NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
«IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE»
INSTITUTE OF PHYSICS AND TECHNOLOGY

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PHYSICS 1. MECHANICS

Problems

For specialties

113 APPLIED MATHEMATICS
125 CYBERSECURITY

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Contents

CHAPTER

0

Vectors

§ 0.1. Adding Vectors by Components

0.1. The position vector for an electron is $\vec{r} = (5.0\,m)\vec{i} + (3.0\,m)\vec{j} + (2.0\,m)\vec{k}$.

(a) Find the magnitude of $\vec{r}$
(b) Sketch the vector on a right-handed coordinate system.

0.2. A positron undergoes a displacement $\Delta\vec{r} = (2.0\,m)\vec{i} - (3.0\,m)\vec{j} + (6.0\,m)\vec{k}$, ending with the position vector $\vec{r} = (3.0\,m)\vec{j} - (4.0\,m)\vec{k}$. What was the positron's initial position vector?

0.3. Two vectors are given by

$$\vec{a} = 4.0\vec{i} - 3.0\vec{j} + 1.0\vec{k}$$
$$\vec{b} = -1.0\vec{i} + 1.0\vec{j} + 4.0\vec{k}$$

In unit-vector notation, find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$.

0.4. In Fig. 0.1, a vector $\vec{a}$ with a magnitude of 17.0 m is directed at angle $\theta = 56.0°$ counterclockwise from the axis. What are the components (a) a_x (b) and a_y of the vector? A second coordinate system is inclined by angle $\theta' = 18.0°$ with respect to the first. What are the components (c) $a_{x'}$ (d) and $a_{y'}$ in this primed coordinate system?

§ 0.2. Multiplying Vectors

0.5. Two particles are emitted from a common source and at a particular time have displacements:

$$\vec{r}_1 = 4.0\vec{i} + 3.0\vec{j} + 8.0\vec{k}$$
$$\vec{r}_2 = 2.0\vec{i} + 10.0\vec{j} + 5.0\vec{k}$$

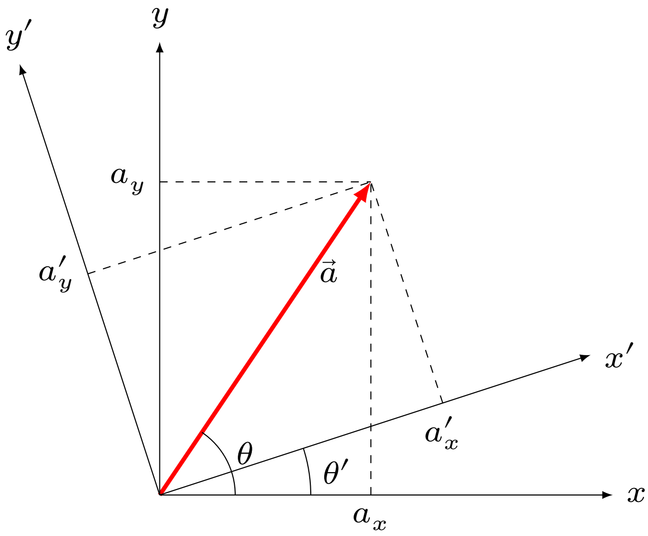


Figure 0.1. Problem 0.4

(a) Sketch the positions of the particles and write the expression for the displacement $\vec{r}$ of particle 2 relative to particle 1.
(b) Use the scalar product to find the magnitude of each vector.
(c) Calculate the angles between all possible pairs of the three vectors.
(d) Calculate the projection of $\vec{r}$ on $\vec{r}_1$.
(e) Calculate the vector product $\vec{r}_1 \times \vec{r}_2$.

0.6. Find x and y such that the vectors $\vec{b} = x\vec{i} + 3\vec{j}$ and $\vec{c} = 2\vec{i} + y\vec{j}$ are each perpendicular to $\vec{a} = 5\vec{i} + 6\vec{j}$. Now prove that $\vec{b}$ and $\vec{c}$ are parallel. Is it true in three dimensions that two vectors perpendicular to a third are necessarily parallel?

0.7. For the vectors in Fig. 0.2, with $a = 4$, $b = 3$, and $c = 5$, what are (a) the magnitude and (b) the direction of $\vec{a} \times \vec{b}$, (c) the magnitude and (d) the direction of $\vec{a} \times \vec{c}$, (e) the magnitude and (f) the direction of $\vec{b} \times \vec{c}$. (The z axis is not shown.)

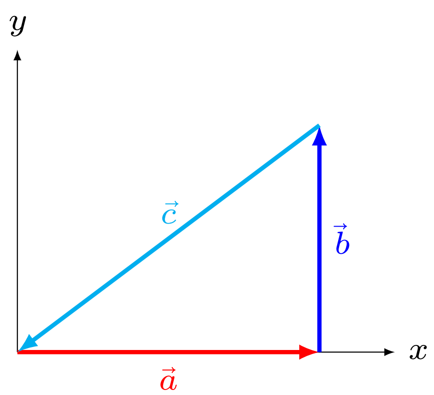


Figure 0.2. Problem 0.7

CHAPTER

1

Kinematics

§ 1.1. Kinematics of the particle

1.1.1 Motion in one dimension

1.1. An object traveling along the x axis with an initial velocity of 5.0 m/s has a constant acceleration of 2.0 m/s^2. When its velocity is 15.0 m/s, how far has it traveled?

1.2. The position of an object moving along an x axis is given by $x = 3t + 4t^2 + t^3$, where x is in meters and t in seconds.

(a) Find the position of the object at the following values of t: 1, 2, 3 and 4 s.
(b) Find the velocity of the object at the following values of t: 1, 2, 3 and 4 s.
(c) What is the object's displacement between $t = 0$ and $t = 4$ s?
(d) What is its average velocity for the time interval from $t = 0$ s to $t = 4$ s?
(e) Graph x versus t for $0 \leq x \leq 4$ s and indicate how the answer for (d) can be found on the graph.

1.3. The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 + 6.0t^2$, with x in meters and t in seconds.

(a) At what time and where does the particle (momentarily) stop?
(b) At what negative time and positive time does the particle pass through the origin?
(c) Graph x versus t for the range -5 s to $+5$ s.
(d) To shift the curve rightward on the graph, should we include the

1.4. At a certain time a particle had a speed of 18 m/s in the positive x direction, and 2.4 s later its speed was 30 m/s in the opposite direction.What is the average acceleration of the particle during this 2.4 s interval?

1.5. An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s^2 in a straight line until it reaches a speed of 20 m/s. The vehicle then slows at a constant rate of 1.0 m/s^2 until it stops.

(a) How much time elapses from start to stop?
(b) How far does the vehicle travel from start to stop?

1.6. A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.

(a) Graph x versus t for the range $t = 0$ s to $t = 4$ s.
(b) Determine the displacement of the particle in the time intervals $t = 0$ s to $t = 1$ s and $t = 1$ s to $t = 3$ s.
(c) Calculate the average velocity during these two time intervals.
(d) Find the instantaneous velocity of the particle at $t = 2.5$ s.

1.7. The velocity of a particle moving in the positive direction of the x axis varies as $v = \alpha\sqrt{x}$, where α is a positive constant. Assuming that at the moment $t = 0$ the particle was located at the point $x = 0$, find:

(a) the time dependence of the velocity and the acceleration of the particle;
(b) the mean velocity of the particle averaged over the time that the particle takes to cover the first s metres of the path.

1.1.2 Motion in three dimension. The coordinate and vector method

1.8. An electron's position is given by $\vec{r} = 3.00t\vec{i} - 4.00t^2\vec{j} + 2.00\vec{k}$, with t in seconds and in meters.

(a) In unit-vector notation, what is the electron's velocity $\vec{v}(t)$?
(b) At $t = 2.00$ s, what is $\vec{v}$ in unit-vector,
(c) a magnitude v,
(d) an angle relative to the positive direction of the x axis?

1.9. Aparticle moves in the xy plane with constant acceleration. At $t = 0$ the particle is at $\vec{r}_1 = (4.00\ m)\vec{i} + (3.00\ m)\vec{j}$, with velocity $\vec{v}_1$. At $t = 2.0$ s, the particle has moved to $\vec{r}_2 = (10.00\ m)\vec{i} + (2.00\ m)\vec{j}$ and its velocity has changed to $\vec{v}_2 = (5.00\ m/s)\vec{i} - (6.00\ m/s)\vec{j}$.

(a) Find $\vec{v}_1$.
(b) What is the acceleration of the particle?
(c) What is the velocity of the particle as a function of time?
(d) What is the position vector of the particle as a function of time?

1.1.3 Projectile motion

1.10. A projectile is launched with speed v_0 at an angle of α above the horizontal. Find an expression for the maximum height it reaches above its starting point in terms of v_0, α and g. (Ignore any effects due to air resistance.)

1.11. A projectile is launched from ground level with an initial speed of 53 m/s. Find the launch angle (the angle the initial velocity vector is above the horizontal) such that the maximum height of the projectile is equal to its horizontal range. (Ignore any effects due to air resistance.)

1.12. A ball launched from ground level lands 2.44 s later on a level field 40.0 m away from the launch point. Find the magnitude of the initial velocity vector and the angle it is above the horizontal. (Ignore any effects due to air resistance.)

1.13. At 1/2 of its maximum height, the speed of a projectile is 3/4 of its initial speed. What was its launch angle? (Ignore any effects due to air resistance.)

1.14. A ball is to be shot from level ground toward a wall at distance x (Fig. 1.1a). Figure 1.1b shows the y component v_y of the ball's velocity just as it would reach the wall, as a function of that distance x. What is the launch angle?

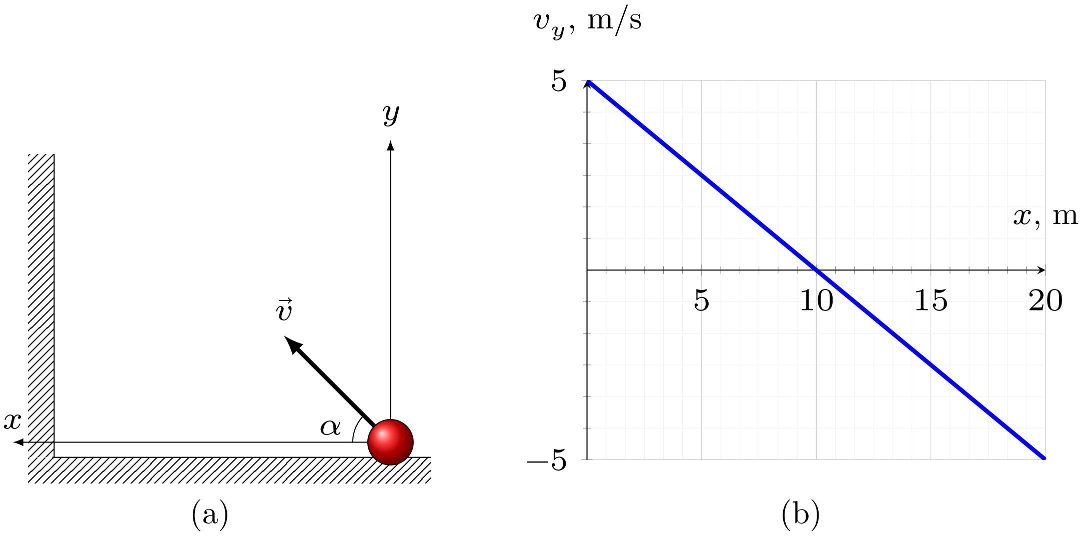


Figure 1.1. Problem 1.14

1.1.4 Circular motion. Natural description of the motion

1.15. A point moves along a circle with a velocity $v = \alpha t$, where $a = 0.50\ \text{m/s}^2$. Find the total acceleration of the point at the moment when it covered the $n = 0.10$ fraction of the circle after the beginning of motion.

1.16. What is the angular speed of Earth in radians per second as it rotates about its axis?

1.17. A point moves in the plane so that its tangential acceleration $a_\tau = \alpha$, and its normal acceleration $a_n = \beta t^4$, where α and β are positive constants, and t is time. At the moment $t = 0$ the point was at rest. Find how the curvature radius R of the point's trajectory and the total acceleration a depend on the distance covered s.

1.18. A point moves in the plane xy according to the law $x = A \sin \omega t$, $y = A(1 - \cos \omega t)$, where A and ω are positive constants. Find:

(a) the distance s traversed by the point during the time τ;

(b) the angle between the point's velocity and acceleration vectors.

1.19. A point moves along an arc of a circle of radius R. Its velocity depends on the distance covered s as $v = \alpha\sqrt{s}$, where α is a constant. Find the angle a between the vector of the total acceleration and the vector of velocity as a function of s.

§ 1.2. Rotational motion and its angular characteristics. Kinematics of a rigid body

1.20. What is the angular speed of (a) the second hand, (b) the minute hand, (c) and the hour hand of a smoothly running analog watch? Answer in radians per second.

1.21. The angular position of a point on a rotating wheel is given by $\varphi = 2.0 + 4.0t^2 + 2.0t^3$, where φ is in radians and t is in seconds. At $t = 0$, what are (a) the point's angular position (b) and its angular velocity? (c) What is its angular velocity at $t = 4.0$ s? (d) Calculate its angular acceleration at $t = 2.0$ s. (e) Is its angular acceleration constant?

1.22. The angular position of a point on the rim of a rotating wheel is given by $\varphi = 4.0t - 3.0t^2 + t^3$, where φ is in radians and t is in seconds. What are the angular velocities at (a) $t = 2.0$ s (b) and $t = 4.0$ s? (c) What is the average angular acceleration for the time interval that begins at $t = 2.0$ s and ends at $t = 4.0$ s? What are the instantaneous angular accelerations at (d) the beginning (e) and the end of this time interval?

1.23. A 12-cm-radius disk that begins to rotate about its axis at $t = 0$, rotates with a constant angular acceleration of 8.0 rad/s^2. At $t = 5$ s find:

(a) what is the angular speed of the disk,
(b) what are the tangential and centripetal components of the acceleration of a point on the edge of the disk?

1.24. A solid body rotates with angular velocity $\vec{\omega} = at\vec{i} + bt^2\vec{j}$, where $a = 0.50$ rad/s^2, $b = 0.060$ rad/s^3. Find:
(a) the magnitude of the angular velocity and the angular acceleration at the moment $t = 10.0$ s;
(b) the angle between the vectors of the angular velocity and the angular acceleration at that moment.

1.25. A solid body rotates about a stationary axis so that its angular velocity depends on the rotation angle $\omega = \omega_0 - a\varphi$, where ω_0 and a are positive constants. At the moment $t = 0$ the angle $\varphi = 0$. Find the time dependence of (a) the rotation angle; (b) the angular velocity.

CHAPTER

2

Dynamics. Newtonian laws

§ 2.1. Applying Newton's Laws

2.1. In Fig. 2.1, let the mass of the block be $m = 8.5$ kg and the angle be $\alpha = 30°$. Find (a) the tension in the cord and (b) the normal force acting on the block. (c) If the cord is cut, find the magnitude of the resulting acceleration of the block.

2.2. In Fig. 2.1 a crate of mass $m = 100$ kg is pushed at constant velocity up a frictionless ramp ($\alpha = 30.0°$) by a horizontal force. What are the magnitudes of (a) force $\vec{F}$ and (b) the force on the crate from the ramp?

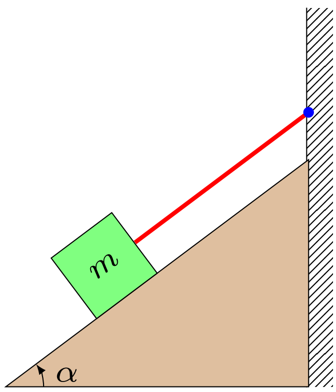


Figure 2.1. Problem 2.1

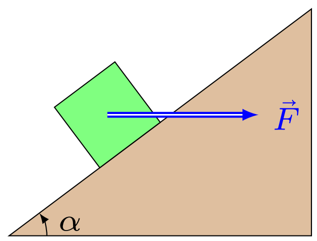


Figure 2.2. Problem 2.2

2.3. Two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass) (see Fig. 2.3). The arrangement is known as *Atwood's machine*. One block has mass $m_1 = 1.30$ kg; the other has mass $m_2 = 2.80$ kg. What are (a) the magnitude of the blocks' acceleration (b) and the tension in the cord?

2.4. A block of mass $m_1 = 3.70$ kg on a frictionless plane inclined at angle $\alpha = 30.0°$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_2 = 2.30$ kg (Fig. 2.4). What are (a) he magnitude

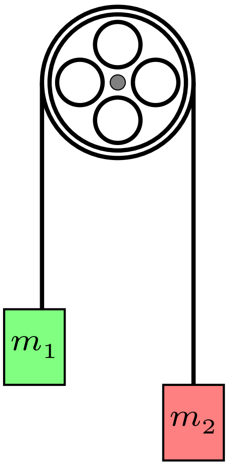


Figure 2.3. Problem 2.3

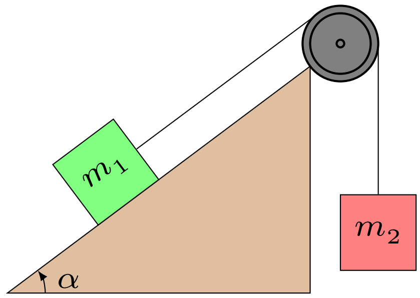


Figure 2.4. Problem 2.4

of the acceleration of each block, (b) he direction of the acceleration of the hanging block, (c) the tension in the cord?

2.5. In Fig. 2.5a, a constant horizontal force is applied to block A, which pushes against block B with a 20.0 N force directed horizontally to the right. In Fig. 2.5b, the same force is applied to block B; now block A pushes on block B with a 10.0 N force directed horizontally to the left.The blocks have a combined mass of 12 kg. What are the magnitudes of (a) their acceleration in Fig. 2.5a (b) and force $\vec{F}$?

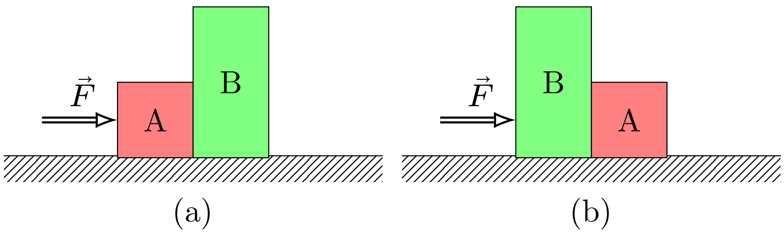


Figure 2.5. Problem 2.5

2.6. A lamp hangs vertically from a cord in a descending elevator that decelerates at 2.4 m/s^2. (a) If the tension in the cord is 89 N, what is the lamp's mass? (b) What is the cord's tension when the elevator ascends with an upward acceleration of 2.4 m/s^2?

2.7. A box of mass 5.00 kg, is sent sliding up a frictionless ramp at an angle of α to the horizontal. Fig. 2.6, as a function of time t, the component v_x of the box's velocity along an x axis that extends directly up the ramp. What is the magnitude of the normal force on the box from the ramp?

2.8. A 3.5 kg block is pushed along a horizontal floor by a force of magnitude 15 N at an angle 40° with the horizontal (Fig. 2.7). The coefficient of kinetic friction between the block and the floor is 0.25. Calculate the magnitudes of (a) the frictional force on the block from the floor and (b) the block's acceleration.

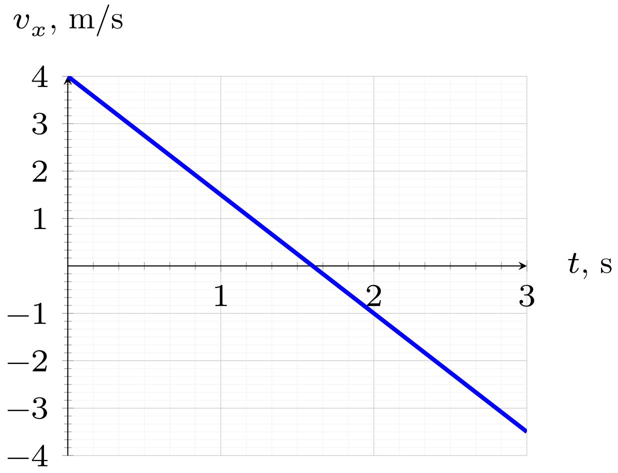


Figure 2.6. Problem 2.7

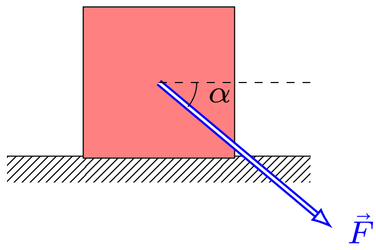


Figure 2.7. Problem 2.8

2.9. A 4.10 kg block is pushed along a floor by a constant applied force that is horizontal and has a magnitude of 40.0 N. Figure 2.8 gives the block's speed v versus time t as the block moves along an x axis on the floor. What is the coefficient of kinetic friction between the block and the floor?

2.10. A 110 g hockey puck sent sliding over ice is stopped in 15 m by the frictional force on it from the ice. (a) If its initial speed is 6.0 m/s, what is the magnitude of the frictional force? (b) What is the coefficient of friction between the puck and the ice?

2.11. A circular curve of highway is designed for traffic moving at 60 km/h. Assume the traffic consists of cars without negative lift. (a) If the radius of the curve is 150 m, what is the correct angle of banking of the road? (b) If the curve were not banked, what would be the minimum coefficient of friction between tires and road that would keep traffic from skidding out of the turn when traveling at 60 km/h?

2.12. As a 40 N block slides down a plane that is inclined at 25° to the horizontal, its acceleration is 0.80 m/s^2, directed up the plane. What is the coefficient of kinetic friction between the block and the plane?

2.13. An initially stationary box of sand is to be pulled across a floor by means of a cable in which the tension should not exceed 1100 N. The coefficient of static friction between the box and the floor is 0.35. (a) What should be the angle between the cable and the horizontal in order to pull the greatest possible amount of sand, and (b) what is the weight of the sand and box in that situation?

2.14. In Fig. 2.9, block 1 of mass $m_1 = 2.0$ kg and block 2 of mass $m_2 = 1.0$ kg are connected by a string of negligible mass. Block 2 is pushed by force of magnitude 20 N and angle $\alpha = 30°$. The coefficient of kinetic friction between each block and the horizontal surface is 0.20. What is the tension in the string?

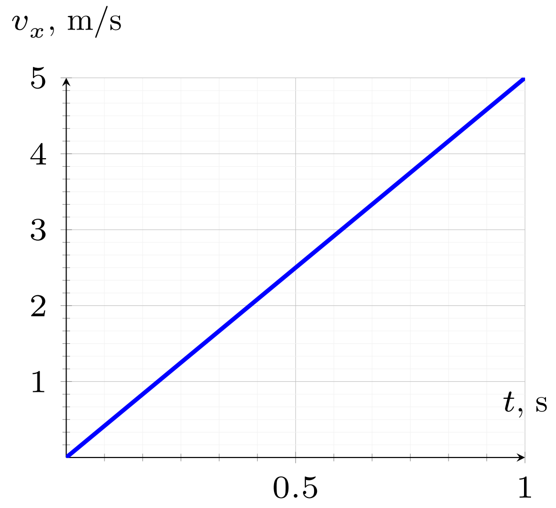


Figure 2.8. Problem 2.9

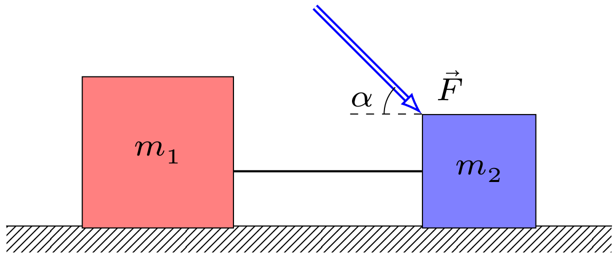


Figure 2.9. Problem 2.14

§ 2.2. Dynamics of motion along a curved path

2.15. In Fig. 2.10, a car is driven at constant magnitude of velocity over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 70.0 kg. What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?

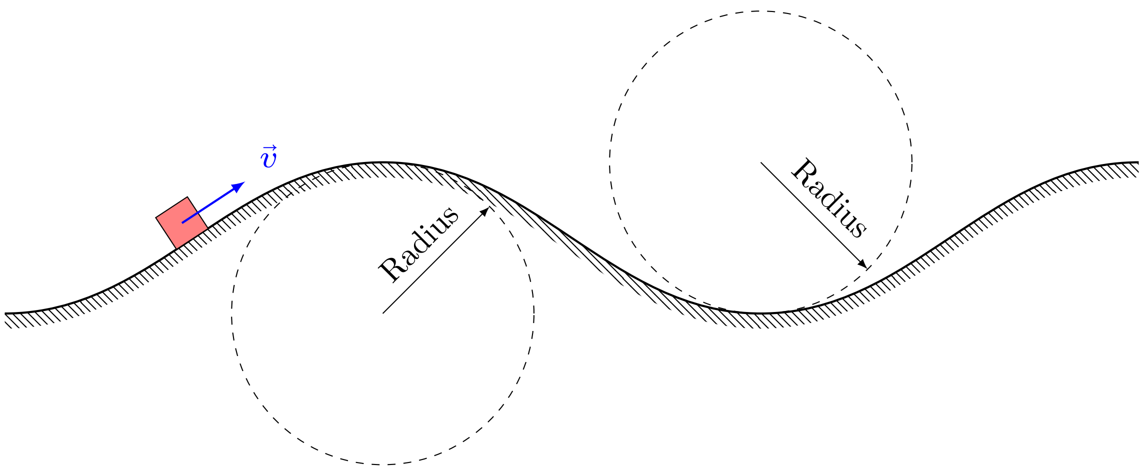


Figure 2.10. Problem 2.15

2.16. A ball suspended by a thread swings in a vertical plane so that its acceleration values in the extreme and the lowest position are equal. Find the thread deflection angle in the extreme position.

2.17. A small sphere of mass m is attached to the end of a cord of length R and set into motion in a vertical circle about a fixed point O as illustrated in Figure 2.11. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the velocity of the sphere is v and the cord makes an angle θ with the vertical.

2.18. Consider a conical pendulum (Fig. 2.12) with a bob of mass $m = 80.0$ kg on a string of length $l = 10.0$ m that makes an angle of $\theta = 5.00°$ with the vertical. Determine (a) the horizontal and vertical components of the force exerted by the string on the pendulum and (b) the radial acceleration of the bob.

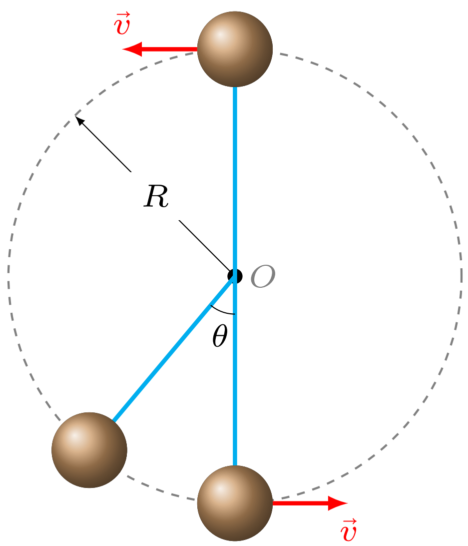


Figure 2.11. Problems 2.17

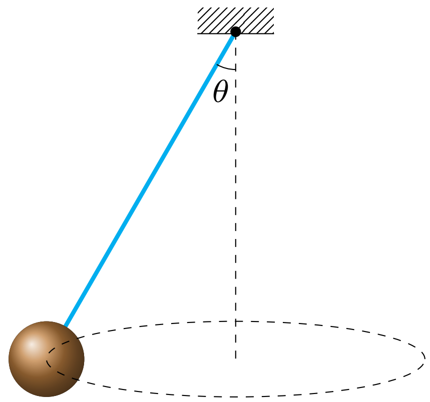


Figure 2.12. Problem 2.18

2.19. A small body A starts sliding off the top of a smooth sphere of radius R (Fig. 2.13). Find (a) the angle θ corresponding to the point at which the body breaks off the sphere and (b) the break-of velocity of the body.

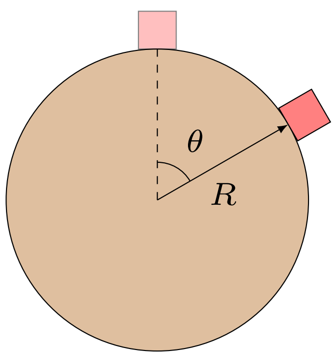


Figure 2.13. Problem 2.19

CHAPTER 3

Non-inertial reference frame. Inertial forces

3.1. A horizontal disc rotates with a constant angular velocity $\omega = 6.0$ rad/s about a vertical axis paissing through its centre. A small body of mass $m = 0.50$ kg moves along a diameter of the disc with a velocity $v' = 50$ cm/s which is constant relative to the disc. Find the force that the disc exerts on the body at the moment when it is located at the distance $r = 30$ cm from the rotation axis.

3.2. A horizontal smooth rod AB rotates with a constant angular velocity $\omega = 2.00$ rad/s about a vertical axis passing through its end A. A freely sliding sleeve of mass $m = 0.50$ kg moves along the rod from the point A with the initial velocity $v_0 = 1.00$ m/s. Find the Coriolis force acting on the sleeve (in the reference frame fixed to the rotating rod) at the moment when the sleeve is located at the distance $r = 50$ cm from the rotation axis.

3.3. A horizontal disc of radius R rotates with a constant angular velocity ω about a stationary vertical axis passing through its edge. Along the circumference of the disc a particle of mass m moves with a velocity that is constant relative to the disc. At the moment when the particle is at the maximum distance from the rotation axis, the resultant of the inertial forces Fin acting on the particle in the reference frame fixed to the disc turns into zero. Find: (a) the acceleration a' of the particle relative to the disc; (b) the dependence of F_{n} on the distance from the rotation axis.

3.4. A small body of mass $m = 0.30$ kg starts sliding down from the top of a smooth sphere of radius $R = 1.00$ m. The sphere rotates with a constant angular velocity $\omega = 6.0$ rad/s about a vertical axis passing through its centre. Find (a) the Centrifugal force of inertia and (b) the Coriolis force at the moment when the body breaks off the surface of the sphere in the reference frame fixed to the sphere.

CHAPTER

4

Work and Energy. Laws of Conservations

§ 4.1. Kinetic Energy and Work

4.1. Figure 4.1 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?

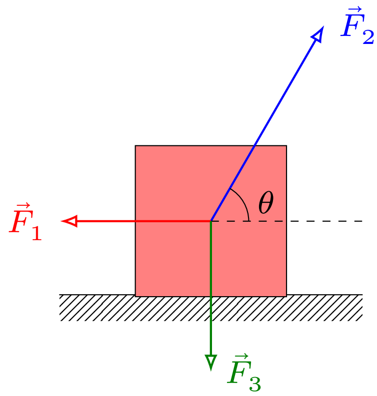


Figure 4.1. Problem 4.1

4.2. A block is sent up a frictionless ramp along which an x axis extends upward. Figure 4.2 gives the kinetic energy of the block as a function of position x. If the block's initial speed is 4.00 m/s, what is the normal force on the block?

4.3. Figure 4.3 gives spring force F_x versus position x for the spring–block arrangement. We release the block at $x = 12$ cm. How much work does the spring do on the block when the block moves from $x_i = 8.0$ cm to (a) $x = 5.0$ cm, (b) $x = -105.0$ cm.

4.4. The kinetic energy of a particle moving along a circle of radius R depends on the distance covered s as $K = \alpha s^2$, a constant. Find the force acting on the particle as a function of s.

4.5. A 10 kg brick moves along an x axis. Its acceleration as a function of its position is shown in Fig. 4.4. What is the net work performed on the brick by the force causing the acceleration as the brick moves from $x = 0$ to $x = 8.0$ m?

4.6. The only force acting on a 2.0 kg body as the body moves along

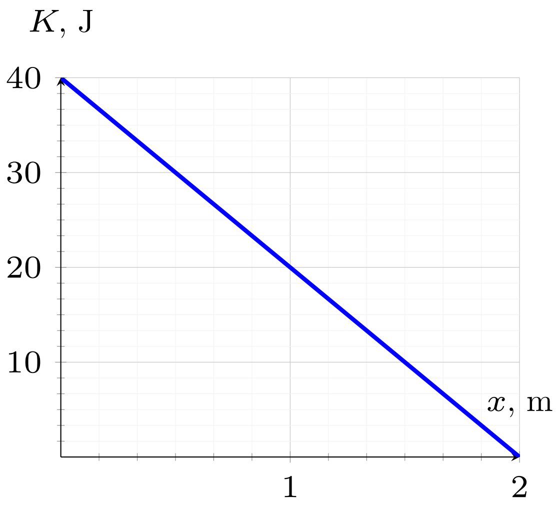


Figure 4.2. Problem 4.2

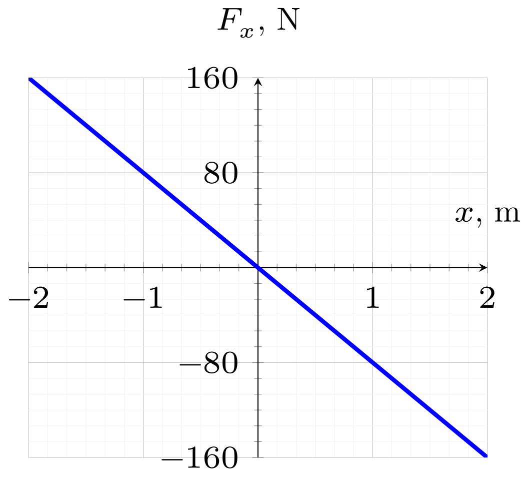


Figure 4.3. Problem 4.3

an x axis varies as shown in Fig. 4.5. The velocity of the body at $x = 0$ is 4.0 m/s. (a) What is the kinetic energy of the body at $x = 3.0$ m? (b) At what value of x will the body have a kinetic energy of 8.0 J? (c) What is the maximum kinetic energy of the body between $x = 0$ and $x = 5.0$ m?

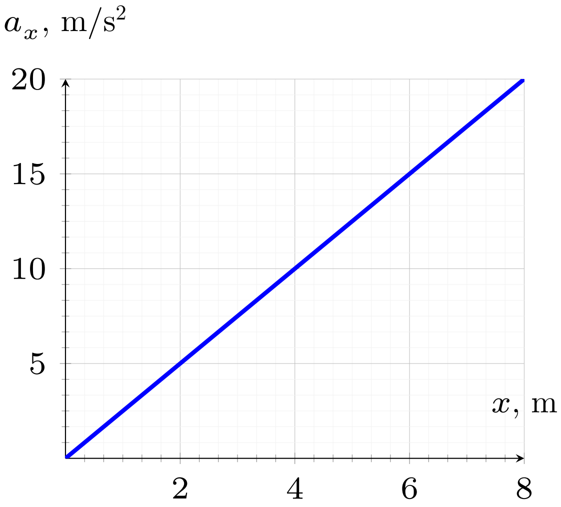


Figure 4.4. Problem 4.5

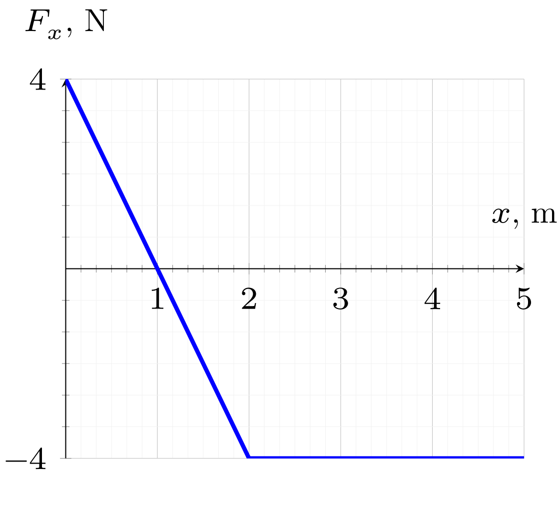


Figure 4.5. Problem 4.6

4.7. A disc of mass $m = 50$ g slides with the zero initial velocity down an inclined plane set at an angle $\theta = 30°$ to the horizontal; having traversed the distance $l = 50$ cm along the horizontal plane, the disc stops. Find the work performed by the friction forces over the whole distance, assuming the friction coefficient $\mu = 0.15$ for both inclined and horizontal planes.

4.8. The force on a particle is directed along an x axis and given by $F = F_0(x/x_0 - 1)$. Find the work done by the force in moving the particle from $x = 0$ to $x = 2x_0$ by (a) plotting $F(x)$ and measuring the work from the graph and (b) integrating F(x).

4.9. An initially stationary 2.0 kg object accelerates horizontally and uniformly to a speed of 10 m/s in 3.0 s. (a) In that 3.0 s interval, how much work is done on the object by the force accelerating it? What is the instantaneous power due to that force (b) at the end of the interval and (c) at the end of the first half of the interval?

§ 4.2. Potential Energy

4.10. Figure 4.6 shows a ball with mass $m = 0.341$ kg attached to the end of a thin rod with length $l = 0.452$ m and negligible mass. The other end of the rod is pivoted so that the ball can move in a vertical circle. The rod is held horizontally as shown and then given enough of a downward push to cause the ball to swing down and around and just reach the vertically up position, with zero speed there. How much work is done on the ball by the gravitational force from the initial point to (a) the lowest point, (b) the highest point, and (c) the point on the right level with the initial point? If the gravitational potential energy of the ball–Earth system is taken to be zero at the initial point, what is it when the ball reaches (d) the lowest point, (e) the highest point, and (f) the point on the right level with the initial point?

4.11. In Fig. 4.7 a small block of mass $m = 0.032$ kg can slide along the frictionless loop-the-loop, with loop radius $R = 12$ cm. The block is released from rest at point P, at height $h = 5.0R$ above the bottom of the loop. How much work does the gravitational force do on the block as the block travels from point P to (a) point Q and (b) the top of the loop? If the gravitational potential energy of the block–Earth system is taken to be zero at the bottom of the loop, what is that potential energy when the block is (c) at point P, (d) at point Q, and (e) at the top of the loop?

4.12. A locomotive of mass m starts moving so that its velocity varies according to the law $v = \alpha\sqrt{s}$, where α is a constant, and s is the distance covered. Find the total work performed by all the forces which are acting on the locomotive during the first t seconds after the beginning of motion.

§ 4.3. Conservation of Mechanical Energy of the Particle

4.13. A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring. (a) What is the change ΔU_g in the gravitational potential energy of the marble–Earth system during the 20 m ascent? (b) What is the change in the elastic potential energy of the spring during its launch of the marble? (c) What is the spring constant of the

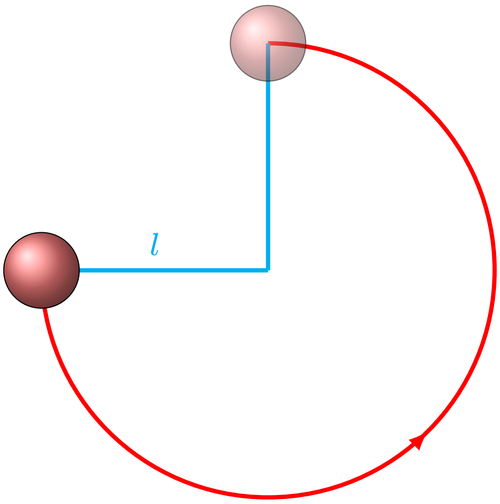


Figure 4.6. Problem 4.10

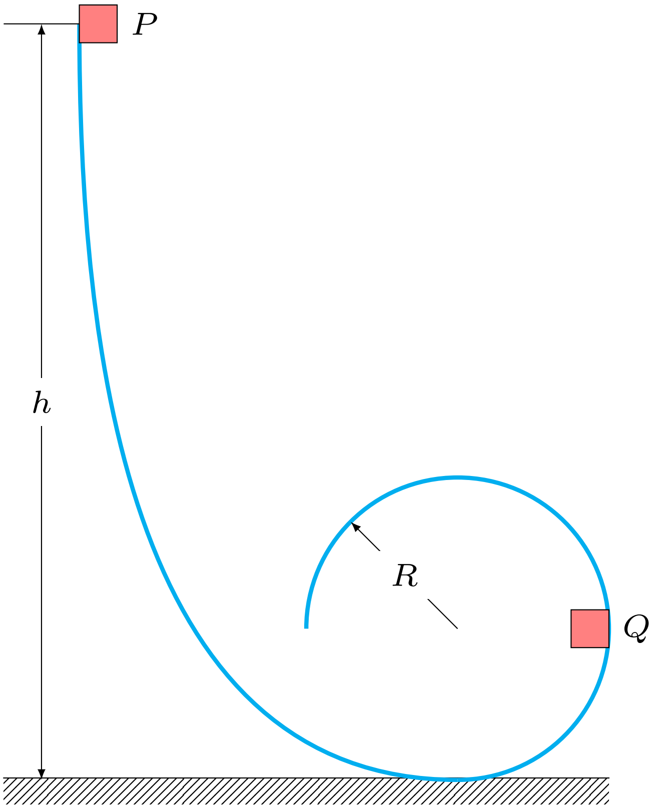


Figure 4.7. Problem 4.11

spring?

4.14. A pendulum consists of a 2.0 kg stone swinging on a 4.0 m string of negligible mass. The stone has a speed of 8.0 m/s when it passes its lowest point. (a) What is the speed when the string is at 60° to the vertical? (b) What is the greatest angle with the vertical that the string will reach during the stone's motion? (c) If the potential energy of the pendulum–Earth system is taken to be zero at the stone's lowest point, what is the total mechanical energy of the system?

4.15. The string in Fig. 4.8 is $l = 120$ cm long, has a ball attached to one end, and is fixed at its other end. The distance d from the fixed end to a fixed peg at point P is 75.0 cm. When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point after the string catches on the peg?

§ 4.4. Relationship Between Conservative. Forces and Potential Energy Reading a Potential Energy Curve

4.16. The potential energy of a system of two particles separated by a distance r is given by $U(r) = \frac{A}{r}$, where A is a constant. Find the radial force

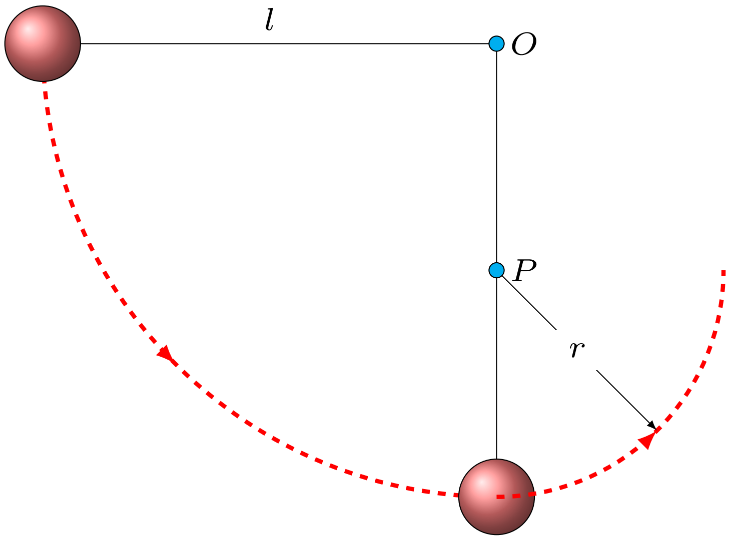


Figure 4.8. Problem 4.15

$\vec{F}_r$ that each particle exerts on the other.

4.17. A potential energy function for a system in which a two dimensional force acts is of the form $U = 3x^3 3y - 7x$. Find the force that acts at the point (x, y).

4.18. Figure 4.9 shows a plot of potential energy U versus position x of a 0.90 kg particle that can travel only along an x axis. (Nonconservative forces are not involved.) The particle is released at $x = 4.5$ m with an initial velocity of 7.0 m/s, headed in the negative x direction. (a) If the particle can reach $x = 1.0$ m, what is its velocity there, and if it cannot, what is its turning point? What are the (b) magnitude and (c) direction of the force on the particle as it begins to move to the left of $x = 4.0$ m? Suppose, instead, the particle is headed in the positive x direction when it is released at $x = 4.5$ m at speed 7.0 m/s. (d) If the particle can reach $x = 7.0$ m, what is its speed there, and if it cannot, what is its turning point? What are the (e) magnitude and (f) direction of the force on the particle as it begins to move to the right of $x = 5.0$ m?

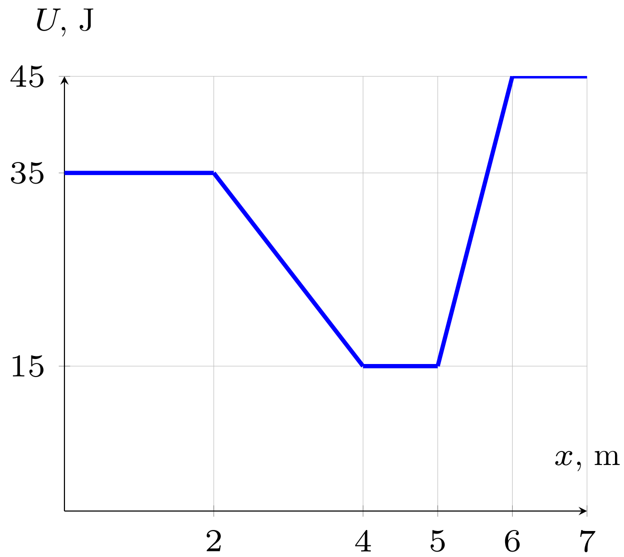


Figure 4.9. Problem 4.18

4.19. The potential energy of a diatomic molecule (a two-atom system like H_2 or O_2) is given by

$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$

where r is the separation of the two atoms of the molecule and a and b are positive constants. This potential energy is associated with the force that binds the two atoms together. (a) Find the equilibrium separation — that is, the distance between the atoms at which the force on each atom is zero. Is the force repulsive (the atoms are pushed apart) or attractive (they are pulled together) if their separation is (b) smaller and (c) larger than the equilibrium separation?

§ 4.5. Center of Mass. Law for a System of Particles and Linear Momentum Conservation Law

4.20. Figure 4.10 shows a three-particle system, with masses $m_1 = 3.0$ kg, $m_2 = 4.0$ kg, and $m_3 = 8.0$ kg. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass? (c) If m_3 is gradually increased, does the center of mass of the system shift toward or away from that particle, or does it remain stationary?

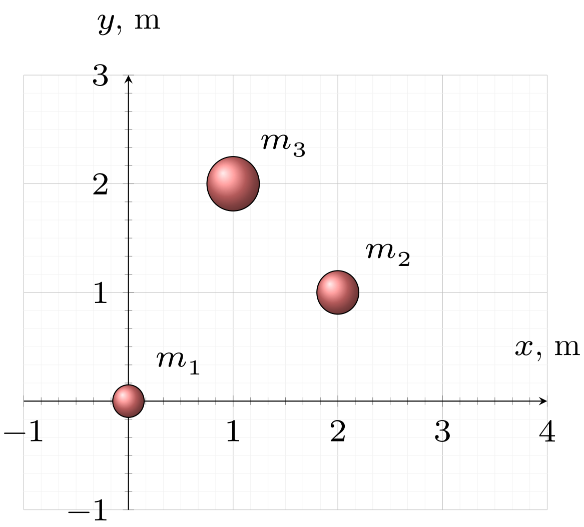


Figure 4.10. Problem 4.20

4.21. A stone is dropped at $t = 0$. A second stone, with twice the mass of the first, is dropped from the same point at $t = 100$ ms. (a) How far below the release point is the center of mass of the two stones at $t = 300$ ms? (Neither stone has yet reached the ground.) (b) How fast is the center of mass of the twostone system moving at that time?

4.22. In Figure 4.11, two particles are launched from the origin of the coordinate system at time $t = 0$. Particle 1 of mass $m_1 = 5.00$ g is shot directly along the x axis on a frictionless floor, with constant speed 10.0 m/s. Particle 2 of mass $m_2 = 3.00$ g is shot with a velocity of magnitude 20.0 m/s, at an upward angle such that it always stays directly above particle 1. (a) What is the maximum height h_{max} reached by the com of the two-particle system? In unit-vector notation, what are the (b) velocity and (c) acceleration of the com when the com reaches h_{max}?

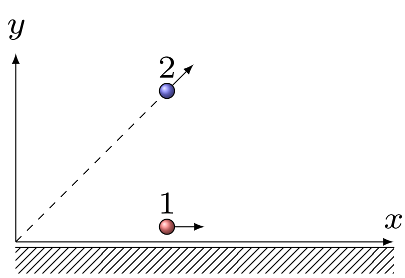


Figure 4.11. Problem 4.22

4.23. A shell is shot with an initial velocity $\vec{v}_0$ of magnitude 20 m/s, at an angle $\alpha = 60°$ of with the horizontal. At the top of the trajectory, the shell explodes into two fragments of equal mass. One fragment, whose velocity immediately after the explosion is zero, falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that air drag is negligible?

4.24. A 0.30 kg softball has a velocity of 15 m/s at an angle of 35° below the horizontal just before making contact with the bat. What is the magnitude of the change in momentum of the ball while in contact with the bat if the ball leaves with a velocity of (a) 20 m/s, vertically downward, and (b) horizontally back toward the pitcher?

4.25. The vector position of a particle of mass 3.50 g moving in the xy plane varies in time according to $\vec{r} = 3\vec{i} + (3t + 2t^2)\vec{j}$, where t is in seconds and $\vec{r}_1$ is in centimeters. At the same time, the vector position of a particle of mass 5.50 g varies as $(3 - 2t^2)\vec{i}6t\vec{j}$. At $t = 2.50$ s, determine (a) the vector position of the center of mass, (b) the linear momentum of the system, (c) the velocity of the center of mass, (d) the acceleration of the center of mass, and (e) the net force exerted on the two-particle system.

4.26. An object, with mass m and speed v relative to an observer, explodes into two pieces, one three times as massive as the other; the explosion takes place in deep space. The less massive piece stops relative to the observer. How much kinetic energy is added to the system during the explosion, as measured in the observer's reference frame?

4.27. A 20.0 kg body is moving through space in the positive direction of an x axis with a speed of 200 m/s when, due to an internal explosion, it breaks into three parts. One part, with a mass of 10.0 kg, moves away from the point of explosion with a speed of 100 m/s in the positive y direction. A second part, with a mass of 4.00 kg, moves in the negative x direction with a speed of 500 m/s. (a) In unit-vector notation, what is the velocity of the third part? (b) How much energy is released in the explosion? Ignore effects due to the gravitational force.

4.28. A 4.0 kg mess kit sliding on a frictionless surface explodes into two 2.0 kg parts: 3.0 m/s, due north, and 5.0 m/s, 30° north of east. What is the original speed of the mess kit?

4.29. A vessel at rest at the origin of an xy coordinate system explodes into three pieces. Just after the explosion, one piece, of mass m, moves with velocity (−30 m/s) and a second piece, also of mass m, moves with velocity (−30 m/s). The third piece has mass $3m$. Just after the explosion, what are the (a) magnitude and (b) direction of the velocity of the third piece?

4.30. Particle A and particle B are held together with a compressed spring between them. When they are released, the spring pushes them apart, and they then fly off in opposite directions, free of the spring. The mass of A is 2.00 times the mass of B, and the energy stored in the spring was 60 J. Assume that the spring has negligible mass and that all its stored energy is transferred to the particles. Once that transfer is complete, what are the kinetic energies of (a) particle A and (b) particle B?

§ 4.6. Equation of motion of a body with a variable mass

4.31. A model rocket engine has an average thrust of 5.26 N. It has an initial mass of 25.5 g, which includes fuel mass of 12.7 g. The duration of its burn is 1.90 s. (a) What is the average exhaust speed of the engine? (b) This engine is placed in a rocket body of mass 53.5 g. What is the final velocity of the rocket if it were to be fired from rest in outer space by an astronaut on a spacewalk? Assume the fuel burns at a constant rate.

4.32. A 6090 kg space probe moving nose-first toward Jupiter at 105 m/s relative to the Sun fires its rocket engine, ejecting 80.0 kg of exhaust at a speed of 253 m/s relative to the space probe.What is the final velocity of the probe?

4.33. A rocket that is in deep space and initially at rest relative to an inertial reference frame has a mass of $2.55 \cdot 10^5$ kg, of which $1.81 \cdot 10^5$ kg is fuel. The rocket engine is then fired for 250 s while fuel is consumed at the rate of 480 kg/s. The speed of the exhaust products relative to the rocket is 3.27 km/s. (a) What is the rocket's thrust? After the 250 s firing, what are (b) the mass and (c) the speed of the rocket?

§ 4.7. Collisions of Particles

4.7.1 Momentum and Kinetic Energy in Collisions

4.34. A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball?

4.35. A particle of mass m having collided with a stationary particle of mass M deviated by an angle $\pi/2$ whereas the particle M recoiled at an angle $\theta = 30°$ to the direction of the initial motion of the particle m. How much (in per cent) and in what way has the kinetic energy of this system changed after the collision, if $M/m = 5.0$?

4.36. A bullet of mass 10 g strikes a ballistic pendulum of mass 2.0 kg.The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

4.37. A 3.50 g bullet is fired horizontally at two blocks at rest on a frictionless table. The bullet passes through block 1 (mass 1.20 kg) and embeds itself in block 2 (mass 1.80 kg). The blocks end up with speeds $v_1 = 0.630$ m/s and $v_2 = 1.40$ m/s. Neglecting the material removed from block 1 by the bullet, find the speed of the bullet as it (a) leaves and (b) enters block 1.

4.7.2 Elastic Collisions in One Dimension

4.38. A cart with mass 340 g moving on a frictionless linear air track at an initial speed of 1.2 m/s undergoes an elastic collision with an initially stationary cart of unknown mass. After the collision, the first cart continues in its original direction at 0.66 m/s. (a) What is the mass of the second cart? (b) What is its speed after impact? (c) What is the speed of the twocart center of mass?

4.39. A body of mass 2.0 kg makes an elastic collision with another body at rest and continues to move in the original direction but with one-fourth of its original speed. (a) What is the mass of the other body? (b) What is the speed of the two-body center of mass if the initial speed of the 2.0 kg body was 4.0 m/s?

4.40. A steel ball of mass 0.500 kg is fastened to a cord that is 70.0 cm long and fixed at the far end. The ball is then released when the cord is horizontal (Fig. 4.12). At the bottom of its path, the ball strikes a 2.50 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.

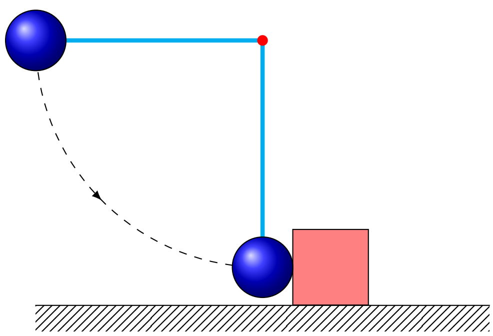


Figure 4.12. Problem 4.40

4.7.3 Collisions in Two Dimensions

4.41. Ball B, moving in the positive direction of an x axis at speed v, collides with stationary ball A at the origin. A and B have different masses. After the collision, B moves in the negative direction of the y axis at speed $v/2$. (a) In what direction does A move? (b) Show that the speed of A cannot be determined from the given information.

4.42. After a completely inelastic collision, two objects of the same mass and same initial speed move away together at half their initial speed. Find the angle between the initial velocities of the objects.

4.43. A projectile proton with a speed of 500 m/s collides elastically with a target proton initially at rest. The two protons then move along perpendicular paths, with the projectile path at 60° from the original direction. After the collision, what are the speeds of (a) the target proton and (b) the projectile proton?

4.44. The three balls in the overhead view of Fig. 4.13 are identical. Balls 2 and 3 touch each other and are aligned perpendicular to the path of ball 1. The velocity of ball 1 has magnitude $v_0 = 10$ m/s and is directed at the contact point of balls 1 and 2. After the collision, what are the (a) speed and (b) direction of the velocity of ball 2, (c) the direction of the velocity of ball 2, (d) the speed and (e) direction of the velocity of ball 3, and (f) the speed and (g) direction of the velocity of ball 1? (Hint:With friction absent, each impulse is directed along the line connecting the centers of the colliding balls, normal to the colliding surfaces.)

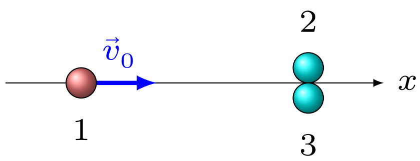


Figure 4.13. Problem 4.44

§ 4.8. Angular Momentum and Torque

4.45. A particle of mass 1.50 kg moves in the xy plane with a velocity of $\vec{v} = 4.20\vec{i} - 3.60\vec{j}$ m/s. Determine the angular momentum of the particle about the origin when its position vector is $\vec{r} = 1.50\vec{i} + 2.20\vec{j}$ m.

4.46. At one instant, force $\vec{F} = 4.0\vec{j}$ N acts on a 0.25 kg object that has position vector $\vec{r} = 2.0\vec{i} - 2.0\vec{k}$ (in meters) and velocity vector $\vec{v} = -5.0\vec{i} + 5.0\vec{k}$ (in m/s). About the origin and in unit-vector notation, what are (a) the object's angular momentum and (b) the torque acting on the object?

4.47. At the instant the displacement of a 2.00 kg object relative to the origin is $\vec{d} = 2.00\vec{i} + 4.00\vec{j} - 3.00\vec{k}$ (in meters) its velocity is $\vec{v} = -6.00\vec{i} + 3.00\vec{j} + 3.00\vec{k}$ (in m/s) and it is subject to a force $\vec{F} = 6.00\vec{i} - 8.00\vec{j} - 4.00\vec{k}$. Find (a) the acceleration of the object, (b) the angular momentum of the object about the origin, (c) the torque about the origin acting on the object, and (d) the angle between the velocity of the object and the force acting on the object.

4.48. In the instant of Fig. 4.16, two particles move in an xy plane. Particle P_1 has mass 6.5 kg and speed $v_1 = 2.2$ m/s, and it is at distance $d_1 = 1.5$ m from point O. Particle P_2 has mass 3.1 kg and speed $v_2 = 3.6$ m/s, and it is at distance $d_2 = 2.8$ m from point O. What are the (a) magnitude and (b) direction of the net angular momentum of the two particles about O?

4.49. A light, rigid rod of length $l = 1.00$ m joins two particles, with masses $m_1 = 4.00$ kg and $m_2 = 3.00$ kg, at its ends. The combination rotates in the xy plane about a pivot through the center of the rod (Fig. 4.14). Determine the angular momentum of the system about the origin when the speed of each particle is 5.00 m/s.

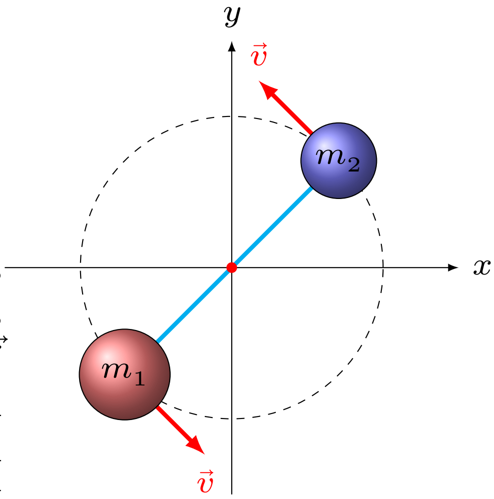


Figure 4.14. Problem 4.49

4.50. A particle of mass 5.00 kg starts from the origin at time zero. Its velocity as a function of time is given by $\vec{v} = 6t^2\vec{\imath} + 2t\vec{\jmath}$ where $\vec{v}$ is in meters per second and t is in seconds. (a) Find its position as a function of time. (b) Describe its motion qualitatively. Find (c) its acceleration as a function of time, (d) the net force exerted on the particle as a function of time, (e) the net torque about the origin exerted on the particle as a function of time, (f) the angular momentum of the particle as a function of time, (g) the kinetic energy of the particle as a function of time, and (h) the power injected into the system of the particle as a function of time.

4.51. At the instant of Fig. 4.15, a 2.0 kg particle P has a position vector of magnitude 3.0 m and angle $\theta_1 = 45°$ and a velocity vector of magnitude 4.0 m/s and angle $\theta_2 = 30°$. Force $\vec{F}$ of magnitude 2.0 N and angle $\theta_3 = 30°$, acts on P. All three vectors lie in the xy plane. About the origin, what are the (a) magnitude and (b) direction of the angular momentum of P and the (c) magnitude and (d) direction of the torque acting on P?

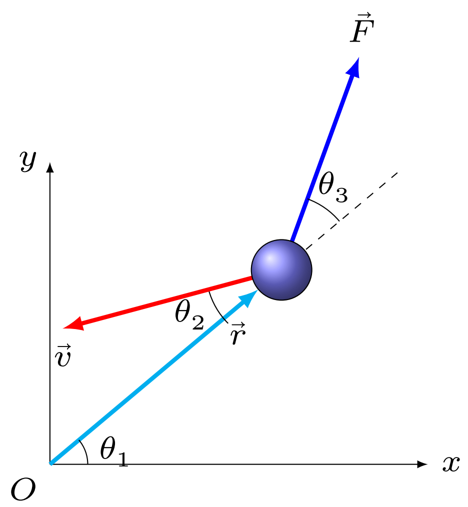


Figure 4.15. Problem 4.51

4.52. At time t, the vector $\vec{r} = 4.0t^2\vec{\imath} - (2.0t + 6.0t^2)\vec{\jmath}$ gives the position of a 3.0 kg particle relative to the origin of an xy coordinate system ($\vec{r}$ is in meters and t is in seconds). (a) Find an expression for the torque acting on the particle relative to the origin. (b) Is the magnitude of the particle's angular momentum relative to the origin increasing, decreasing, or unchanging?

4.53. A ball of mass m moving with velocity v_0 experiences a head-on elastic collision with one of the spheres of a stationary rigid dumbbell as whown in Fig. 4.17. The mass of each sphere equals $m/2$, and the distance between them is l. Disregarding the size of the spheres, find the proper angular momentum of the dumbbell after the collision, i.e. the angular momentum in the reference frame moving translationally and fixed to the dumbbell's centre of mass.

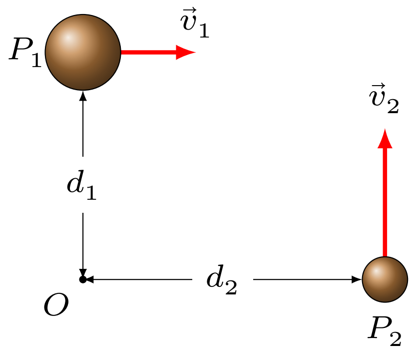


Figure 4.16. Problem 4.48

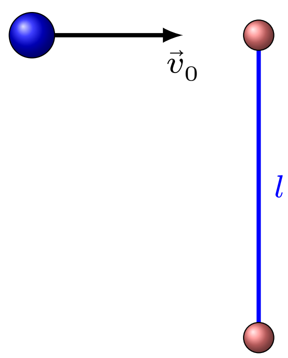


Figure 4.17. Problem 4.53

CHAPTER 5

Universal Gravitation. Central-force problem

5.1. In Fig. 5.1, three 5.00 kg spheres are located at distances $d_1 = 0.300$ m and $d_2 = 0.400$ m. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net gravitational force on sphere B due to spheres A and C? (c) What is the gravitational potential energy of the system?

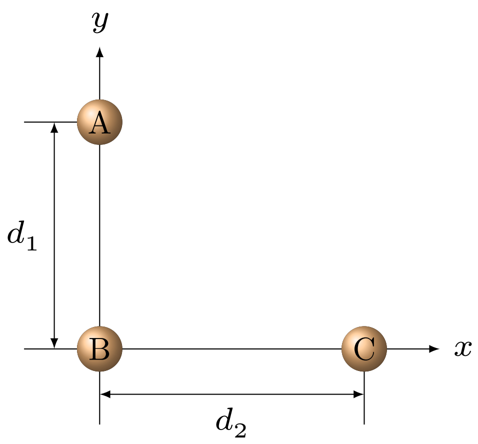


Figure 5.1. Problem 5.1

5.2. Planet Roton, with a mass about of $7.0 \cdot 10^{24}$ kg and a radius of 1600 km, gravitationally attracts a meteorite that is initially at rest relative to the planet, at a distance great enough to take as infinite. The meteorite falls toward the planet. Assuming the planet is airless, find the speed of the meteorite when it reaches the planet's surface.

5.3. Consider a pulsar, a collapsed star of extremely high density, with a mass M equal to that of the Sun ($1.98 \cdot 10^{30}$ kg), a radius R of only 12 km, and a rotational period T of 0.041 s. By what percentage does the free-fall acceleration differ from the gravitational acceleration at the equator of this spherical star?

5.4. A projectile is fired vertically from Earth's surface with an initial speed of 10 km/s. Neglecting air drag, how far above the surface of Earth will it go?

5.5. Figure 5.2 gives the potential energy function $U(r)$ of a projectile, plotted outward from the surface of a planet of radius R_s. (a) What least kinetic energy is required of a projectile launched at the surface if the projectile is to «escape» the planet? If the projectile is launched radially outward from the surface with a mechanical energy of $-2.0 \cdot 10^9$ J, what are (b) its kinetic energy at radius $r = 2R_s$ and (c) its turning point in terms of R_s?

5.6. Figure 5.3 is a graph of the kinetic energy K of an asteroid versus its distance r from Earth's center, as the asteroid falls directly in toward that center. (a) What is the (approximate) mass of the asteroid? (b) What is its speed at $r = 1.945 \cdot 10^7$ m?

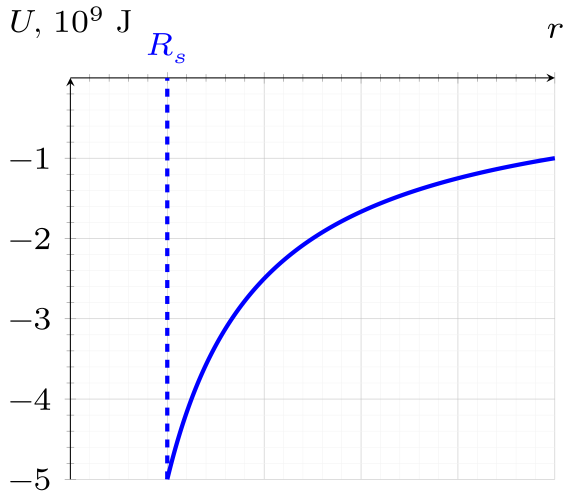


Figure 5.2. Problem 5.5

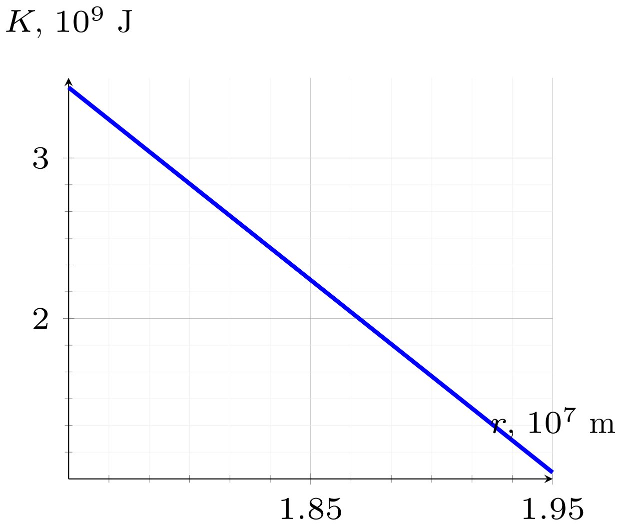


Figure 5.3. Problem 5.6

5.7. In a binary star system, two stars follow circular orbits about their common center of mass. If the stars have masses m_1 and and m_2 are separated by a distance r, show that the period of rotation is related to r by

$$T^2 = \frac{4\pi^2 r^3}{G(m_1 + m_2)}.$$

5.8. Using the conservation laws, demonstrate that the total mechanical energy of a planet of mass m moving around the Sun along an ellipse depends only on its semi-major axis a. Find this energy as a function of a.

5.9. A double star is a system of two stars moving around the centre of inertia of the system due to gravitation. Find the distance between the components of the double star, if its total mass equals M and the period of revolution T.

5.10. A certain quaternary star system consists of three stars, each of mass m, moving in the same circular orbit of radius r about a central star of mass M. The stars orbit in the same sense and are positioned one-third of a revolution apart from one another (Fig. 5.4). Derive an expression for the period of revolution of the stars.

5.11. A certain triple-star system consists of two stars, each of mass m, revolving in the same circular orbit of radius r around a central star of mass M (Fig. 5.5). The two orbiting stars are always at opposite ends of a diameter of the orbit. Derive an expression for the period of revolution of the stars.

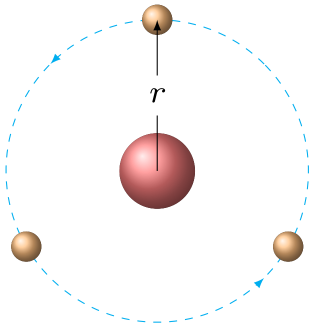


Figure 5.4. Problem 5.10

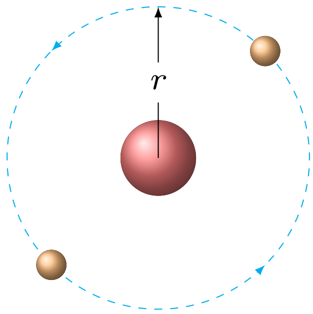


Figure 5.5. Problem 5.11

5.12. A 50 kg satellite circles planet Cruton every 6.0 h.The magnitude of the gravitational force exerted on the satellite by Cruton is 80 N. (a) What is the radius of the orbit? (b) What is the kinetic energy of the satellite? (c) What is the mass of planet Cruton?

5.13. A cosmic body A moves to the Sun with velocity v_0 (when far from the Sun) and aiming parameter l the arm of the vector $\vec{v}_0$ relative to the centre of the Sun (Fig. 5.6). Find the minimum distance by which this body will get to the Sun.

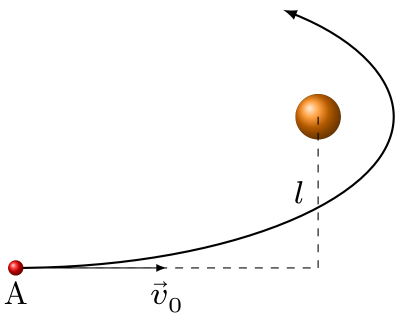


Figure 5.6. Problem 5.13

5.14. Comet Halley moves about the Sun in an elliptical orbit, with its closest approach to the Sun being about 0.590 AU and its greatest distance 35.0 AU (1 AU is the Earth–Sun distance). The comet's speed at closest approach is 54.0 km/s. What is its speed when it is farthest from the Sun?

CHAPTER

6

Rigid body dynamics

§ 6.1. Calculating the Rotational Inertia

6.1. Figure 6.1a shows a disk that can rotate about an axis at a radial distance h from the center of the disk. Figure 6.1b gives the rotational inertia (momentum of inertia) I of the disk about the axis as a function of that distance h, from the center out to the edge of the disk. (a) What is the mass of the disk? (b) What is the radius of the disk?

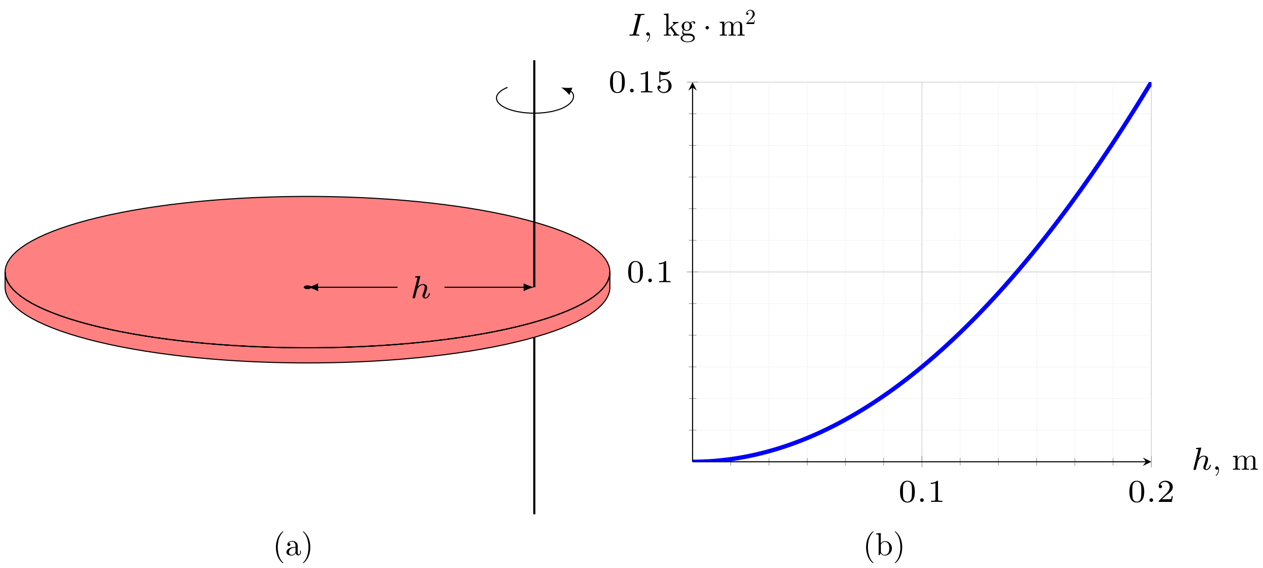


Figure 6.1. Problem 6.1

6.2. In Fig. 6.2, two particles, each with mass $m = 0.85$ kg, are fastened to each other, and to a rotation axis at O, by two thin rods, each with length $d = 5.6$ cm and mass $M = 1.2$ kg. The combination rotates around the rotation axis with the angular speed $v = 0.30$ rad/s. Measured about O, what are the combination's (a) rotational inertia and (b) kinetic energy?

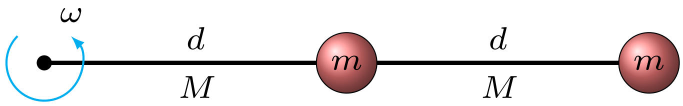


Figure 6.2. Problem 6.2

§ 6.2. Newton's Second Law for Rotation

6.3. Figure 6.3 shows a uniform disk that can rotate around its center. The disk has a radius of 2.00 cm and a mass of 20.0 g and is initially at rest. Starting at time $t = 0$, two forces are to be applied tangentially to the rim as indicated, so that at time $t = 1.25$ s the disk has an angular velocity of 250 rad/s counterclockwise. Force $\vec{F}_1$ has a magnitude of 0.100 N. What is magnitude F_2?

6.4. In Fig. 6.4, block 1 has mass $m_1 = 460$ g, block 2 has mass $m_2 = 500$ g, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius $R = 5.00$ cm. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension T_2 and (c) tension T_1? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?

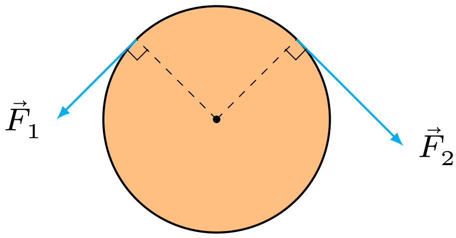


Figure 6.3. Problem 6.3

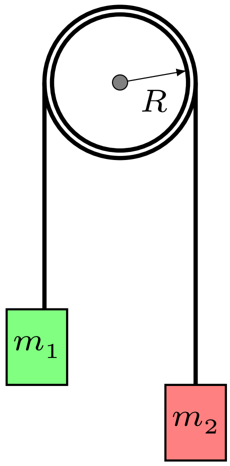


Figure 6.4. Problem 6.4

6.5. A pulley, with a rotational inertia of $1.0 \cdot 10^{-3}$ kg $\cdot$ m^2 about its axle and a radius of 10 cm,is acted on by a force applied tangentially at its rim. The force magnitude varies in time as $F = 0.50t + 0.30t^2$, with F in newtons and t in seconds. The pulley is initially at rest. At $t = 3.0$ s what are its (a) angular acceleration and (b) angular speed?

6.6. A string is wound around a uniform disk of radius R and mass M. The disk is released from rest with the string vertical and its top end tied to a fixed bar (Fig. 6.5). Find (a) the tension in the string, (b) the magnitude of the acceleration of the center of mass, (c) the speed of the center of mass after the disk has descended through distance h. Verify your answer to part (c) using the energy approach.

6.7. A spool of wire of mass M and radius R is unwound under a constant force $\vec{F}$ (Fig. 6.6). Assuming the spool is a uniform, solid cylinder that doesn't slip, show that (a) the acceleration of the center of mass and (b) the direction of force of friction it magnitude. (c) If the cylinder starts from rest and rolls without slipping, what is the speed of its center of mass after it has rolled through a distance d?

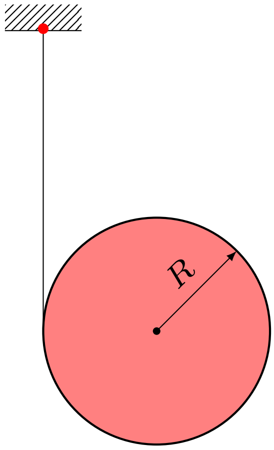


Figure 6.5. Problem 6.6

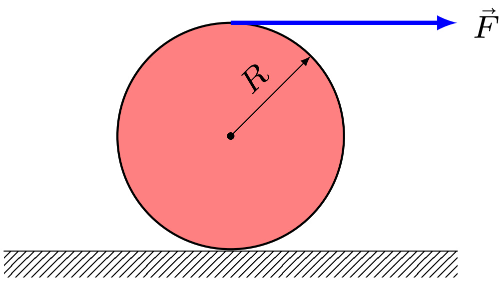


Figure 6.6. Problem 6.7

§ 6.3. Work and Rotational Kinetic Energy

6.8. A 32.0 kg wheel, essentially a thin hoop with radius 1.20 m, is rotating at 280 rev/min. It must be brought to a stop in 15.0 s. (a) How much work must be done to stop it? (b) What is the required average power?

6.9. The thin uniform rod in Fig. 6.7 has length 2.0 m and can pivot about a horizontal, frictionless pin through one end. It is released from rest at angle 40° above the horizontal. Use the principle of conservation of energy to determine the angular speed of the rod as it passes through the horizontal position.

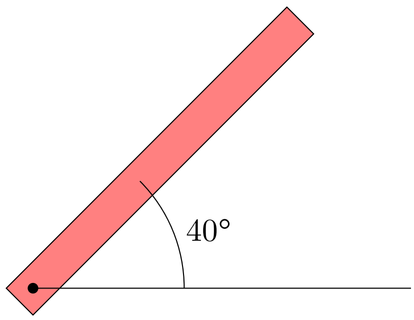


Figure 6.7. Problem 6.9

6.10. A thin rod of length 0.75 m and mass 0.42 kg is suspended freely from one end. It is pulled to one side and then allowed to swing like a pendulum, passing through its lowest position with angular speed 4.0 rad/s. Neglecting friction and air resistance, find (a) the rod's kinetic energy at its lowest position and (b) how far above that position the center of mass rises.

6.11. A uniform cylinder of radius 10 cm and mass 20 kg is mounted so as to rotate freely about a horizontal axis that is parallel to and 5.0 cm from the central longitudinal axis of the cylinder. (a) What is the rotational inertia of the cylinder about the axis of rotation? (b) If the cylinder is released from rest with its central longitudinal axis at the same height as the axis about which the cylinder rotates, what is the angular speed of the cylinder as it passes through its lowest position?

§ 6.4. Angular Momentum of a Rigid Body and its Conservation Law

6.12. A sanding disk with rotational inertia $1.2 \cdot 10^{-3}$ kg $\cdot$ m^2 is attached to an electric drill whose motor delivers a torque of magnitude 16 N $\cdot$ m about the central axis of the disk. About that axis and with the torque applied for 33 ms, what is the magnitude of the (a) angular momentum and (b) angular velocity of the disk?

6.13. A disk with a rotational inertia of 7.00 kg $\cdot$ m^2 rotates while undergoing a time-dependent torque given by $\tau = 5.00 + 2.00t$ (in N $\cdot$ m). At time $t = 1.00$ s, its angular momentum is 5.00 kg $\cdot$ m^2/s. What is its angular momentum at $t = 3.00$ s?

6.14. A horizontal vinyl record of mass 0.10 kg and radius 0.10 m rotates freely about a vertical axis through its center with an angular speed of 4.7 rad/s and a rotational inertia of $5.0 \cdot 10^{-4}$ kg $\cdot$ m^2/s. Putty of mass 0.020 kg drops vertically onto the record from above and sticks to the edge of the record.What is the angular speed of the record immediately afterwards?

6.15. A horizontal platform in the shape of a circular disk rotates on a frictionless bearing about a vertical axle through the center of the disk. The platform has a mass of 150 kg, a radius of 2.0 m, and a rotational inertia of 300 kg $\cdot$ m^2/s about the axis of rotation. A 60 kg student walks slowly from the rim of the platform toward the center. If the angular speed of the system is 1.5 rad/s when the student starts at the rim, what is the angular speed when she is 0.50 m from the center?

6.16. In Fig. 6.8, a 1.0 g bullet is fired into a 0.50 kg block attached to the end of a 0.60 m nonuniform rod of mass 0.50 kg. The block–rod–bullet system then rotates in the plane of the figure, about a fixed axis at A. The rotational inertia of the rod alone about that axis at A is 0.060 kg $\cdot$ m^2/s. Treat the block as a particle. (a) What then is the rotational inertia of the block–rod–bullet system about point A? (b) If the angular speed of the system aboutA just after impact is 4.5 rad/s, what is the bullet's speed just before impact?

6.17. In Fig. 6.9, a small 50 g block slides down a frictionless surface through height $h = 20$ cm and then sticks to a uniform rod of mass 100 g and length 40 cm.The rod pivots about point O through angle θ before momentarily stopping. Find θ.

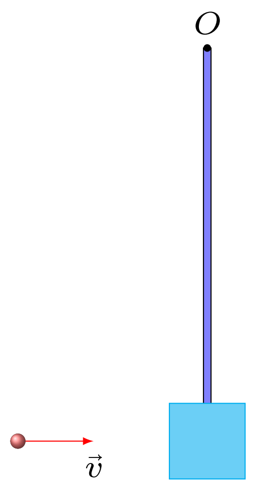


Figure 6.8. Problem 6.16

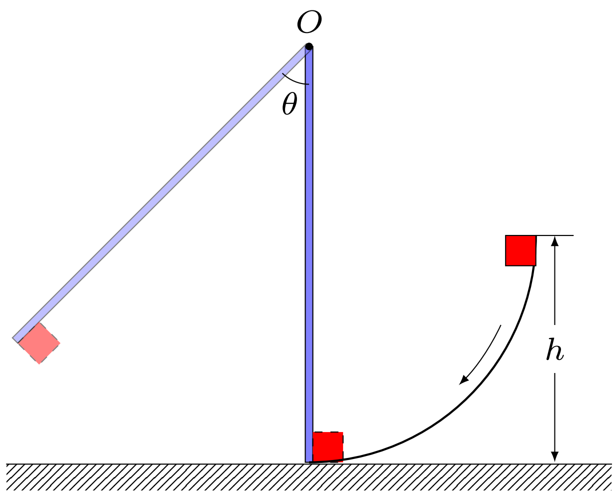


Figure 6.9. Problem 6.17

6.18. A projectile of mass m moves to the right with a speed $\vec{v}_i$ (Fig. 6.10a). The projectile strikes and sticks to the end of a stationary rod of mass M, length d, pivoted about a frictionless axle perpendicular to the page through O (Fig. 6.10b). (a) What is the angular momentum of the system before the collision about an axis through O? (b) What is the moment of inertia (rotational inertia) of the system about an axis through O after the projectile sticks to the rod? (c) If the angular velocity of the system after the collision is ω, what is the angular momentum of the system after the collision? (d) Find the angular velocity ω after the collision in terms of the given quantities. (e) What is the kinetic energy of the system before the collision? (f) What is the kinetic energy of the system after the collision? (g) Determine the fractional change of kinetic energy due to the collision.

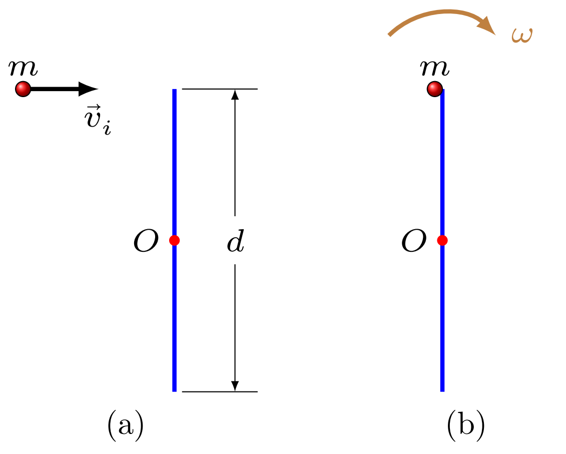


Figure 6.11. Problem 6.18

CHAPTER 7

Theory of Special Relativity

§ 7.1. Kinematics of Special Relativity

7.1.1 Simultaneity and Time Dilation

7.1. The mean lifetime of stationary muons is measured to be 2.2000 μs. The mean lifetime of high-speed muons in a burst of cosmic rays observed from Earth is measured to be 16.000 μs. To five significant figures, what is the speed parameter b of these cosmic- ray muons relative to Earth?

7.2. An unstable high-energy particle enters a detector and leaves a track of length 1.05 mm before it decays. Its speed relative to the detector was $0.992c$. What is its proper lifetime? That is, how long would the particle have lasted before decay had it been at rest with respect to the detector?

7.1.2 The Relativity of Length

7.3. A rod lies parallel to the x axis of reference frame K, moving along this axis at a speed of $0.630c$. Its rest length is 1.70 m.What will be its measured length in frame K?

7.4. A moving rod is observed to have a length of 2.00 m and to be oriented at an angle of 30.0° with respect to the direction of motion. The rod has a speed of $0.995c$. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame?

7.1.3 The Relativity of Velocities

7.5. A spaceship whose rest length is 350 m has a speed of $0.82c$ with respect to a certain reference frame. A micrometeorite, also with a speed of $0.82c$ in this frame, passes the spaceship on an antiparallel track. How long does it take this object to pass the ship as measured on the ship?

7.6. Galaxy A is reported to be receding from us with a speed of $0.35c$. Galaxy B, located in precisely the opposite direction, is also found to be receding from us at this same speed. What gives the recessional speed an observer on Galaxy A would find for (a) our galaxy and (b) Galaxy B?

§ 7.2. Dynamics of Special Relativity

7.2.1 Relativistic Energy and Momentum

7.7. A proton ($m_p = 1.6726219 \cdot 10^{-27}$ kg) moves at $0.950c$. Calculate in eV (1 eV $= 1.60217662 \cdot 10^{-19}$J) its (a) rest energy, (b) total energy, and (c) kinetic energy.

7.8. What is the momentum in MeV/c of an electron ($m_e = 9.10938356 \cdot 10^{-31}$) with a kinetic energy of 2.00 MeV?

7.9. How much work must be done to increase the speed of an electron from rest to (a) $0.500c$, (b) $0.990c$, and (c) $0.9990c$?

7.10. An unstable particle at rest spontaneously breaks into two fragments of unequal mass. The mass of the first fragment is $2.50 \cdot 10^{-28}$ kg, and that of the other is $1.67 \cdot 10^{-27}$ kg. If the lighter fragment has a speed of $0.893c$ after the breakup, what is the speed of the heavier fragment?

7.11. Particle A (with rest energy 200 MeV) is at rest in a lab frame when it decays to particle B (rest energy 100 MeV) and particle C (rest energy 50 MeV). What are the (a) total energy and (b) momentum of B and the (c) total energy and (d) momentum of C?

7.2.2 Relativistic Equation of Motion

7.12. A particle of rest mass m starts moving at a moment $t = 0$ due to a constant force F. Find the time dependence of the particle's (a) velocity and (b) of the distance covered.

7.13. A particle of rest mass m moves along the x axis of the frame K in accordance with the law $x = \sqrt{a^2 + c^2t^2}$, where a is a constant, c is the velocity of light, and t is time. Find the force acting on the particle in this reference frame.

CHAPTER

8

Mechanical Oscillations

§ 8.1. Kinematics of Simple Harmonic Motion

8.1. A simple harmonic oscillator takes 12.0 s to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in hertz, and (c) the angular frequency in radians per second.

8.2. A object with mass 0.500 kg attached to a spring with a force constant of 8.00 N/m vibrates in simple harmonic motion with an amplitude of 10.0 cm. Calculate the maximum value of its (a) speed and (b) acceleration, (c) the speed and (d) the acceleration when the object is 6.00 cm from the equilibrium position, and (e) the time interval required for the object to move from $x = 0$ to $x = 8.00$ cm.

8.3. In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression

$$x = 5.00 \cos\left(2t + \frac{\pi}{6}\right)$$

where x is in centimeters and t is in seconds. At $t = 0$, find (a) the position of the particle, (b) its velocity, and (c) its acceleration. Find (d) the period and (e) the amplitude of the motion.

8.4. A 1.00 kg object is attached to a horizontal spring. The spring is initially stretched by 0.100 m, and the object is released from rest there. It proceeds to move without friction. The next time the speed of the object is zero is 0.500 s later. What is the maximum speed of the object?

8.5. An object attached to a spring vibrates with simple harmonic motion as described by Figure 8.1. For this motion, find (a) the amplitude, (b) the period, (c) the angular frequency, (d) the maximum speed, (e) the maximum acceleration, and (f) an equation for its position x as a function of time.

8.6. What is the phase constant for the harmonic oscillator with the velocity function $v(t)$ given in Fig. 8.2 if the position function $x(t)$ has the form

$$x = A\cos(\omega t + \varphi)$$

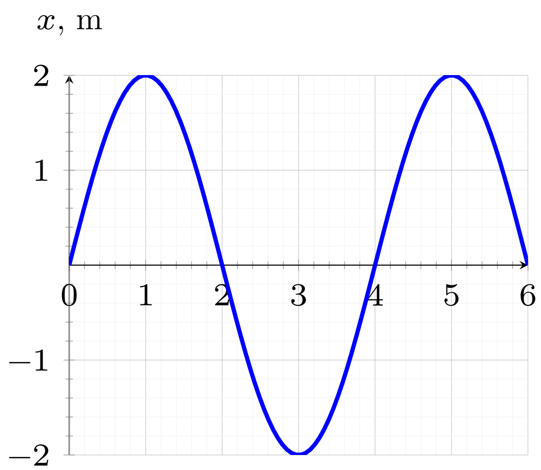


Figure 8.1. Problem 8.5

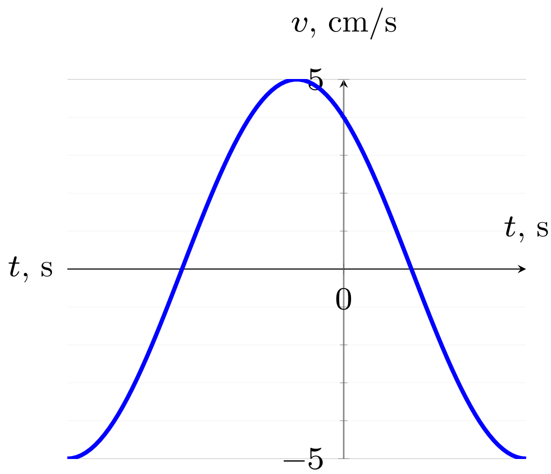


Figure 8.2. Problem 8.6

8.7. A particle with a mass of $1.00 \cdot 10^{-20}$ kg is oscillating with simple harmonic motion with a period of $1.00 \cdot 10^{-5}$ s and a maximum speed of $1.00 \cdot 10^{3}$ m/s. Calculate (a) the angular frequency and (b) the maximum displacement of the particle.

§ 8.2. Energy of the Simple Harmonic Oscillator

8.8. A 50.0 g object connected to a spring with a force constant of 35.0 N/m oscillates with an amplitude of 4.00 cm on a frictionless, horizontal surface. Find (a) the total energy of the system and (b) the speed of the object when its position is 1.00 cm. Find (c) the kinetic energy and (d) the potential energy when its position is 3.00 cm.

8.9. A simple harmonic oscillator of amplitude A has a total energy E. Determine (a) the kinetic energy and (b) the potential energy when the position is one-third the amplitude. (c) For what values of the position does the kinetic energy equal one-half the potential energy? (d) Are there any values of the position where the kinetic energy is greater than the maximum potential energy?

8.10. Figure 8.3 gives the one dimensional potential energy well for a 2.0 kg particle. (a) If the particle passes through the equilibrium position with a velocity of 85 cm/s, will it be turned back before it reaches $x = 15$ cm? (b) If yes, at what position, and if no, what is the speed of the particle at

$x = 15$ cm?

8.11. Figure 8.4 shows the kinetic energy K of a simple harmonic oscillator versus its position x. What is the spring constant?

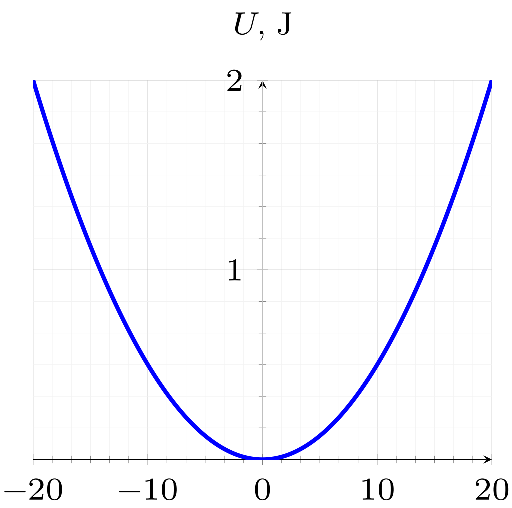


Figure 8.3. Problem 8.10

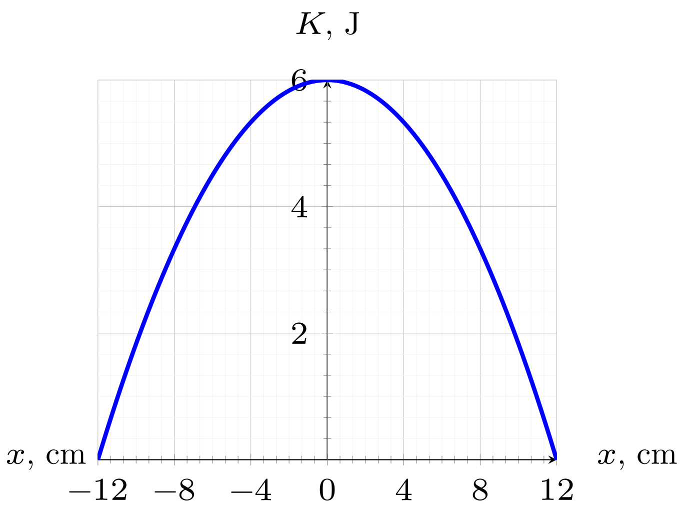


Figure 8.4. Problem 8.11

8.12. A simple harmonic oscillator consists of a 0.50 kg block attached to a spring. The block slides back and forth along a straight line on a frictionless surface with equilibrium point $x = 0$. At $t = 0$ the block is at $x = 0$ and moving in the positive x direction. A graph of the magnitude of the net force F on the block as a function of its position is shown in Fig. 8.5. What are (a) the amplitude and (b) the period of the motion, (c) the magnitude of the maximum acceleration, and (d) the maximum kinetic energy?

8.13. Figure 8.6 gives the position of a 20 g block oscillating on the end of a spring. What are (a) the maximum kinetic energy of the block and (b) the number of times per second that maximum is reached? (Hint: Measuring a slope will probably not be very accurate. Find another approach.)

8.14. A block of mass $M = 5.4$ kg, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant $k = 6000$ N/m. A bullet of mass $m = 9.5$ g and velocity of magnitude 630 m/s strikes and is embedded in the block (Fig. 8.8). Assuming the compression of the spring is negligible until the bullet is embedded, determine (a) the speed of the block immediately after the collision and (b) the amplitude of the resulting simple harmonic motion.

8.15. In Fig. 8.7, a solid cylinder attached to a horizontal spring ($k = 3.00$ N/m) rolls without slipping along a horizontal surface. If the system is released from rest when the spring is stretched by 0.250 m, find (a) the translational kinetic energy and (b) the rotational kinetic energy of the cylinder

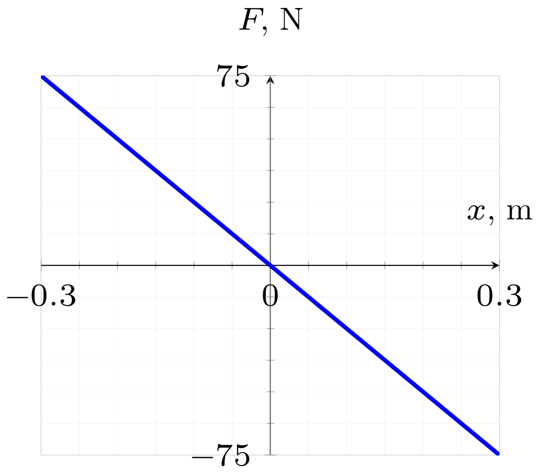


Figure 8.5. Problem 8.12

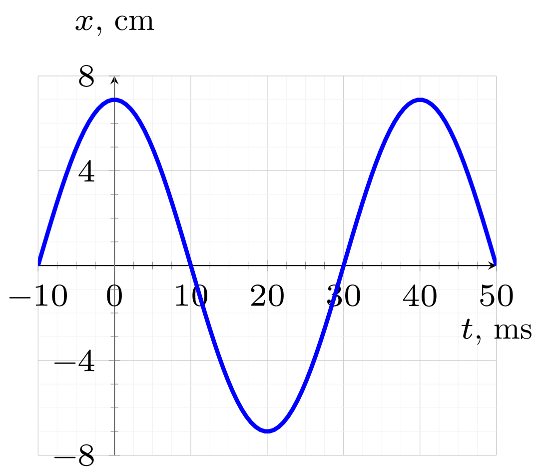


Figure 8.6. Problem 8.13

as it passes through the equilibrium position. (c) Show that under these conditions the cylinder's center of mass executes simple harmonic motion with period

$$T = 2\pi\sqrt{\frac{3M}{2k}},$$

where M — is the cylinder mass. (Hint: Find the time derivative of the total mechanical energy.)

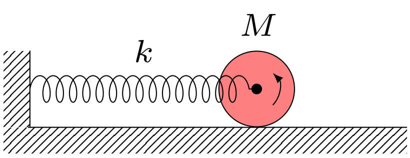


Figure 8.7. Problem 8.15

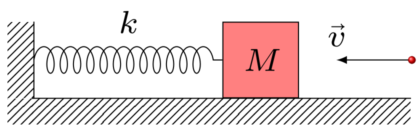


Figure 8.8. Problem 8.14

§ 8.3. Period of Small Oscillations of Physicsl Pendulum

8.16. In Fig. 8.9, a physical pendulum consists of a uniform solid disk (of radius $R = 2.35$ cm) supported in a vertical plane by a pivot located a distance $d = 1.75$ cm from the center of the disk.The disk is displaced by a small angle and released. What is the period of the resulting simple harmonic motion?

8.17. In Fig. 8.10, a stick oflength $L = 1.85$ m oscillates as a physical pendulum. (a) What value of distance x between the stick's center of mass and its pivot point O gives the least period? (b) What is that least period?

8.18. A uniform circular disk whose radius $R = 12.6$ cm is suspended as a physical pendulum from a point on its rim. (a) What is its period? (b) At what radial distance $r < R$ is there a pivot point that gives the same period?

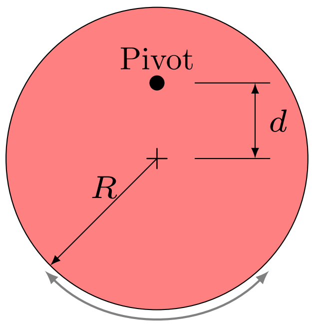


Figure 8.9. Problem 8.16

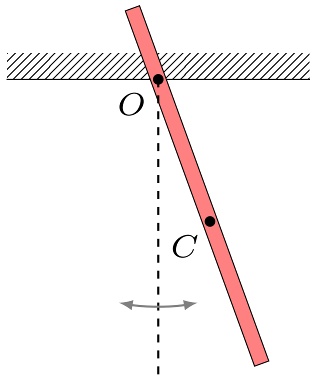


Figure 8.10. Problem 8.17

8.19. The 3.00 kg cube in Fig. 8.11 has edgelengths $d = 6.00$ cm and is mounted on an axle through its center. A spring (k1200 N/m) connects the cube's upper corner to a rigid wall. Initially the spring is at its rest length. If the cube is rotated 3 and released, what is the period of the resulting system?

8.20. In the overhead view of Fig. 8.12, a long uniform rod of mass 0.600 kg is free to rotate in a horizontal plane about a vertical axis through its center. A spring with force constant $k = 1850$ N/m is connected horizontally between one end of the rod and a fixed wall. When the rod is in equilibrium, it is parallel to the wall. What is the period of the small oscillations that result when the rod is rotated slightly and released?

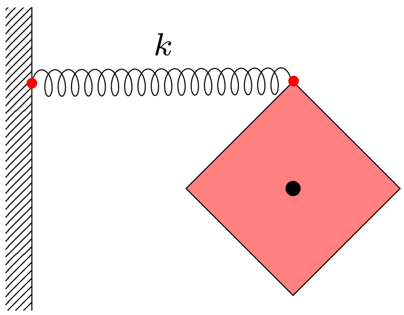


Figure 8.11. Problem 8.19

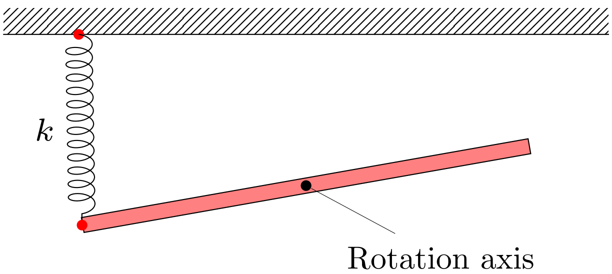


Figure 8.12. Problem 8.20

§ 8.4. Damped Oscillations

8.21. The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

8.22. A point performs damped oscillations with frequency co and damping coefficient β. Find the velocity amplitude of the point as a function of time t if at the moment $t = 0$ (a) its displacement amplitude is equal to a_0; (b) the displacement of the point a 0 and its velocity projection v_0.

8.23. For the damped oscillator system, the block has a mass of 1.50 kg and the spring constant is 8.00 N/m. The damping force is given by $-b(dx/dt)$, where $b = 230$ g/s. The block is pulled down 12.0 cm and released. (a) Calculate the time required for the amplitude of the resulting oscillations to fall to one-third of its initial value. (b) How many oscillations are made by the block in this time?

§ 8.5. Forced Oscillations and Resonance

8.24. A 2.00 kg object attached to a spring moves without friction and is driven by an external force given by the expression $F = 3.00 \sin(2\pi t)$, where F is in newtons and t is in seconds. The force constant of the spring is 20.0 N/m. Find (a) the resonance angular frequency of the system, (b) the angular frequency of the driven system, and (c) the amplitude of the motion.

8.25. A block weighing 40.0 N is suspended from a spring that has a force constant of 200 N/m. The system is undamped and is subjected to a harmonic driving force of frequency 10.0 Hz, resulting in a forced-motion amplitude of 2.00 cm. Determine the maximum value of the driving force.

8.26. Damping is negligible for a 0.150 g object hanging from a light, 6.30 N/m spring. A sinusoidal force with an amplitude of 1.70 N drives the system. At what frequency will the force make the object vibrate with an amplitude of 0.440 m?

§ 8.6. Superposition of Harmonic Oscillations

8.6.1 Superposition of Harmonic Oscillations of the same direction

8.27. Two simple harmonic oscillations are represented by

$$x_1 = 3\sin(20\pi t + {}^{\pi}/_{6})$$
$$x_2 = 4\sin(20\pi t + {}^{\pi}/_{3})$$

Find the (a) amplitude, (b) phase constant and (c) the period of resultant vibration.

8.28. The superposition of two harmonic oscillations of the same direction results in the oscillation of a point according to the law $x = a\cos(2.1t)\cos 50.0t$, where t is expressed in seconds. Find the (a) angular frequencies of the constituent oscillations and (b) the period with which they beat.

8.6.2 Superposition of of Harmonic Oscillations of the mutually perpendicular directions

8.29. Find the trajectory equation $y(x)$ of a point if it moves according to the following laws:

$$x = a\sin(\omega t)$$
$$y = a\sin(2\omega t)$$

Plot these trajectories.

8.30. Find the trajectory equation $y(x)$ of a point if it moves according to the following laws:

$$x = a\sin(\omega t)$$
$$y = a\cos(2\omega t)$$

Plot these trajectories.

Answers

Vectors

0.4. (a) 9.51 m; (b) 14.1 m; (c) 13.4 m; (d) 10.5 m.

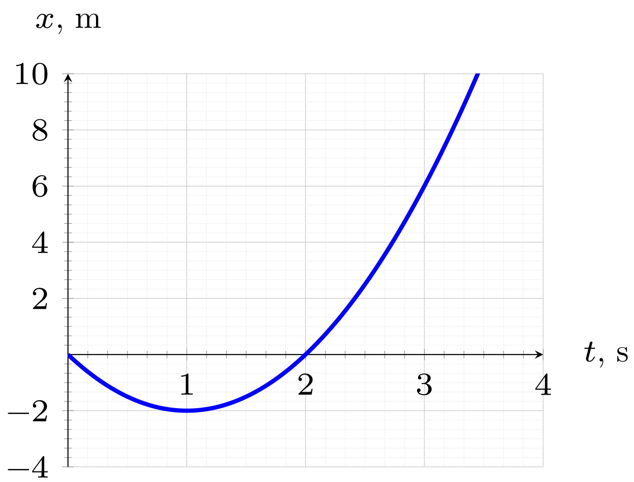
0.5. (a) $\vec{r} = \vec{r}_2 - \vec{r}_1 = -2.0\vec{i} + 7.0\vec{j} - 3.0\vec{k}$ (b) $r_1 = 9.4$, $r_2 = 11.4$, $r = 7.9$. (c) $\angle(\vec{r}_1, \vec{r}_2) = \arccos 0.7$, $\angle(\vec{r}_1, \vec{r}) = \arccos -0.15$, $\angle(\vec{r}_2, \vec{r}) = \arccos 0.6$. (d) -1.2. (e) $\vec{r}_1 \times \vec{r}_2 = -65\vec{i} - 4\vec{j} + 34\vec{k}$.

0.7. (a) 12; (b) $+z$; (c) 12; (d) $-z$; (e) (12; (f) $+z$.

Kinematics

1.6.

(a) Graph x versus t for the range $t = 0$ s to $t = 4$ s:



(b) Displacement of the particle in the time intervals $t = 0$ s to $t = 1$ s is $\Delta x = -2$ m, displacement of the particle in the time intervals $t = 1$ s to $t = 5$ s is $\Delta x = +8$ m.

(c) $\langle v \rangle = -2$ m/s.

(d) $v_x = 6$ m/s.

1.7. (a) $v = \frac{\alpha t^2}{2}$, $a = \frac{\alpha^2}{2}$; (b) $\langle v \rangle = \frac{\sqrt{s}}{2}$.

1.13. $\alpha = 69.3°$.

1.15. $a = \alpha\sqrt{1 + (4\pi n)^2}$ $0.8\ \mathrm{m/s^2}$.

1.17. $R = \frac{\alpha^3}{2\beta s}$, $a = \alpha\sqrt{1 + \left(\sqrt{\frac{4\beta s^2}{\alpha^3}}\right)}$.

1.18. (a) $s = A\omega\tau$, (b) $\pi/2$.

1.19. $\tan\alpha = \frac{2s}{R}$

1.24. (a) $\omega = 8.0$ rad/s, $\beta = 1.3$ rad/s^2; (b) 17°.

1.25. (a) $\varphi = \frac{\omega_0}{a}(1 - e^{-at})$; (b) $\omega = \omega_0 e^{-at}$.

Dynamics. Newtonian laws

2.5. (a) $a = 2.5\ \mathrm{m/s^2}$ (b) and $F = 30$ N.

2.12. 0.56

2.13. (a) 19°, (b) 3.3 kN.

2.15. $1.37 \cdot 10^3$ N.

2.16. $\approx 53°$.

2.17. $T = mg\left(\frac{v^2}{gR} + \cos\theta\right)$

2.19. (a) $\theta = \arccos\frac{2}{3}$, (b) $v = \sqrt{\frac{2}{3}gR}$.

Non-inertial reference frame. Inertial forces

3.1. $F = m\sqrt{g^2\omega^4 r^2 + 2 + (2v'\omega)^2} = 8$ N.

3.2. $F = 2m\omega^2 r\sqrt{1 + \left(\frac{v_0}{\omega r}\right)^2} = 2.8$ N.

3.3. (a) $a' = \omega^2 R$; (b) $F_\mathrm{n} = m\omega^2 r\sqrt{\left(\frac{2R}{r}\right) - 1}$.

3.4. (a) $F_\mathrm{cf} = m\omega^2 R\sqrt{\frac{5}{9}} = 8$ N; (b) $F_\mathrm{cor} = {}^2/_3 m\omega^2 R\sqrt{5 + \frac{8g}{3\omega^2 R}} = 17$ N.

Work and Energy. Laws of Conservations

4.1. (a) 1.50 J; (b) increases.

4.4. $F = 2\alpha s\sqrt{1 + (r/R)^2}$.

4.7. $W = -\frac{\mu mgl}{1-\mu\cot\theta} = -0.05$ J.

4.8. $W = 0$.

4.12. $W = \frac{m\alpha^4 t^2}{8}$.

4.16. $\frac{A}{r^2}$a way from the other particle.

4.18. (a) 2.1 m/s; (b) 10 N; (c) $+x$ direction; (d) 5.7 m; (e) 30 N; (f) $-x$ direction.

4.21. (a) 28 cm; (b) 2.3 m/s.

4.23. 53 m.

4.24. (a) 5.0 kg · m/s; (b) 10.0 kg · m/s.

4.25.

4.27. (a) $(1.00\vec{i} - 0.167\vec{j})$ km/s; (b) 3.23 MJ.

4.29. (a) 14 m/s; (b) 45°.

4.31. (a) 787 m/s; (b) 138 m/s.

4.33. (a) $1.57 \cdot 10^6$ N; (b) $1.35 \cdot 10^5$ kg; (c) 2.08 km/s.

4.34. (a) 42 N · s; (b) 2.1 kN

4.35. 40%.

4.36. $3.1 \cdot 10^2$ m/s.

4.37. (a) 721 m/s; (b) 937 m/s.

4.38. (a) 99 g; (b) 1.9 m/s; (c) 0.93 m/s.

4.39. (a) 1.2 kg; (b) 2.5 m/s.

4.42. 120°.

4.43. (a) 433 m/s; (b) 250 m/s.

4.44. (a) 6.9 m/s; (b) 30°; (c) 6.9 m/s; (d) −30°; (e) 2.0 m/s; (f) 180°.

4.46. (a) 0; (b) $8.0\vec{i} + 8.0\vec{k}$ (in N · m).

4.48. (a) 9.8 kg · m^2/s; (b) $+z$ direction.

4.49. $17.5\vec{k}$ kg · m^2/s.

4.52. (a) $48t$ N · m; (b) increasingю

4.53. $L = \frac{1}{3} m v_0 l$.

Universal Gravitation. Central-force problem

5.2. $2.4 \cdot 10^4$ m/s.

5.3. 0.03%.

5.4. $2.5 \cdot 10^4$ km.

5.5. (a) $5.0 = 10^9$ J; (b) $0.5 = 10^9$ J; (c) $3.5R_s$.

5.6. (a) $1.0 \cdot 10^3$ kg; (b) 1.5 km/s.

5.8. $E = -G\frac{Mm}{2a}$, where M is the mass of the Sun.

5.9. $\sqrt[3]{GM\left(T/2\pi\right)^2}$.

5.10. $T = \frac{2\pi r^{3/2}}{\sqrt{G(M + m/\sqrt{3})}}$.

5.11. $T = \frac{2\pi r^{3/2}}{\sqrt{G(M + m/4)}}$.

5.13. $r_{\min} = G\frac{M}{v_0^2}\left(\sqrt{1 + \left(\frac{l v_0^2}{GM}\right)^2} - 1\right)$, where M — is the mass of Sun.

5.14. 0.910 km/s.

Rigid body dynamics

6.1. (a) 2.5 kg; (b) ≈ 0.63 m.
6.2. (a) 0.023 $\text{kg}\cdot\text{m}^2$, (b) 1.1 mJ.
6.3. 0.140 N.
6.4. (a) 6.00 cm/s^2; (b) 4.87 N; (c) 4.54 N; (d) 1.20 rad/s^2; (e) 0.0138 $\text{kg}\cdot\text{m}^2$.
6.5. (a) $4.2\cdot10^2$ rad/s^2; (b) $5.0\cdot10^2$ rad/s.
6.6. (a) $\frac{1}{3}Mg$; (b) $\frac{2}{3}g$; (c) $\sqrt{\frac{4}{3}gh}$;
6.7. (a) $\frac{4}{3}\frac{\vec{F}}{M}$; (b) direction to the right, magnitude $\frac{F}{3}$.
6.8. (a) −19.8 kJ; (b) 1.32 kW.
6.9. 3.1 rad/s.
6.14. 3.4 rad/s.
6.18. (a) $\frac{mv_i d}{2}$; (b) $\left(\frac{M}{12}+\frac{m}{4}\right)d^2$; (c) $\left(\frac{M}{12}+\frac{m}{4}\right)d^2\omega$; (d) $\frac{6mv_i}{(M+3m)d}$; (e) $\frac{mv_i^2}{2}$; (f) $\frac{3m^2v_i^2}{2(M+3m)}$; (g) $-\frac{M}{M+3m}$.

Theory of Special Relativity

7.1. 0.99050.
7.2. 0.446 ps.
7.3. 1.32 m.
7.4. (a) 17.4 m; (b) 3.30°.
7.5. 1.2 µs.
7.6. (a) 0.35c; (b) 0.62c.
7.7. (a) 938 MeV; (b) 3.00 GeV; (c) 2.07 GeV.
7.8. 2.46 MeV/c.
7.10. 0.285c.
7.11. (a) 119 MeV; (b) 64.0 MeV/c; (c) 81.3 MeV; (d) 64.0 MeV/c.
7.12. (a) $v(t)=\frac{Fct}{\sqrt{m^2c^2+F^2t^2}}$; (b) $s(t)=\sqrt{\left(\frac{mc^2}{F}\right)^2+c^2t^2}-\frac{mc^2}{F}$.
7.13. $F=\frac{mc^2}{a}$.

Mechanical Oscillations

8.2. (a) 40.0 cm/s; (b) 160 cm/s^2; (c) 32.0 cm/s; (d) 296.0 cm/s^2; (e) 0.232 s.
8.4. 0.628m/s.
8.8. (a) 28.0 mJ; (b) 1.02 m/s; (c) 12.2 mJ; (d) 15.8 mJ.

8.9. (a) $\frac{8}{9}E$; (b) $\frac{1}{9}E$; (c) $x = \pm\sqrt{\frac{2}{3}}A$.

8.12. (a) 0.30 m; (b) 0.28 s; (c) $1.5 \cdot 10^2$ m/s^2; (d) 11 J.

8.13. (a) 1.2 J; (b) 50.

8.14. (a) 1.1 m/s; (b) 3.3 cm.

8.16. 0.366 s.

8.17. (a) 0.53 m; (b) 2.1 s.

8.20. 0.0653 s.

8.21. 7%.

8.22. (a) $v = a_0\sqrt{\omega^2 + \beta^2}e^{-\beta t}$; (b) $v = |v_0|\sqrt{1 + \left(\frac{\beta}{\omega}\right)^2}e^{-\beta t}$.

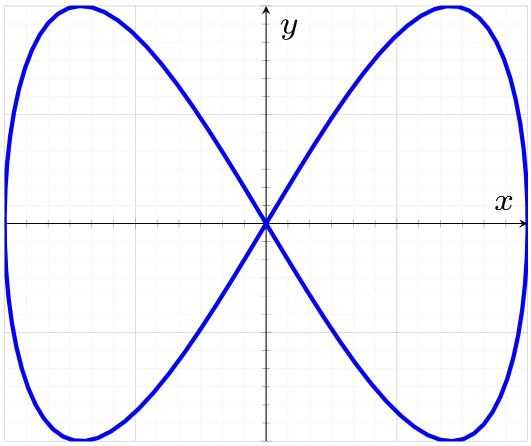
8.23. (a) 14.3 s; (b) 5.27.

8.24. (a) 3.16 1/s; (b) 6.28 1/s; (c) 5.09 cm.

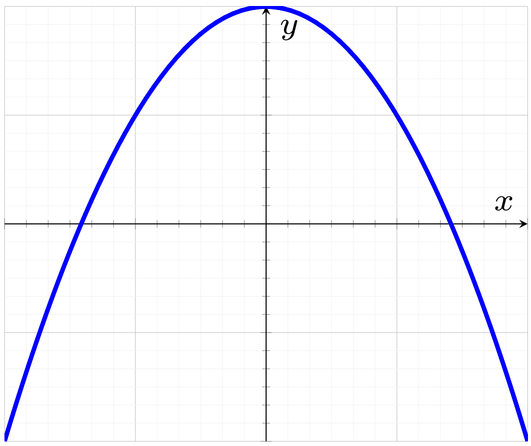
8.26. 0.641 Hz or 1.31 Hz.

8.28. (a) 47.9 1/s and 52.1 1/s; (b) 1.5 s.

8.29. $y^2 = 4x^2(1 - \frac{x^2}{a^2})$.



8.30. $y^2 = a(1 - \frac{2x^2}{a^2})$.



Bibliography

Textbooks

1. *Crawford F. S.* Waves, (Berkeley Physics Course, Vol. 3). — McGraw-Hill Book Company, 1968.
2. *Crowell B.* Mechanics. — Light, Matter. — eprint: `http://www.lightandmatter.com/mechanics/`.
3. *Feynman R., Lejton R., Sends M.* Lectures on physics. Vol. 1. Mainly mechanics, radiation, and heat. — New Millenium Edition. — Basic Books, 2010. — 968 p.
4. *Halliday D., Resnik R., Walker J.* Fundamentals of Physics. — 10th ed. — 1450 p.
5. *Irodov I. E.* Fundamental Laws of Mechanics. — CBS publishers & distributors, 2004. — ISBN 9788123903040.
6. Mechanics (Berkeley Physics Course, Vol. 1) / C. Kittel [et al.]. — McGraw-Hill Book Company, 1973. — ISBN 0070048800.

Problem books

7. *Irodov I. E.* Problems in general physics. — Revised. — Mir Publishers, 1988. — ISBN 9785030008004.