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INVESTIGATION OF THE INFLUENCE OF LOGARITHMIC CORRECTIONS AGAINST THE BACKGROUND OF TWO-PHOTON EFFECTS IN ELASTIC ELECTRON-DEUTERON SCATTERING

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Abstract

The aim of this work is to investigate the simultaneous influence of two-photon effects and logarithmic Quantum Chromodynamics(QCD) corrections on certain observed experimental quantities (structure factors A and B) in elastic electron-deuteron scattering. Analyzing these effects will expand our understanding of elementary particle physics, particularly the manifestations of quark-gluon degrees of freedom in the deuteron.

Keywords: elastic electron-deuteron scattering, deuteron, QCD, QED, quark, gluon, experimental data

Introduction

To date, a vast amount of experimental data on the interaction of polarized and unpolarized deuterons with large momentum transfer has been accumulated. This opens up new opportunities for studying the structure of the deuteron at distances smaller than the nucleon size. However, the comparison of predictions of perturbative Quantum Chromodynamics (pQCD), obtained from the analysis of quark-quark interactions, with available experimental data and other approaches to elastic scattering of electrons by polarized and unpolarized deuterons, remains insufficiently explored. In the asymptotic region, where the absolute value of the momentum transfer squared significantly exceeds the deuteron's mass squared, according to pQCD, predictions of the deuteron's form factor can be obtained based on the phenomena of asymptotic freedom and the factorization theorem. In this case, the deuteron is considered as a system of 6 quarks moving collinearly, each contributing to the deuteron's momentum fraction.

However, these pQCD predictions can be conditionally divided into two parts: predictions based on the "quark counting rules" (taking into account "cascade" pQCD diagrams), which are relatively well-supported by experimental comparisons [1, 2, 3, 4], and more subtle corrections, which, although derived from analyzing the dominant contributions of simpler ladder pQCD diagrams ("quark interchange" diagrams) [5, 6, 7, 8, 9], their contribution lacks stable experimental confirmation and therefore has not received as wide recognition as the predictions based on the "quark counting rules" [10, 11].

On the other hand, in recent decades, the role of higher-order perturbation theory beyond the single-photon approximation in electron scattering on hadronic systems has been widely discussed within the framework of Quantum Electrodynamics (QED). This is mainly associated with precise measurements of the electric and magnetic form factors of the proton conducted at the Thomas Jefferson Laboratory and their theoretical interpretation.

As a result of the aforementioned studies, it has been shown that taking into account two-photon contributions leads to qualitative changes in both the differential cross-section and polarization observables [12, 13].

However, considering individually both the two-photon contributions and the logarithmic corrections in pQCD, as mentioned earlier, has not led to a significant breakthrough in the experimental description of elastic electron-deuteron scattering [13, 5].

1. A brief theoretical description and problem statement

In the perturbative pQCD approach, at high energies, the masses of quarks and hadrons are neglected. In this case, the amplitude of the investigated process is expressed through the amplitude of hard electron-quark scattering, multiplied by the nonperturbative part (which can be associated with parameters N_i, as described in detail below), which is associated with the distribution functions of quarks and gluons in the deuteron in the initial and final states [14, 8, 9].

When calculating the amplitude of hard (specifically perturbative) scattering, the deuteron is considered as a system of 6 quarks moving collinearly, each of which contributes to the deuteron's momentum fraction: $x_i = p_i^+/P^+$, where

$$p_i^+ = p_i^0 + p_i^3, 0 < x_i < 1, \sum_i x_i = 1.$$

It is assumed that the main deuteron form factor in the perturbative region can be represented as the product of three factors, which exhibit dipole, power-law, and logarithmic behaviors, respectively [8, 9, 5].

$$F_1^d(Q^2) \sim N_1 G^2 \left(\frac{Q^2}{4}\right) \left[\frac{\alpha_s^5(Q^2)}{Q^2}\right] \left(\frac{\ln^{-2\gamma_d}(Q^2/\Lambda^2_{QCD})}{\ln^{-4\gamma_N}(Q^2/4\Lambda^2_{QCD})}\right).$$

where N_1 is the fitting parameter, $\alpha_s(Q^2)$ is the running strong coupling constant, and γ^d and γ^N is the so-called "anomalous dimensions" of deuteron and nucleon, which depends on the number of flavors and colors and Λ_{QCD} is the characteristic scale factor of QCD. It is assumed that the form factor $G(Q^2/4) = 1/(1 + Q^2/(4\mu^2))^2$ exhibits dipole behavior. The parameter μ^2 differs from the value 0.71 $(GeV/c)^2$ for a free nucleon and, consequently, takes into account the influence of the nuclear environment.

In the experimental data, two kinematic regions can be conventionally distinguished, separated by a characteristic stitching parameter Q_0^2. The first region is the "pQCD region" with relatively large values of the momentum transfer squared ($Q^2 > Q_0^2$). The second region is the low-energy "meson region" ($Q^2 < Q_0^2$). The high-energy region is parameterized according to pQCD predictions [8], [9], [5]. A "meson" approximation has been proposed for describing experimental data in the low-energy region [7], [11] [15]. The value of the parameter Q_0^2, as well as the validity of the adopted parameterizations of the low-energy and high-energy regions, are obtained from the analysis of experimental data.

On the other hand, in the two-photon approach, when virtual photons interact with nucleons, two types of two-photon exchange are calculated [12]:

$$M_2 = M_2^I + M_2^{II}. \tag{1}$$

The first amplitude M_2^I can be associated with Feynman diagrams in which two photons interact with the same nucleon, and the second amplitude M_2^{II} with diagrams in which each of the two virtual photons interacts with different nucleons. As shown in the studies [13], [16], these effects exhibit a conditionally logarithmic behavior.

2. Model description and obtained results.

As noted in the introduction, accounting for higher orders of perturbation theory in pQCD and QED separately did not lead to a significant improvement in describing experimental data. At the same time, it would be interesting to see how the degrees of freedom of quarks and gluons manifest in elastic electron-deuteron scattering against the background of two-photon contributions. For example, this can be done by comparing the theoretical description of experimental data on elastic $e - d$ scattering with and without consideration of two-photon corrections. It would be interesting to choose the most advanced theoretical model, taking into account the logarithmic corrections of pQCD, as the baseline theoretical model.

Based on the theoretical and experimental analysis conducted previously beyond the single-photon order of perturbation theory, we propose the following phenomenological parameterization of the two-photon corrections:

$$A(Q^2) = A_{one}(Q^2)(1 + a\ln^b(Q^2/\lambda^2_{EM})) \tag{2}$$

and

$$B(Q^2) = B_{one}(Q^2)(1 + a\ln^b(Q^2/\lambda^2_{EM})) \tag{3}$$

In this study, pQCD predictions are compared with experimental data for the structure functions $A(Q^2)$ and $B(Q^2)$. The first and second figures show the comparison of the asymptotic "logarithmic" prediction of pQCD with the proposed parameterization of two-photon corrections against the experiment (Figure 1, Figure 2). For comparison, the third and fourth figures present the same logarithmic pQCD prediction, but without considering the two-photon corrections (Figure 3, Figure 4). It has been found that the asymptotic logarithmic behavior predicted by pQCD slightly improves in the presence of two-photon corrections compared to without them (the chi-square value considering two-photon contributions $\chi^2_{2ph} = 3.801 \cdot 10^{-15}$ is lower than the chi-square value obtained without considering these contributions $\chi^2_{1ph} = 5.284 \cdot 10^{-14}$).

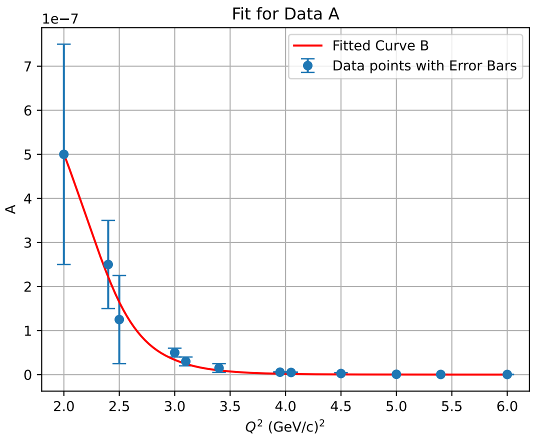

Figure 1. Optimal parameters for A:$N_1 = 4.934 \cdot 10^{-4}, N_2 = 1.499 \cdot 10^{-4}, N_3 = 9. \cdot 10^{-4}, a = 99.999, b = 6.133$ Error for A : $2.043 \cdot 10^{-8}$

The model utilizes the most reliable results based on a critical analysis of known experimental data. Comparison was conducted for models with 5 and 3 independent fitting parameters, respectively (plus two common parameters, which we fixed in both cases with the values $\lambda_{EM} = 1.1$ and $n_f = 4.91$). A sample of 17 points was taken, and a fitting criterion was obtained.

It is interesting to compare these results with the works of other authors [15, 4, 10, 7, 11]. To justify the proposed baseline model with logarithmic pQCD corrections, it should be noted that, as indicated in [5], in the work of [7], where anomalous dimensions were not considered, a poorer description was obtained with

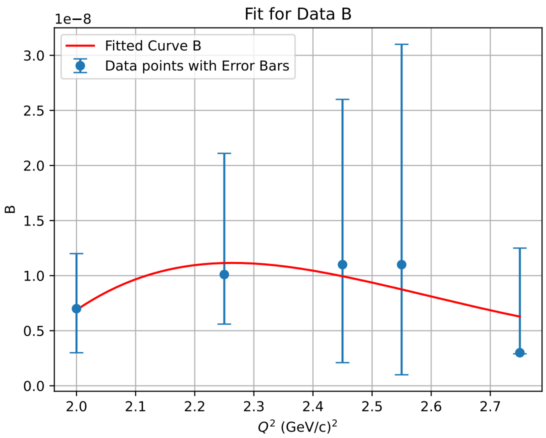

Figure 2. Optimal parameters for B: $N_1 = 1.622$, $N_2 = -2.444 \cdot 10^{-3}$, $N_3 = 7.947$, $a = 2.714$, $b = 1.000$ Error for B : $4.359 \cdot 10^{-8}$. Total χ^2_{2ph} : $3.801 \cdot 10^{-15}$

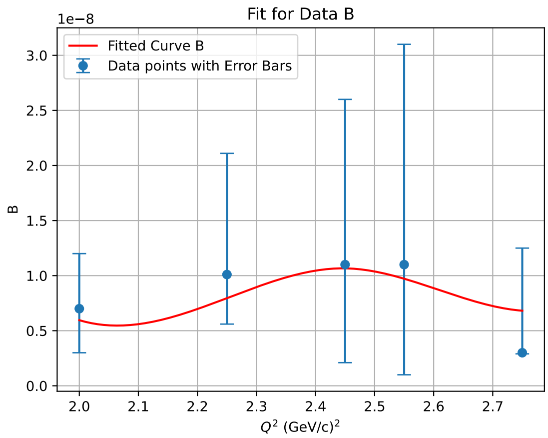

Figure 4. Optimal parameters for B: $N_1 = 2.848$, $N_2 = -1.902 \cdot 10^3$, $N_3 = 9.459$.
Error for B: $1.625 \cdot 10^{-7}$. Total χ^2_{1ph} : $5.284 \cdot 10^{-3}$

a larger number of fitting parameters (the number of fitting parameters was 13, as opposed to 10 in [5]). It can be argued that subsequent fittings also did not lead to significantly better results [10, 11].

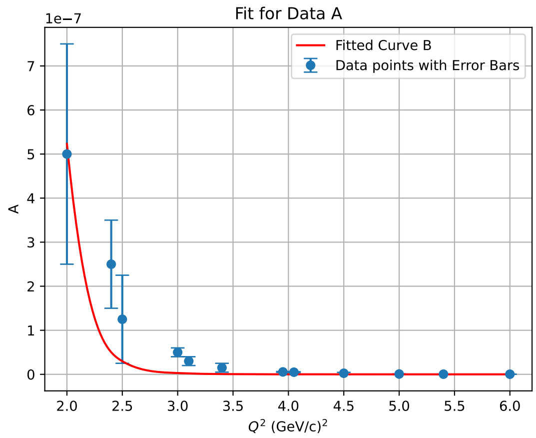

Figure 3. Optimal parameters for A: $N_1 = 1.300 \cdot 10^{-3}$, $N_2 = 1.487 \cdot 10^{-7}$, $N_3 = 6.812 \cdot 10^{-4}$ Error for A: $7.662 \cdot 10^{-8}$

Therefore, in our study, we used the parameterization that takes into account the logarithmic corrections of pQCD from [5] as the base, rather than the simple power-law parameterization from [7].

Summary

During this study, a review of the results of works [1, 12, 13, 5, 4, 10, 7, 11] on investigating the influence of two-photon effects and logarithmic corrections in elastic electron-deuteron scattering was conducted. The analysis of these works allows us to conclude the significance of these effects in studying the interaction between electrons and deuterons. Two-photon effects arising from the interaction of electrons and deuterons, largely due to their interference nature, demonstrate the importance of their consideration for achieving high precision in describing experimental data. Logarithmic corrections, as another key aspect of the research, have shown their unique impact on studying the internal structure of the deuteron.

As indicated by the above study, logarithmic corrections are more significant than two-photon effects, due to a smaller number of parameters (3 independent fitting parameters compared to 5 parameters), but substantially improve the fitting criterion only when combined with two-photon corrections (see Figure 1, Figure 2), Figure 3, Figure 4)).

The results of our comparison with experimental observations emphasize the importance of two-photon effects and logarithmic corrections in the process of electron-deuteron interaction, indicating the necessity of improving theoretical models to ensure consistency with experimental observations. They also point to the direction for further research in this area, namely conducting detailed experiments and expanding theoretical approaches to comprehensively account for all defining physical phenomena in elastic electron-deuteron scattering.

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