EXISTENCE, UNIQUENESS SOLUTION AND INVARIANT MEASURE RESULTS FOR NEUTRAL FSDES IN HILBERT SPACES

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We study the large time behaviour of the solutions of neutral type stochastic functional-differential equations of the form

$$\begin{aligned} &d[u(t)+g(u_t)] = [Au + f(u_t)]dt + \sigma(u_t)dW(t) \text{ for } t>0; \qquad (1)\\ &u(t)=\varphi(t), t\in[-h,0),\ h>0. \end{aligned}$$

Here A is an infinitesimal generator of a strong continuous semigroup $\{S(t), t \geqslant 0\}$ of bounded linear operators in real separable Hilbert space H. The noise $W(t)$ is a Q-Wiener process on a separable Hilbert space K. For any $h>0$ denote $C_h := C([-h,0],H)$ to be a space of continuous H-valued functions $\varphi : [-h,0] \to H$, equipped with the norm

$$\|\varphi\|_{C_h} := \sup_{t\in[-h,0]} \|\varphi(t)\|_H,$$

where $\|\cdot\|_H$ stands for the norm in H. The functionals f and g map C_h to H, and $\sigma : C_h \to \mathcal{L}_2^0$, where $\mathcal{L}_2^0 = \mathcal{L}(Q^{1/2}K, H)$ is the space of Hilbert-Schmidt operators from $Q^{1/2}K$ to H. In our studies, the maps f and g do not satisfy the Lipschitz condition. Therefore, it is important for applications.

Finally, $\varphi : [-h,0]\times\Omega \to H$ is the initial condition, where $(\Omega, \mathcal{F}, P)$ is the probability space.

We study the existence and uniqueness of the solution to the initial problem without the Lipschitz condition. Then we establish the Markov and Feller properties in the shift spaces for such equations, and using the compactness approach we establish the existence of invariant measures in the shift spaces for such equations. The obtained abstract results are applied to stochastic partial differential equations of the reaction-diffusion type. The issue of approximate control of such equations is also studied.

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