} = p_l$, and error linear rises with A. As A rises more, the quadratic estimation begins to work, and give suitable results. Unobvios, but the (3) and (4) working areas practically not intersects.

To determine, is model (1) is suitable for complex dynamic system representation, the same simulations was carried out for well-known Lorenz system:

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = x(r - z) - y \\ \dot{z} = xy - bz \end{cases}, \qquad (6)$$

where x, y, z – system state variables, r, b, σ – parameters. Parameter r will is assumed as identification target, and in this paper $p = r$.

In the fig. the results of simulation are represented for the system (6).

Comparison between fig. , and shows the similarity of the functions shapes. Naturally, the shapes for the Lorenz system have many complex details, which is conditioned by the chaotic nature of this system. But, in general, approximation in (1) gives adequate results even for such complex system.

To determine relationships between suitable values of the parameters A and q_γ, we assume, that we can measure or at some extent estimate the values of a_l and a_a for the real system, and we can neglect changes of this values in the parameter working region.

As a first step, we conduct $\overline{e_{eg}}(A, q_\gamma)$ measurement while simulation, but in this case we adjust scales, to $A_{\max}/q_{\gamma\max}$ value be constant. The results are shown in fig. .

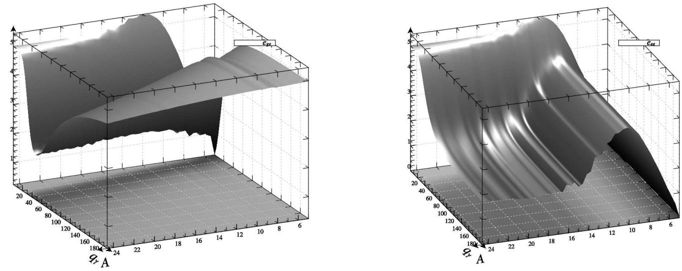

Figure 3 – Dependencies $\overline{e_{ge}}(A, q_\gamma)$ and $\overline{e_{ee}}(A, q_\gamma)$ for Lorenz system

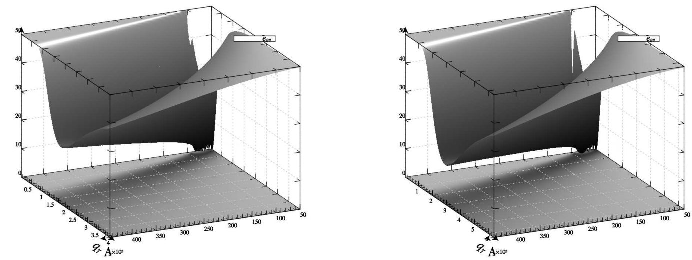

Figure 4 – Dependencies $\overline{e_{eg}}(A, q_\gamma)$ with different a_l values and fixed $A_{\max}/q_{\gamma\,\max}$ value

These simulation results shows, that in such conditions the angle of the "valley bottom" projection to the (A, q_γ) plane is near constant. After series of such simulations it can be possible to determine rough working q_γ value for the p_{ge} estimation:

$$q_\gamma \approx 0.86 A a_l. \tag{7}$$

One of the our tasks is to estimate suitable range of A values. Obviously, in the minimal measurement noise and precisely symmetrical shape of the $F(p)$ (i.e. $a_a = 0$), we can use nearly arbitrary A value. Otherwise, the working range will be limited. To determine appropriate A values, a new series of simulation was performed. In this simulations, the value of a_a parameter was varied. The results are shown in fig. 5.

The results indicates, that if the shape of $F(p)$ function has essential non-symmetrical shape, the suitable range of the "A" parameter is limited. In post cases the following restriction must be met: $A < 4|p_o - p_m|$. Real tasks will require more strict restrictions.

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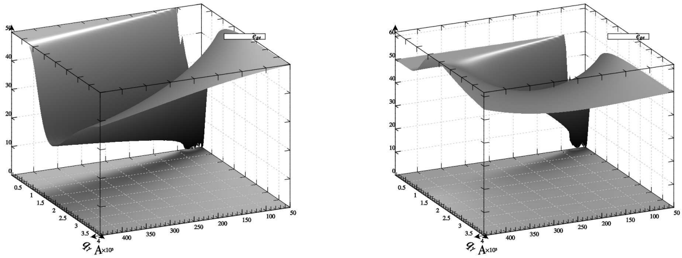

Figure 5 – Dependencies $\overline{e_{eg}}(A, q_\gamma)$ with different a_a values

Conclusions

The result of simulations allow us to made certain conclusions:

- The representation of the criterion and quality functions in form of (1) and (2) is adequate for quite complex systems, like Lorenz system.
- It is possible to use both COG (3) and QAE (4) approaches.The former method allows us to achieve results in tighter area in (A, q_γ) axis plane, but simple in realization and predictable in results. The late one, in spite on more wide working area, may lead to non-uniform results in some areas.
- If the results of preliminary simulations can provide values of a_l and a_a, it is possible to determine suitable values of identification system parameters itself, namely A and q_γ.
- The QAE appreach require further development.

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