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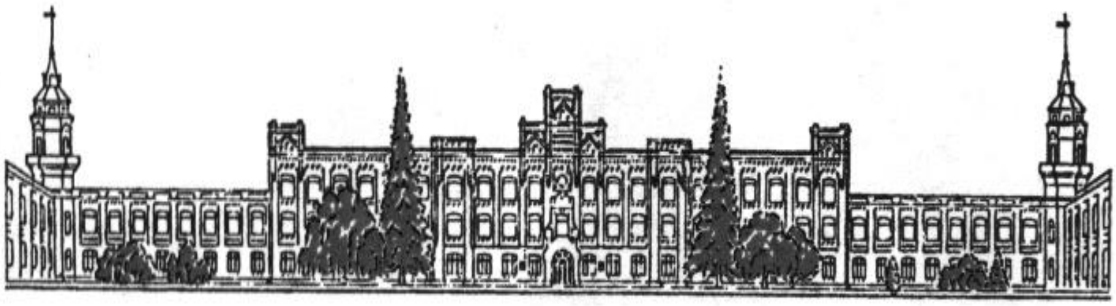

HIGHER MATHEMATICS

SERIES

Practice exercises collection

Recommended by the Methodological Council
of the Igor Sikorsky Kyiv Polytechnic Institute
as a study aid for bachelor's degree applicants
on the technical specialties

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The practice book offers additional individual exercises for university students studying Series in the course of Higher Mathematics of Igor Sikorsky KPI. The book contains 30 different variants and each variant consists of 12 exercises (21 tasks). Students master the material being studied and consolidate the acquired knowledge by solving such individual tasks.

The practice book can be recommended as an individual work on the Series section for first-year students of technical specialties.

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CONTENT

INTRODUCTION

The Series section is included in the course of Higher Mathematics for engineering students of Igor Sikorsky KPI. An important factor in the successful assimilation of the educational material by the students is solving practical tasks on their own.

The practice book offers a systematized set of exercises that students of technical specialties should be able to solve when studying Series. The book contains 30 different variants and each variant consists of 12 exercises (21 tasks).

This practice book helps students to develop practical skills in solving basic exercises: to determine convergence or divergence of positive series, to establish whether alternating series converge absolutely or conditionally, to identify the interval and radius of convergence of power series, to find the Fourier sine and cosine series for given functions.

GENERAL RECOMMENDATIONS

The practice book is designed to control and improve the knowledge of university students in the study of Series in the course of Higher Mathematics. The main goal is to develop and consolidate the skills of independent work of students in the study of educational material.

In order to successfully complete the exercises, students need to thoroughly study the lecture material and analyze the examples solved in practical classes. Only after that students can start solving their individual tasks.

Students have to adhere to the following requirements:

1. The number of the variant of the individual exercises corresponds to the ordinal number of the student in the list of the study group;
2. Individual work is written in a separate notebook, which should contain:
   - the title page;
   - the results table;
   - solved exercises (the solution of each exercise starts from a new page).
3. Before solving each exercise, the condition and all specific data for the corresponding variant are completely rewritten.
4. The solution of each task must contain detailed explanations and necessary formulas.
5. Completed work must be handed over to the teacher for verification within the prescribed time limit.

Students who do not submit their completed individual work on time will not be allowed to take the exam.

Series

Variant №1

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=11}^{\infty} \frac{3}{n^2 - 17n + 70}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{\operatorname{arctg}^2 n}{n(n+1)(n+2)}$; b) $\sum_{n=1}^{\infty} (n+1)^2 \left(e^{\frac{1}{n^3}} - 1\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty} \frac{n!(n+3)!}{(2n+1)!}$; b) $\sum_{n=1}^{\infty} \left(\frac{3n^2+6}{2n^2+4}\right)^{n^2}$; c) $\sum_{n=1}^{\infty} \frac{2n}{(n^2+4)\ln(n^2+4)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \sin\frac{1}{n} \cdot \operatorname{tg}\frac{1}{n^2}$; b) $\sum_{n=0}^{\infty} (-1)^{n-1} \frac{2n}{\sqrt[3]{3n^3+2}}$; c) $\sum_{n=3}^{\infty} (-1)^n \frac{1}{n\sqrt{\ln(n-1)}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n(3n+1)}$ to within $\alpha = 0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=0}^{\infty} (-1)^n \frac{(x^2-x-1)^n}{5^n(n+1)}$; b) $\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{\ln^n(n+1)}$; c) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(n!)^2}{(2n)!}(x-3)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x+4)^n}{(n^2+1)\sqrt{n}}$ on the closed interval $[-5;-3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n+1}}{2n+1}$; b) $\sum_{n=1}^{\infty} (-1)^{n+1}(n+2)x^{n+1}$.

Exercise 9. Find the Taylor series of the function $f(x) = \ln\frac{1}{x^2+4x+13}$ at $x = -2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,5} x\ln(1+x^2)\,dx$ to within $\alpha = 0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'' + y\cos x = \sin x$, $y(\pi) = 1$, $y'(\pi) = 0$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} -1, & -1 < x < 0, \\ x-1, & 0 < x < 1 \end{cases}$ on the interval $(-1,1)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = x - \frac{\pi}{2}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №2

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty} \frac{2}{n^2+4n+3}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=2}^{\infty} \frac{n(2-\sin n)}{(n-1)^2}$; **b)** $\sum_{n=1}^{\infty} \frac{1}{(n+1)} \operatorname{tg} \frac{(n+3)^2}{n^3}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty} \frac{5^n n!}{2^{2n}(n+1)!}$; **b)** $\sum_{n=1}^{\infty} 2^n \left(\frac{n+6}{4n-2}\right)^n$; **c)** $\sum_{n=2}^{\infty} \frac{3}{(3n-2)\sqrt{\ln^3(3n-2)}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty} (-1)^n n\left(1-\cos\frac{3}{\sqrt{n}}\right)$; **b)** $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n}{n!}$; **c)** $\sum_{n=0}^{\infty} (-1)^{n+1} \arcsin\sqrt{\frac{2}{n+3}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(n+1)^{2n}}$ to within $\alpha = 0,00001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(n+\mathrm{e})}{\ln^n(x+\mathrm{e})}$; **b)** $\sum_{n=1}^{\infty} (-1)^{n+1}(n+1)!\,x^{n+2}$; **c)** $\sum_{n=1}^{\infty} \frac{(x+1)^n}{3^n+2^n}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-4)^n}{(n+2)\ln^2(n+2)}$ on the closed interval $[3;5]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty} (-1)^n (n+1)x^n$; **b)** $\sum_{n=2}^{\infty} \frac{x^{2n-3}}{2n-3}$.

Exercise 9. Find the Taylor series of the function $f(x) = (x \operatorname{ctg} x - 1)\sin x$ at $x = 0$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^1 \cos x^2 dx$ to within $\alpha = 0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'' = xyy'$, $y(0) = 1$, $y'(0) = 1$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} x+2, & -2 < x < 0, \\ 2, & 0 < x < 2 \end{cases}$ on the interval $(-2, 2)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = 2x - \pi$ on the interval $(0, \pi)$; graph the function and the sum of each series.

Series

Variant №3

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=10}^{\infty}\dfrac{6}{n^2-15n+54}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=1}^{\infty}\dfrac{\sqrt{n}\ln n}{n^3+1}$; **b)** $\sum\limits_{n=1}^{\infty}\dfrac{\sqrt{n-1}}{n+1}\sin\dfrac{\pi}{n^2}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=1}^{\infty}\dfrac{n^n}{2^n(n+2)!}$; **b)** $\sum\limits_{n=1}^{\infty}\left(\dfrac{4n+1}{3n-2}\right)^{2n}$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{1}{n\left(\ln^2(3n)+1\right)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{(2-\cos n)}{5n}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n n\ln\left(1+\dfrac{1}{n}\right)$; **c)** $\sum\limits_{n=0}^{\infty}(-1)^{n+1}\left(\dfrac{2n-1}{3n+1}\right)^n$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=2}^{\infty}\dfrac{(-1)^{n+1}}{3^n(n-1)!}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{nx^2}{e^{n(x^2-x)}}$; **b)** $\sum\limits_{n=1}^{\infty}\dfrac{3^n n!\,x^{3n}}{(3n)!}$; **c)** $\sum\limits_{n=1}^{\infty}(-1)^n\left(\dfrac{n+1}{n}\right)^{n^2}(x-4)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=0}^{\infty}\dfrac{(\pi-x)}{\sqrt[3]{n^8+2}}\cos^2 nx$ on the closed interval $[0;\pi]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=1}^{\infty}\dfrac{x^{2n-1}}{2n-1}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n 4n\,x^{4n-1}$.

Exercise 9. Find the Taylor series of the function $f(x)=(x+2)e^{4x-x^2}$ at $x=2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{0,5}\dfrac{dx}{\sqrt[3]{x^3+1}}dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''+y=e^x+xy'$, $y(0)=1$, $y'(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}3, & -3<x<0,\\ 3-x, & 0<x<3\end{cases}$ on the interval $(-3,3)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=x-\dfrac{\pi}{4}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №4

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=0}^{\infty}\frac{4}{n^2+6n+8}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=2}^{\infty}\frac{\sqrt{n+1}(3+\cos n)}{n-1}$; **b)** $\sum\limits_{n=1}^{\infty} n\ln\frac{n^2+3}{n^2+2}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=2}^{\infty}\frac{n^5(n+1)!}{4^n(n-1)!}$; **b)** $\sum\limits_{n=1}^{\infty} n^3\left(\frac{2n}{3n+2}\right)^n$; **c)** $\sum\limits_{n=1}^{\infty}\frac{2}{(2n+3)\sqrt{\ln(2n+3)}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty}(-1)^n\frac{1}{n!}\operatorname{arctg}\frac{1}{2n}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n^3+2}}{2n^2+n}$; **c)** $\sum\limits_{n=0}^{\infty}(-1)^{n-1}\left(\frac{5n-1}{4n+7}\right)^{2n}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=0}^{\infty}\frac{(-1)^n}{2+5n^4}$ to within $\alpha=0{,}01$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=0}^{\infty}\frac{12^n(n+1)}{(x^2-5x-2)^n}$; **b)** $\sum\limits_{n=0}^{\infty}(-1)^n(n+2)!\,x^{n+3}$; **c)** $\sum\limits_{n=1}^{\infty}\frac{(x+2)^n}{2n-1}\sin\frac{1}{3^n}$;

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\frac{2^{n-1}}{(4n-1)^2}x^{2n-1}$ on the closed interval $[-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=1}^{\infty}(-1)^n n\,x^{n-1}$; **b)** $\sum\limits_{n=0}^{\infty}\frac{x^{2n+2}}{2n+2}$.

Exercise 9. Find the Taylor series of the function $f(x)=\ln(8+4x)$ at $x=1$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{0,5}\frac{\sin x^2}{x}dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'=y^3+x^2$, $y(1)=1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-x-4, & -4<x<0,\\ -4, & 0<x<4\end{cases}$ on the interval $(-4,4)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=x-\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №5

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=9}^{\infty}\frac{9}{n^2-13n+40}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty}\frac{\sqrt{n-1}}{n(n+1)^2(\pi-\operatorname{arctg} n)}$; **b)** $\sum_{n=1}^{\infty} n^2\left(1-\cos\frac{1}{\sqrt{n^3+3}}\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{(n!)^2}{2^n(2n-1)!}$; **b)** $\sum_{n=1}^{\infty}\frac{1}{3^n}\left(1+\frac{1}{n}\right)^{2n^2}$; **c)** $\sum_{n=1}^{\infty}\frac{1}{n\sqrt[3]{\ln(2n)+1}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{\sin^2 n}{3n^2+2}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n-1} n \operatorname{tg}\frac{1}{5n-1}$; **c)** $\sum_{n=1}^{\infty}\frac{(-1)^n}{n\ln(n+4)}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{(n+2)^n}$ to within $\alpha = 0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}\frac{2^n\sin^n x}{3^{\frac{n}{2}}n^2}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{5^n x^n}{n!}$; **c)** $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(x-1)^n}{(n+3)\ln^2(n+3)}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}\frac{(x+1)^n}{(n^3+4)\sqrt[3]{n}}\cos^2 nx$ on the closed interval $[-2;0]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{n+1}}{n+1}$; **b)** $\sum_{n=2}^{\infty}(-1)^n(2n-3)x^{2n-4}$.

Exercise 9. Find the Taylor series of the function $f(x)=\sin\frac{\pi x}{4}$ at $x=2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,2}\frac{1-e^{-x}}{x}dx$ to within $\alpha = 0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'' = yy' - x^2$, $y(0)=1$, $y'(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-5, & -5<x<0,\\ x-5, & 0<x<5\end{cases}$ on the interval $(-5,5)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\frac{x}{2}-\frac{\pi}{2}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №6

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=0}^{\infty} \frac{6}{n^2+8n+15}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=1}^{\infty} \frac{n+2}{n\sqrt{n}(3-\sin n)}$; **b)** $\sum\limits_{n=1}^{\infty} \sqrt{n^2+1}-\sqrt{n^2-1}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=2}^{\infty} \frac{3^n n!}{e^{2n}(n-1)!}$; **b)** $\sum\limits_{n=2}^{\infty} \frac{n^2}{\ln^n n}$; **c)** $\sum\limits_{n=1}^{\infty} \frac{n}{(n^2+5)\ln\sqrt{n^2+5}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty} (-1)^{n-1}\left(\frac{7n^3-1}{5n^3+2}\right)^n$; **b)** $\sum\limits_{n=1}^{\infty} (-1)^{n+1} n^2 \operatorname{arctg} \frac{1}{2^n}$; **c)** $\sum\limits_{n=0}^{\infty} (-1)^n \frac{2n+1}{n^2+4}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n!(3n-2)}$ to within $\alpha = 0{,}01$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{4n^2} e^{n(x^2-5x)+x\sqrt{n}}$; **b)** $\sum\limits_{n=1}^{\infty} (-1)^{n+1} n^n (x+4)^n$; **c)** $\sum\limits_{n=1}^{\infty} \frac{((n+1)!)^2}{(2n)!}(x-2)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=0}^{\infty} (-1)^n (x-2)^n \sin\frac{\pi}{5^n}$ on the closed interval $[1;3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=2}^{\infty} (-1)^n (n-1)x^{n-2}$; **b)** $\sum\limits_{n=2}^{\infty} (-1)^{n-1} \frac{x^n}{n}$.

Exercise 9. Find the Taylor series of the function $f(x) = \frac{1}{2x+6}$ at $x = 4$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{0,5} \frac{\operatorname{arctg} x}{x}\,dx$ to within $\alpha = 0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y' = \cos y + 2x, \quad y(0) = 0$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} x+6, & -6 < x < 0, \\ 6, & 0 < x < 6 \end{cases}$ on the interval $(-6, 6)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = x - 2\pi$ on the interval $(0, \pi)$; graph the function and the sum of each series.

Series

Variant №7

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=8}^{\infty}\frac{12}{n^2-11n+28}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=3}^{\infty}\frac{\sqrt{n-3}}{(n^2+4)\ln n}$; **b)** $\sum_{n=1}^{\infty}\frac{1}{n^2}\arcsin\frac{n}{\sqrt{n^3+5}}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{(2n+1)!}{(n+1)!(n+3)!}$; **b)** $\sum_{n=1}^{\infty}\left(\frac{4n^2+3}{5n^2-4}\right)^{n^2}$; **c)** $\sum_{n=1}^{\infty}\frac{1}{n\left(\ln(5n)+7\right)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\sqrt{n}\left(1-\cos\frac{2}{\sqrt{n}}\right)$; **b)** $\sum_{n=0}^{\infty}(-1)^n\sqrt[3]{\frac{n^2-1}{3n^2+2n+1}}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(n!)^2}{(2n)!}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^n\frac{5n+8}{6^n}$ to within $\alpha=0{,}01$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}\frac{(x^2-2x-3)^n}{5^n(n+2)}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{\ln^n(n^2+1)}$; **c)** $\sum_{n=1}^{\infty}(x-3)^n\operatorname{arctg}\frac{1}{5^n}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+3)^n}{4^n(2n-5)}$ on the closed interval $[-4;-2]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=2}^{\infty}(-1)^n\frac{x^{n-1}}{n-1}$; **b)** $\sum_{n=1}^{\infty}5n\;x^{5n-1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\ln(x^2+6x+12)$ at $x=-3$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,1}\frac{\ln(1+x)}{x}dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'=xy+e^y$, $y(0)=1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}7, & -7<x<0,\\ 7-x, & 0<x<7\end{cases}$ on the interval $(-7,7)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\frac{x}{2}-2\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №8

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty}\frac{8}{n^2+10n+24}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty}\frac{n-1}{\sqrt{n^5}(2-\cos n)}$; **b)** $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}\operatorname{arctg}\frac{1}{\sqrt{n+1}}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{3^{n-1}(2n)!}{2^n(2n+1)!}$; **b)** $\sum_{n=2}^{\infty}\frac{1}{4^n}\left(\frac{5n+2}{n-1}\right)^n$; **c)** $\sum_{n=1}^{\infty}\frac{2\ln n}{n(\ln^4 n+9)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=0}^{\infty}(-1)^{n+1}n\ln\frac{n^3+2}{n^3+1}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n-1}n^2\sin\frac{\pi}{\sqrt{n^4+2}}$; **c)** $\sum_{n=3}^{\infty}(-1)^n\frac{\sqrt{n+3}-\sqrt{n-3}}{\sqrt{n}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n+3}{n^5(n+1)}$ to within $\alpha=0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=3}^{\infty}(-1)^{n-1}\frac{\ln^n(x-\mathrm{e})}{n-\mathrm{e}}$; **b)** $\sum_{n=2}^{\infty}(-1)^n(n+5)!(x-1)^{n-2}$; **c)** $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}(x+3)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^n\frac{(x+1)^{2n}}{9^n\sqrt{n+3}}$ on the closed interval $[-3;1]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=2}^{\infty}(2n-3)x^{2n-4}$; **b)** $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{x^{2n-3}}{2n-3}$.

Exercise 9. Find the Taylor series of the function $f(x)=(x-\operatorname{tg}x)\cos x$ at $x=0$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,5}\sin(4x^2)\,dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''=xy+(y')^2,\quad y(1)=1,\quad y'(1)=0.$

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-8, & -8<x<0,\\ x-8, & 0<x<8\end{cases}$ on the interval $(-8,8)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\frac{1}{4}(x-\pi)$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №9

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=7}^{\infty}\frac{15}{n^2-9n+18}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=2}^{\infty}\frac{\sqrt{n}(\pi+\operatorname{arctg} n)}{\sqrt{n^3-1}}$; **b)** $\sum\limits_{n=1}^{\infty}\frac{1}{\sqrt{n+1}}\left(3^{\frac{1}{n}}-1\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=1}^{\infty}\frac{(2n)^n}{3^n(n+4)!}$; **b)** $\sum\limits_{n=1}^{\infty}\left(\frac{3n+2}{4n-3}\right)^{3n}$; **c)** $\sum\limits_{n=1}^{\infty}\frac{1}{n\left(\ln(6n)+3\right)^2}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=0}^{\infty}(-1)^{n+1}\arcsin\frac{\pi}{\sqrt{3n+1}}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n\frac{1}{4^n}\left(1+\frac{1}{n}\right)^{n^2}$; **c)** $\sum\limits_{n=1}^{\infty}(-1)^{n-1}n^2\ln\left(1+\frac{1}{n^2}\right)$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}(-1)^n\frac{n^4}{(2n)!\,n!}$ to within $\alpha=0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\frac{(n+1)x}{2^{n(4x-x^2)}}$; **b)** $\sum\limits_{n=1}^{\infty}\frac{x^{2n-1}}{(2n-1)(2n-1)!}$; **c)** $\sum\limits_{n=1}^{\infty}\frac{(x-5)^n}{(n+1)\sqrt{\ln(n+1)}}$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=1}^{\infty}(-1)^n\frac{(x+2)^{n^2}}{n^n}$ on the closed interval $[-3;-1]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=1}^{\infty}(-1)^n\frac{x^{2n+3}}{2n+3}$; **b)** $\sum\limits_{n=1}^{\infty}(n+2)x^{n+1}$.

Exercise 9. Find the Taylor series of the function $f(x)=x\,\mathrm{e}^{-2x+6}$ at $x=1$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^1\frac{dx}{\sqrt[3]{x^3+8}}$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'=y^2+x,\quad y(0)=1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}x+1, & -1<x<0,\\ 1, & 0<x<1\end{cases}$ on the interval $(-1,1)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\frac{x}{2}-\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №10

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=0}^{\infty}\frac{2}{n^2+12n+35}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=1}^{\infty}\frac{\sin^2 n}{\sqrt{n^3+3n+1}}$; **b)** $\sum\limits_{n=1}^{\infty}\sqrt{n}\left(e^{\frac{n}{n^2+1}}-1\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=1}^{\infty}\frac{2^n(n+2)!}{n^3 n!}$; **b)** $\sum\limits_{n=1}^{\infty}n^4\left(\frac{4n}{2n+3}\right)^n$; **c)** $\sum\limits_{n=2}^{\infty}\frac{2n}{(n^2-2)\ln^3(n^2-2)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}}{n\sqrt{\ln(n+7)}}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n n^3\left(1-\cos\frac{1}{\sqrt{n^3}}\right)$; **c)** $\sum\limits_{n=0}^{\infty}(-1)^{n+1}\frac{n^2 3^n}{4^n}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{2^{2n}n!}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=0}^{\infty}\frac{10^n n^3}{(x^2-4x-2)^n}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n-1}(n+3)!(x-3)^{2n}$; **c)** $\sum\limits_{n=1}^{\infty}(x+4)^n\arcsin\frac{1}{\sqrt{n+1}}$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{n^n}(x+3)^n$ on the closed interval $[-5;-1]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=1}^{\infty}(-1)^n 6n\, x^{6n-1}$; **b)** $\sum\limits_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n+1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\ln(20-5x)$ at $x=2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{0,5}\frac{1-\cos x}{x^2}dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''=x+y\cos x,\quad y(0)=1,\quad y'(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}2, & -2<x<0,\\ 2-x, & 0<x<2\end{cases}$ on the interval $(-2,2)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\frac{x}{2}-\frac{\pi}{4}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №11

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=6}^{\infty}\frac{18}{n^2-7n+10}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=3}^{\infty}\frac{\sqrt{n^2+6}}{(n-2)\ln n}$; **b)** $\sum_{n=1}^{\infty}(n+2)^3\operatorname{tg}\frac{\sqrt{n+1}}{n^4}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{(n!)^2(e^n+1)}{(2n)!}$; **b)** $\sum_{n=1}^{\infty}\ln^n\left(\frac{2n-1}{2n+2}\right)$; **c)** $\sum_{n=1}^{\infty}\frac{3}{(n+5)\sqrt{\ln(n+5)^3}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=0}^{\infty}(-1)^{n+1}n^3\sin\frac{\pi}{3^n}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3n}{\sqrt{5n^2+n}}$; **c)** $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{2n+1}-\sqrt{2n-1}}{\sqrt{n}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^n\frac{n^3}{(2n-1)!}$ to within $\alpha=0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^{\frac{n}{2}}}{2^n\cos^n x}$; **b)** $\sum_{n=1}^{\infty}\frac{x^{3n}}{\ln^n(n^3+4)}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+2)^n}{3^n+4^n}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=0}^{\infty}(-1)^n\frac{(x+\pi)}{(n^2+1)\sqrt[3]{n}}\cos^2 nx$ on the closed interval $[-2\pi;0]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}$; **b)** $\sum_{n=1}^{\infty}(-1)^n 2n\,x^{2n-1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\cos\frac{\pi x}{6}$ at $x=-3$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,3}e^{-2x^2}dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''+y\,e^x+4y'=0$, $y(0)=1$, $y'(0)=-1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-x-3, & -3<x<0,\\ -3, & 0<x<3\end{cases}$ on the interval $(-3,3)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=x+\frac{\pi}{2}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №12

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty}\frac{4}{n^2+14n+48}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty}\frac{n^3+2}{n^4(2+\cos n)}$; **b)** $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n^2-1}}\ln\frac{n+1}{n}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{n^n}{e^{2n}(n+1)!}$; **b)** $\sum_{n=1}^{\infty}\frac{3^n}{n}\left(\frac{2n+5}{8n+1}\right)^n$; **c)** $\sum_{n=1}^{\infty}\frac{3\ln^2 n}{n(\ln^3 n+2)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(n+3)!}{2^n n!}$; **b)** $\sum_{n=1}^{\infty}(-1)^n(3n-1)\arcsin\frac{1}{n}$; **c)** $\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{n^2+1}{4n^3-3n+2}\right)$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^5+6n}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}\left(1+\frac{2}{n^2}\right)^n e^{x\sqrt[3]{n}-\frac{n}{x^2-4}}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{\ln^{2n}(n+2)}$; **c)** $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n}{n+1}\right)^{n^2}(x+1)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^{n-1}(x+1)^n\operatorname{tg}\frac{\pi}{4^n}$ on the closed interval $[-2;0]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty}(-1)^{n-1}(n-1)x^{n-2}$; **b)** $\sum_{n=2}^{\infty}\frac{x^{2n-1}}{2n-1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\frac{1}{3x-12}$ at $x=-2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,5}\frac{\operatorname{arctg}x^2}{x^2}\,dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''+y+xy'=0,\quad y(0)=1,\quad y'(0)=0.$

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-4, & -4<x<0,\\ x-4, & 0<x<4\end{cases}$ on the interval $(-4,4)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=2x+\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №13

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=5}^{\infty}\frac{21}{n^2-5n+4}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty}\frac{(n^2+1)(\pi-\operatorname{arctg} n)}{\sqrt{n^5}}$; **b)** $\sum_{n=2}^{\infty}\frac{n^2}{\sqrt{n^3+4}}\sin\frac{n}{n^2-1}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{n!(2n-1)!}{(3n)!}$; **b)** $\sum_{n=1}^{\infty}5^n\left(1+\frac{1}{n}\right)^{-n^2}$; **c)** $\sum_{n=2}^{\infty}\frac{4}{(4n-3)\sqrt{\ln^5(4n-3)}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\sqrt{n}\ln\left(1+\frac{1}{\sqrt{n}}\right)$; **b)** $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^n}{3^{n+1}n!}$; **c)** $\sum_{n=2}^{\infty}(-1)^n\frac{3+\cos 2n}{n-1}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=0}^{\infty}(-1)^n\frac{n+5}{n^6+6}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}\frac{(x^2-3x-3)^n}{7^n n^2}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}(n+1)^n x^{n+1}$; **c)** $\sum_{n=1}^{\infty}\frac{((n+2)!)^2(x+4)^n}{(2n)!}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^{2n}}{8^n(n+1)}$ on the closed interval $[-1;3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=2}^{\infty}(-1)^n\frac{x^{2n-1}}{2n-1}$; **b)** $\sum_{n=1}^{\infty}(-1)^n(n-1)x^{n-2}$.

Exercise 9. Find the Taylor series of the function $f(x)=\sin 2x+2x\cos 2x$ at $x=0$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,25}\ln(1+\sqrt{x})dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'=y^2+x^2+x,\quad y(0)=1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}x+5, & -5<x<0,\\ 5, & 0<x<5\end{cases}$ on the interval $(-5,5)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=x+\frac{\pi}{4}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №14

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty}\frac{6}{n^2+16n+63}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=2}^{\infty}\frac{n\sqrt{n}(3+\sin n)}{n^2-1}$; **b)** $\sum_{n=1}^{\infty}\frac{n^2-4}{n}\left(\operatorname{arctg}\frac{1}{n+2}\right)^2$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{5^n(n+2)!}{2^{3n}n!}$; **b)** $\sum_{n=1}^{\infty}\frac{1}{4^n}\left(\frac{7n+2}{2n-1}\right)^n$; **c)** $\sum_{n=1}^{\infty}\frac{1}{n(\ln(4n)+5)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n\sqrt{\ln(n+2)}}$; **b)** $\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{\sqrt{5n^6+2}}{n^3-2}\right)^{2n}$; **c)** $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}\left(e^{\frac{1}{\sqrt{n^3}}}-1\right)$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{(n+5)^n}$ to within $\alpha=0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\ln^n x}{3^n n^3}$; **b)** $\sum_{n=1}^{\infty}\frac{2^n n!\,x^{2n}}{(2n)!}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+5)^n}{n+2}\operatorname{tg}\frac{1}{n^2}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{(n+3)\sqrt[3]{n}}\sin^2 nx$ on the closed interval $[1;3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty}(-1)^{n-1}(2n-1)x^{2n-2}$; **b)** $\sum_{n=1}^{\infty}\frac{x^{2n+3}}{2n+3}$.

Exercise 9. Find the Taylor series of the function $f(x)=\ln\frac{1}{x^2+2x+10}$ at $x=-1$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^1 \sin x^2\,dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''=\cos y'+y$, $y(0)=1$, $y'(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}6, & -6<x<0,\\ 6-x, & 0<x<6\end{cases}$ on the interval $(-6,6)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=x+\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №15

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=3}^{\infty} \frac{24}{n^2 - n - 2}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{(n-1)\ln n}{n^4 + n^2 + 3}$; **b)** $\sum_{n=2}^{\infty} \sqrt{n-1}\left(1 - \cos\frac{\pi}{n-1}\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty} \frac{e^{2n}\, n!}{2^n (n+5)!}$; **b)** $\sum_{n=3}^{\infty} 3^n \ln^n\left(\frac{n+2}{n-2}\right)$; **c)** $\sum_{n=2}^{\infty} \frac{2n}{(n^2-1)\sqrt{\ln(n^2-1)}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=0}^{\infty} (-1)^{n+1} \sqrt{\frac{4n^2+1}{n^4+3}}$; **b)** $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sin^2 n}{\sqrt{n^3+1}}$; **c)** $\sum_{n=1}^{\infty} (-1)^n (n^2+1)\operatorname{arctg}\frac{1}{3n^2+2}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{5^n}$ to within $\alpha = 0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n\, e^n (x-1)}{e^{n(x^2-x)}}$; **b)** $\sum_{n=1}^{\infty} \frac{x^{n+2}}{(n^2+1)n!}$; **c)** $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+2}\left(\frac{x+1}{5}\right)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^{n-1}}{(5n-1)^3} x^{2n-1}$ on the closed interval $[-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=2}^{\infty} \frac{x^{n+1}}{n+1}$; **b)** $\sum_{n=2}^{\infty} (-1)^n 3n\, x^{3n-1}$.

Exercise 9. Find the Taylor series of the function $f(x) = (x-3)e^{6x-x^2}$ at $x = 3$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{1,5} \frac{dx}{\sqrt[3]{x^3+3^3}}$ to within $\alpha = 0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'' = yy' - x^2$, $y(0) = 1$, $y'(0) = 2$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} -x-7, & -7 < x < 0, \\ -7, & 0 < x < 7 \end{cases}$ on the interval $(-7, 7)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = \frac{1}{2}(x+\pi)$ on the interval $(0, \pi)$; graph the function and the sum of each series.

Series

Variant №16

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty}\frac{3}{n^2+17n+70}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty}\frac{\sqrt{n}\cos^2 n}{2n^3+n+1}$; **b)** $\sum_{n=2}^{\infty}\frac{1}{n}\left(\sqrt{n+2}-\sqrt{n-2}\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{3^n n!}{n^4(n+3)!}$; **b)** $\sum_{n=1}^{\infty}\frac{n}{2^n}\left(\frac{3n}{n+1}\right)^n$; **c)** $\sum_{n=1}^{\infty}\frac{1}{n(\ln^2(2n)+4)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=0}^{\infty}(-1)^n\operatorname{arctg}^n\frac{1}{3^n}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2-1}{2n^3+3n^2}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n-1}n^3\left(e^{\frac{3}{n^3}}-1\right)$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^n\frac{n}{(n+1)!(2n-1)}$ to within $\alpha=0,01$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=0}^{\infty}\frac{8^n(n+1)}{(x^2-3x-2)^n}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n-1}(n^2+1)^n(x-4)^n$; **c)** $\sum_{n=1}^{\infty}\frac{(n!)^2(x+3)^n}{(2n)!}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}\frac{(x+3)^n}{(n+1)\sqrt{\ln^3(n+1)}}$ on the closed interval $[-4;-2]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty}(-1)^{n+1}(n+2)x^{n+1}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{2n+1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\ln(5x+10)$ at $x=1$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,5}\frac{1-\cos x}{x}dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'=y^2-x,\quad y(0)=1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}8, & -8<x<0,\\ 8-x, & 0<x<8\end{cases}$ on the interval $(-8,8)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=x+2\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №17

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=4}^{\infty}\dfrac{4}{n^2-4n+3}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=2}^{\infty}\dfrac{\sqrt{n-2}}{(n+5)^2(\pi+\operatorname{arctg} n)}$; **b)** $\sum\limits_{n=1}^{\infty}\sqrt{n}\left(2^{\frac{n}{(n+2)^2}}-1\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=1}^{\infty}\dfrac{e^n(n!)^3}{(3n-1)!}$; **b)** $\sum\limits_{n=1}^{\infty}\dfrac{1}{5^n}\left(1+\dfrac{1}{n}\right)^{n^2}$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{2}{(n+1)\ln(n+1)^2}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=0}^{\infty}(-1)^{n+1}\dfrac{(n+3)!}{2^n(n+1)!}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n n^4\left(1-\cos\dfrac{5}{n^2}\right)$; **c)** $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{\sqrt{n+2}-\sqrt{n-1}}{\sqrt{3n}}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{3n-1}{n^3(n+7)}$ to within $\alpha=0{,}01$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=0}^{\infty}\dfrac{3^n\operatorname{tg}^{2n}x}{(n+1)^2}$; **b)** $\sum\limits_{n=3}^{\infty}(-1)^n\dfrac{x^{2n}}{\ln^n(\sqrt[3]{n}-1)}$; **c)** $\sum\limits_{n=1}^{\infty}\left(\dfrac{n+1}{n}\right)^{n^2}(x+4)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{(x-2)^n}{n^2\sqrt[3]{n}}$ on the closed interval $[1;3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=2}^{\infty}\dfrac{x^{2n-3}}{2n-3}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n(n+1)x^n$.

Exercise 9. Find the Taylor series of the function $f(x)=\sin\dfrac{\pi x}{6}$ at $x=3$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{0,1}\dfrac{1-e^{-2x}}{x}\,dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''+x^3+y^3=0,\quad y(0)=-2,\quad y'(0)=-1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}1, & -1<x<0,\\ 1-x, & 0<x<1\end{cases}$ on the interval $(-1,1)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\dfrac{x}{2}+2\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series..

Series

Variant №18

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty}\frac{6}{n^2+15n+54}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=2}^{\infty}\frac{n(2-\cos n)}{\sqrt{n^3-1}}$; **b)** $\sum_{n=1}^{\infty}\sqrt{n+5}\arcsin\frac{1}{(n+1)^2}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=2}^{\infty}\frac{n^n}{2^{2n}(n-1)!}$; **b)** $\sum_{n=1}^{\infty}\frac{\ln^n n}{n^5}$; **c)** $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{\ln(5n)+1}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{4n^2+2n-1}{3n^2-3n+2}\right)^n$; **b)** $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{\cos^2 n}{n^2+2n+1}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n-1}\operatorname{tg}\frac{1}{\sqrt[3]{n^3+3}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^n\frac{n^2+5}{(2n)!\,n}$ to within $\alpha=0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{5n^2}\mathrm{e}^{x\sqrt[3]{n}+n(x^2+6x)}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n-1}n^{2n}(x-2)^{2n}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{2^n+3^n}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n!}{n^n}(x+2)^n$ on the closed interval $[-4;0]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty}(-1)^n 4n\, x^{4n-1}$; **b)** $\sum_{n=2}^{\infty}(-1)^n\frac{x^{2n-1}}{2n-1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\frac{1}{2x+8}$ at $x=1$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,5}\frac{x-\operatorname{arctg}x}{x^2}\,dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'=x\,\mathrm{e}^y+y,\quad y(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-x-2, & -2<x<0,\\ -2, & 0<x<2\end{cases}$ on the interval $(-2,2)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\frac{x}{4}+\frac{\pi}{4}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №19

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=5}^{\infty}\frac{6}{n^2-6n+8}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=3}^{\infty}\frac{n\ln n}{(n-2)^2}$; **b)** $\sum_{n=1}^{\infty}\sqrt{n+2}\left(e^{\frac{\pi}{n^2}}-1\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=2}^{\infty}\frac{(3n-1)!}{(n-1)!(2n)!}$; **b)** $\sum_{n=1}^{\infty}\left(\frac{2n+5}{4n-3}\right)^{4n}$; **c)** $\sum_{n=1}^{\infty}\frac{2n}{(n^2+1)\ln^4(n^2+1)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}(-1)^n n^2\arcsin\frac{1}{2^n}$; **b)** $\sum_{n=2}^{\infty}(-1)^{n-1}\frac{2+\cos n}{\sqrt{n^2-2}}$; **c)** $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{n^2}{\sqrt{2n^4-n^2+1}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{3^n n!}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=0}^{\infty}(-1)^n\frac{(x^2-4x-3)^n}{9^n(n+4)}$; **b)** $\sum_{n=2}^{\infty}\frac{n^3x^{n-1}}{(n-1)!}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-2)^n}{(5n-4)}\sin\frac{1}{3^n}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+3)^{n^2}}{n^n}$ on the closed interval $[-4;-2]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=0}^{\infty}\frac{x^{2n+2}}{2n+2}$; **b)** $\sum_{n=1}^{\infty}(-1)^n n\,x^{n-1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\left(\operatorname{ctg}x-\frac{1}{x}\right)\sin x$ at $x=0$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,1}\frac{\ln(1-x)}{x}dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''=xy^2$, $y(1)=1$, $y'(1)=-1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-3, & -3<x<0,\\ x-3, & 0<x<3\end{cases}$ on the interval $(-3,3)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\frac{x}{2}+\pi$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №20

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty} \frac{9}{n^2+13n+40}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3(3+\sin n)}$; **b)** $\sum_{n=2}^{\infty} \sqrt{n+1} \ln \frac{n^3}{n^3-1}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty} \frac{4^n (2n)!}{3^{n+3}(2n+1)!}$; **b)** $\sum_{n=1}^{\infty} 3^n \left(\frac{n+2}{2n+3}\right)^n$; **c)** $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{(\ln(3n)+2)^3}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty} (-1)^{n+1} n \left(1-\cos\frac{2}{\sqrt{n^3}}\right)$; **b)** $\sum_{n=1}^{\infty} (-1)^n n (e^{\frac{1}{2n}} - 1)$; **c)** $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \ln(n+3)}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ to within $\alpha = 0{,}01$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(n+1)}{\ln^n(x+1)}$; **b)** $\sum_{n=1}^{\infty} (n+5)!(x-2)^{4n}$; **c)** $\sum_{n=0}^{\infty} (-1)^n (x+3)^n \operatorname{arctg} \frac{1}{4^n}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=0}^{\infty} (-1)^n \frac{(x+2)^n}{3^n(5n+1)}$ on the closed interval $[-4;0]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=2}^{\infty} (-1)^n (2n-3) x^{2n-4}$; **b)** $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n+1}$.

Exercise 9. Find the Taylor series of the function $f(x) = \ln(x^2 - 6x + 18)$ at $x = 3$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,5} \cos(4x^2)\, dx$ to within $\alpha = 0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y' = e^y + xy, \quad y(0) = 0$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} x+4, & -4 < x < 0, \\ 4, & 0 < x < 4 \end{cases}$ on the interval $(-4, 4)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = \frac{x}{2} + \frac{\pi}{4}$ on the interval $(0, \pi)$; graph the function and the sum of each series.

Series

Variant №21

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=6}^{\infty}\dfrac{5}{n^2-8n+15}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=1}^{\infty}\dfrac{n^2+2}{n^3(\pi-\operatorname{arctg} n)}$; **b)** $\sum\limits_{n=2}^{\infty}\dfrac{n^3}{n^4-1}\operatorname{tg}^2\dfrac{1}{n}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=1}^{\infty}\dfrac{n^n}{3^n(n+1)!}$; **b)** $\sum\limits_{n=1}^{\infty}\dfrac{1}{2^n}\left(\ln\dfrac{9n-1}{n+1}\right)^n$; **c)** $\sum\limits_{n=2}^{\infty}\dfrac{3}{(3n-4)\ln(3n-4)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{(n!)^2}{(2n+1)!}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{\sqrt{n+3}-\sqrt{n-1}}{\sqrt{n+2}}$; **c)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}n^4\left(1-\cos\dfrac{1}{n^2}\right)$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{3^n}{(n+4)^n}$ to within $\alpha=0{,}0001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=1}^{\infty}\dfrac{3^{4n}(x+2)}{3^{n(x^2-3x)}}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{n^2x^n}{(n+2)!}$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{(x+1)^n}{(n+2)\ln^3(n+2)}$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=2}^{\infty}(-1)^n(x+3)^n\operatorname{arctg}\dfrac{\pi}{3^n}$ on the closed interval $[-4;-2]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=2}^{\infty}(-1)^{n-1}\dfrac{x^n}{n}$; **b)** $\sum\limits_{n=2}^{\infty}(-1)^n(n-1)x^{n-2}$.

Exercise 9. Find the Taylor series of the function $f(x)=x\,\mathrm{e}^{-3x+1}$ at $x=-2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^2\dfrac{dx}{\sqrt[3]{x^3+4^3}}$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'+\cos x=xy,\quad y(0)=1,\quad y'(0)=-1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}5, & -5<x<0,\\ 5-x, & 0<x<5\end{cases}$ on the interval $(-5,5)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\dfrac{\pi}{2}-x$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №22

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty} \frac{12}{n^2+11n+28}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}(3-\cos n)}{\sqrt{n^4+3}}$; **b)** $\sum_{n=2}^{\infty} \frac{n+1}{\sqrt{n}} \sin^2 \frac{1}{\sqrt{n-1}}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty} \frac{n!n^2}{2^n(n+2)!}$; **b)** $\sum_{n=1}^{\infty} \frac{n}{3^n}\left(\frac{n+3}{n}\right)^n$; **c)** $\sum_{n=1}^{\infty} \frac{2n\ln(n^2+2)}{n^2+2}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{3n+5}{4n^2-2n+1}$; **b)** $\sum_{n=1}^{\infty} (-1)^n n \ln\left(1+\frac{2}{n^3}\right)$; **c)** $\sum_{n=1}^{\infty} (-1)^{n-1} \operatorname{arctg}^n \frac{\sqrt{3}\, n}{\sqrt{n^2+4}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4+3}$ to within $\alpha = 0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=0}^{\infty} \frac{6^n n^3}{(x^2-2x-2)^n}$; **b)** $\sum_{n=1}^{\infty} (-1)^{n+1} n^{2n} (x-4)^{2n}$; **c)** $\sum_{n=1}^{\infty} \frac{((n+1)!)^2 (x+2)^n}{(2n)!}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=3}^{\infty} (-1)^n \frac{(x+2)^n}{(n-1)\sqrt[3]{\ln^4(n-1)}}$ on the closed interval $[-3;-1]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty} (-1)^{n+1} 5n\, x^{5n-1}$; **b)** $\sum_{n=2}^{\infty} (-1)^n \frac{x^{n-1}}{n-1}$.

Exercise 9. Find the Taylor series of the function $f(x) = \ln(3-3x)$ at $x = -3$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^1 \frac{\operatorname{sh} x}{x} dx$ to within $\alpha = 0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'' = 2xy - y - 1$, $y(0) = 0$, $y'(0) = 1$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} -x-6, & -6 < x < 0, \\ -6, & 0 < x < 6 \end{cases}$ on the interval $(-6, 6)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = \pi - 2x$ on the interval $(0, \pi)$; graph the function and the sum of each series.

Series

Variant №23

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=7}^{\infty}\dfrac{10}{n^2-10n+24}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=4}^{\infty}\dfrac{n-4}{\sqrt{n^6+4\ln n}}$; **b)** $\sum\limits_{n=1}^{\infty}\operatorname{arctg}\dfrac{n+2}{n^2\sqrt{n+3}}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=2}^{\infty}\dfrac{\mathrm{e}^n(n!)^2}{(n-1)!(n+5)!}$; **b)** $\sum\limits_{n=1}^{\infty}\left(\dfrac{3n-1}{2n+2}\right)^{5n}$; **c)** $\sum\limits_{n=2}^{\infty}\dfrac{2}{(2n-1)\sqrt{\ln^7(2n-1)}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\left(\dfrac{3n^3}{4n^3-n}\right)^{n^2}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{n+1}{\sqrt{3n^2-n}}$; **c)** $\sum\limits_{n=0}^{\infty}(-1)^n n\arcsin\dfrac{2}{\sqrt{n^3+1}}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{n}{6^n}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{3n+1}{2^n\sin^{2n}x}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{4^n n!x^n}{(4n)!}$; **c)** $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{(x+5)^n}{(n+3)\ln(n+3)}$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=0}^{\infty}\dfrac{(\pi-x)}{\sqrt[5]{n^7+4}}\sin^2 nx$ on the closed interval $[0;\pi]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=2}^{\infty}(-1)^{n+1}\dfrac{x^{2n-3}}{2n-3}$; **b)** $\sum\limits_{n=2}^{\infty}(-1)^n(2n-3)x^{2n-4}$.

Exercise 9. Find the Taylor series of the function $f(x)=\cos\dfrac{\pi x}{4}$ at $x=-2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{0,2}\mathrm{e}^{-3x^2}dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''+y'=xy^2$, $y(0)=2$, $y'(0)=1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-7, & -7<x<0,\\ x-7, & 0<x<7\end{cases}$ on the interval $(-7,7)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\dfrac{\pi}{4}-x$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №24

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty} \frac{15}{n^2+9n+18}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^3+1}}{n^2(2+\sin n)}$; **b)** $\sum_{n=1}^{\infty} (n^3+3n)\left(1-\cos\frac{1}{n^2}\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty} \frac{2^n(2n+1)!}{\mathrm{e}^n(2n)!}$; **b)** $\sum_{n=1}^{\infty} \frac{n^3}{\ln^n(n+2)}$; **c)** $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{\ln(6n)+2}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty} (-1)^n \frac{3n^2-1}{2n^3+2n^2-n}$; **b)** $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\operatorname{arctg} n}{\sqrt{n^4+4n}}$; **c)** $\sum_{n=0}^{\infty} (-1)^{n+1} n^3\left(e^{\frac{1}{n^3+1}}-1\right)$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(3n-1)n!}$ to within $\alpha = 0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty} \left(1+\frac{3}{n^2}\right)^n \mathrm{e}^{x\sqrt{n}-\frac{n}{x^2-9}}$; **b)** $\sum_{n=1}^{\infty} (n+5)!\,x^{n+5}$; **c)** $\sum_{n=2}^{\infty} (-1)^n \frac{(x-4)^n}{\sqrt[5]{n^2-1}}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-2)^n}{5^n\sqrt{n+1}}$ on the closed interval $[1;3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty} (-1)^n (n+2) x^{n+1}$; **b)** $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n+3}}{2n+3}$.

Exercise 9. Find the Taylor series of the function $f(x) = \frac{1}{3x-3}$ at $x=3$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,5} \frac{\arcsin x}{x}\,dx$ to within $\alpha = 0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'' = x + \mathrm{e}^y$, $y(0)=1$, $y'(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} -x-8, & -8 < x < 0, \\ -8, & 0 < x < 8 \end{cases}$ on the interval $(-8, 8)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = \pi - x$ on the interval $(0, \pi)$; graph the function and the sum of each series.

Series

Variant №25

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=8}^{\infty} \frac{12}{n^2 - 12n + 35}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}(\pi - \operatorname{arctg} n)}{\sqrt[3]{n^5 + 2}}$; **b)** $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left(\sqrt{n+3} - \sqrt{n+1} \right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty} \frac{(n+1)!(2n)!}{(3n+1)!}$; **b)** $\sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{3n+2}{5n+1} \right)^n$; **c)** $\sum_{n=3}^{\infty} \frac{2n}{(n^2 - 3)\sqrt{\ln(n^2 - 3)}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=2}^{\infty} (-1)^{n-1} \frac{n^n}{3^n (n-1)!}$; **b)** $\sum_{n=1}^{\infty} (-1)^n n^2 \ln \frac{n^2 + 2}{n^2 + 1}$; **c)** $\sum_{n=1}^{\infty} (-1)^{n+1} \sin \frac{1}{\sqrt{3n^2 + 7}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n (n+2)}$ to within $\alpha = 0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=0}^{\infty} \frac{(x^2 - 5x - 3)^n}{11^n (n+3)}$; **b)** $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)^2 n!}$; **c)** $\sum_{n=0}^{\infty} (-1)^n \left(\frac{n}{n+1} \right)^{n^2} (x-3)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x+5)}{(n+2)\sqrt[3]{n^2}}$ on the closed interval $[-6; -4]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n+1}$ **b)** $\sum_{n=1}^{\infty} (-1)^{n+1} 6n\, x^{6n-1}$.

Exercise 9. Find the Taylor series of the function $f(x) = \left(1 - \frac{\operatorname{tg} x}{x} \right) \cos x$ at $x = 0$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0{,}1} \frac{\ln(1+2x)}{x} dx$ to within $\alpha = 0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'' + y \cos x = 0$, $y(0) = 1$, $y'(0) = 0$.

Exercise 12. a) Find the Fourier series for the function $f(x) = \begin{cases} -x - 1, & -1 < x < 0, \\ -1, & 0 < x < 1 \end{cases}$ on the interval $(-1, 1)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y = \frac{\pi}{2} - \frac{x}{2}$ on the interval $(0, \pi)$; graph the function and the sum of each series.

Series

Variant №26

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=0}^{\infty}\frac{18}{n^2+7n+10}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty}\frac{(n+1)^2}{3n^3(3-\cos n)}$; **b)** $\sum_{n=2}^{\infty}\sqrt{n-2}\arcsin\frac{n}{n^2-2}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{3^{n+2}(3n-1)!}{2^{2n}(3n)!}$; **b)** $\sum_{n=1}^{\infty}\left(\ln\frac{3n-2}{n+1}\right)^n$; **c)** $\sum_{n=1}^{\infty}\frac{2\ln n}{n(\ln^4 n+4)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{5n^5+1}{3n^5-2n^2}\right)^{2n}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}\sin\frac{1}{n+1}\cdot\operatorname{tg}\frac{2}{n}$; **c)** $\sum_{n=2}^{\infty}(-1)^n\frac{\sqrt{n+5}-\sqrt{n-2}}{\sqrt{2n}}$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^n\frac{n^4}{(2n+1)!}$ to within $\alpha=0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=0}^{\infty}(-1)^n\frac{\ln^n(x-1)}{n+1}$; **b)** $\sum_{n=1}^{\infty}(n+3)!(x+1)^{3n-1}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-2)^n}{5^n+3^n}$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x+4)^{2n}}{4^n(n^2+1)}$ on the closed interval $[-6;-2]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty}(-1)^{n+1}2n\,x^{2n-1}$; **b)** $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\ln\frac{1}{x^2-4x+8}$ at $x=2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,2}\cos(25x^2)\,dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''+(1+x^2)y=0$, $y(0)=-2$, $y'(0)=2$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-2, & -2<x<0,\\ x-2, & 0<x<2\end{cases}$ on the interval $(-2,2)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=2\pi-x$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №27

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=9}^{\infty}\dfrac{14}{n^2-14n+48}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=2}^{\infty}\dfrac{(n^2+2)\ln n}{n^2(n-1)}$; **b)** $\sum\limits_{n=1}^{\infty}(n+3)\left(5^{\frac{1}{\sqrt{n^3}}}-1\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=2}^{\infty}\dfrac{n^n}{7^n(n-1)!}$; **b)** $\sum\limits_{n=1}^{\infty}\dfrac{4^n}{n}\left(\dfrac{n+1}{3n-2}\right)^n$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{2n}{(n^2+3)\ln^2(n^2+3)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{4n+3}{\sqrt[3]{n^3+2n}}$; **b)** $\sum\limits_{n=3}^{\infty}(-1)^n\dfrac{2-\sin n}{\sqrt{n^2-2n}}$; **c)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}3^n\,\mathrm{tg}^n\dfrac{1}{4n}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{4^n}{(2n+1)^n}$ to within $\alpha=0{,}01$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{(n+1)\,\mathrm{e}^{2n}}{\mathrm{e}^{n(x^2+x)}}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}\dfrac{x^n}{\ln^{3n}(2n-1)}$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{((n+2)!)^2}{(2n)!}(x-4)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{(x-2)^{n^2}}{n^n}$ on the closed interval $[1;3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=2}^{\infty}(-1)^n\dfrac{x^{2n-1}}{2n-1}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}(n-1)x^{n-2}$.

Exercise 9. Find the Taylor series of the function $f(x)=(x+1)\,\mathrm{e}^{2x-x^2}$ at $x=1$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{2{,}5}\dfrac{dx}{\sqrt[3]{x^3+5^3}}$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''=y\cos x+x$, $y(0)=1$, $y'(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}x+3, & -3<x<0,\\ 3, & 0<x<3\end{cases}$ on the interval $(-3,3)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=2\pi-\dfrac{x}{2}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №28

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=0}^{\infty}\dfrac{21}{n^2+5n+4}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=1}^{\infty}\dfrac{\sqrt{n}(2+\sin n)}{(n+1)(n+3)}$; **b)** $\sum\limits_{n=1}^{\infty}\left(e^{\frac{\sqrt{n+5}}{n(n+1)^2}}-1\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=1}^{\infty}\dfrac{2^n n!}{n^5(n+4)!}$; **b)** $\sum\limits_{n=1}^{\infty}2^n\left(1+\dfrac{1}{n}\right)^{-n^2}$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{1}{n\sqrt{\ln(7n)+3}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=0}^{\infty}(-1)^{n+1}\operatorname{tg}\dfrac{n+1}{n^2+3}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n n\ln\dfrac{n+5}{n+4}$; **c)** $\sum\limits_{n=2}^{\infty}(-1)^{n-1}\dfrac{(2n)!(n-1)!}{(3n)!}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{3n}{(n+3)^n}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=0}^{\infty}\dfrac{4^n(n+1)^2}{(x^2-x-2)^n}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}(n+2)!(x+5)^{n+5}$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{(x-5)^n}{n^2+1}\sin\dfrac{1}{n}$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=2}^{\infty}(-1)^n\dfrac{4^{n-1}}{\sqrt{(n-1)^5}}x^{2n-1}$ on the closed interval $\left[-\frac{1}{2};\frac{1}{2}\right]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n+1}(n-1)x^{n-2}$; **b)** $\sum\limits_{n=2}^{\infty}(-1)^{n+1}\dfrac{x^{2n-1}}{2n-1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\ln(16-2x)$ at $x=2$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^1\cos\sqrt{x}\,dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y'=y^2+x^2$, $y(0)=1$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}4, & -4<x<0,\\ 4-x, & 0<x<4\end{cases}$ on the interval $(-4,4)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\dfrac{\pi}{4}-\dfrac{x}{4}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №29

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum_{n=10}^{\infty}\frac{16}{n^2-16n+63}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum_{n=1}^{\infty}\frac{\operatorname{arctg}^2 n}{\sqrt[4]{n^5+1}}$; **b)** $\sum_{n=1}^{\infty}(n^2+2n)\arcsin^2\frac{1}{\sqrt{n^3}}$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum_{n=1}^{\infty}\frac{(2n+1)!}{3^n(n!)^2}$; **b)** $\sum_{n=1}^{\infty}\frac{1}{3^n}\ln^n\left(\frac{3n-1}{3n+2}\right)$; **c)** $\sum_{n=1}^{\infty}\frac{6}{(6n+1)\sqrt[3]{\ln(6n+1)}}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{5n^2+1}{3n^2-2}\right)^{n^2}$; **b)** $\sum_{n=0}^{\infty}(-1)^n\sqrt[3]{\frac{n-1}{3n^2+1}}$; **c)** $\sum_{n=1}^{\infty}(-1)^{n+1}n^4\left(1-\cos\frac{3}{n^3}\right)$.

Exercise 5. Calculate approximately the sum of the series $\sum_{n=1}^{\infty}(-1)^n\frac{n^7}{(3n-2)!\,n!}$ to within $\alpha=0,001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum_{n=1}^{\infty}\frac{2^n\cos^{2n}x}{(n+2)^2}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{\ln^n(\sqrt[3]{n+1})}$; **c)** $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}(x-1)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{n^n}(x-2)^n$ on the closed interval $[1;3]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n+3}}{2n+3}$; **b)** $\sum_{n=1}^{\infty}(-1)^{n-1}(2n-1)x^{2n-2}$.

Exercise 9. Find the Taylor series of the function $f(x)=\sin\frac{\pi x}{8}$ at $x=4$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int_0^{0,4}\frac{1-e^{-\frac{x}{2}}}{x}dx$ to within $\alpha=0,001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''+xy=y'$, $y(0)=1$, $y'(0)=0$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-x-5, & -5<x<0,\\ -5, & 0<x<5\end{cases}$ on the interval $(-5,5)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\pi-\frac{x}{2}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

Series

Variant №30

Exercise 1. Find the partial sum S_n and calculate the sum S of the series: $\sum\limits_{n=2}^{\infty}\dfrac{27}{n^2+n-2}$.

Exercise 2. Use the comparison tests to determine whether each series converges or diverges:

a) $\sum\limits_{n=2}^{\infty}\dfrac{n^2+4}{n\sqrt{n}\ln n}$; **b)** $\sum\limits_{n=1}^{\infty}\dfrac{n}{(n+1)^2}\left(1-\cos\dfrac{1}{\sqrt{n}}\right)$.

Exercise 3. Determine the convergence or divergence (use the ratio test, root test or integral test):

a) $\sum\limits_{n=1}^{\infty}\dfrac{3^n(3n)!}{e^n(3n+1)!}$; **b)** $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^5}\left(\dfrac{5n-1}{4n+2}\right)^n$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{1}{n(\ln^2 4n+16)}$.

Exercise 4. Determine whether each series converges absolutely, conditionally, or diverges:

a) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{\operatorname{arctg} n}{\sqrt{n^4+n+5}}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^n n(2^{\frac{1}{n}}-1)$; **c)** $\sum\limits_{n=0}^{\infty}(-1)^{n+1}\arcsin\dfrac{n}{\sqrt{n^3+2}}$.

Exercise 5. Calculate approximately the sum of the series $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{n}{(1+n^3)^2}$ to within $\alpha=0{,}001$.

Exercise 6. Find the domain of convergence of each series:

a) $\sum\limits_{n=1}^{\infty}\left(1+\dfrac{1}{n}\right)^{6n^2} e^{n(x^2-5x)+x\sqrt[4]{n}}$; **b)** $\sum\limits_{n=1}^{\infty}(-1)^{n+1}(2n)!\,x^{2n}$; **c)** $\sum\limits_{n=1}^{\infty}\dfrac{n}{(n+1)^2}\left(\dfrac{x-1}{4}\right)^n$.

Exercise 7. Prove the uniform convergence of the series $\sum\limits_{n=0}^{\infty}\dfrac{(\pi-x)\cos^2 nx}{\sqrt[4]{n^7+1}}$ on the closed interval $[0;\pi]$.

Exercise 8. Find the sum of each power series on its interval of convergence:

a) $\sum\limits_{n=2}^{\infty}(-1)^{n+1}3n\,x^{3n-1}$; **b)** $\sum\limits_{n=2}^{\infty}(-1)^n\dfrac{x^{n+1}}{n+1}$.

Exercise 9. Find the Taylor series of the function $f(x)=\dfrac{1}{2x+4}$ at $x=1$ and determine its radius of convergence.

Exercise 10. Calculate the integral $\int\limits_0^{0,5}\ln(1+x^3)dx$ to within $\alpha=0{,}001$.

Exercise 11. Find the first four nonzero terms of the power series for the solution of the Cauchy problem: $y''=(y')^2+xy$, $y(0)=4$, $y'(0)=-2$.

Exercise 12. a) Find the Fourier series for the function $f(x)=\begin{cases}-6, & -6<x<0,\\ x-6, & 0<x<6\end{cases}$ on the interval $(-6,6)$; determine the sum of the series at the discontinuity points of the function and at the ends of the interval; graph the function and the sum of the corresponding series.

b) Find the Fourier sine and cosine series for the function $y=\dfrac{\pi}{4}-\dfrac{x}{2}$ on the interval $(0,\pi)$; graph the function and the sum of each series.

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Електронне мережне навчальне видання

ВИЩА МАТЕМАТИКА

РЯДИ

Практикум

(Англійською мовою)

Укладачі: Массалітіна Є.В., Пилипенко В.А.

Практикум до розділу «Ряди» з курсу «Вища математика» для студентів технічних спеціальностей містить 30 варіантів, кожен варіант складається з 12 завдань (21 задачі). Самостійне виконання цих завдань забезпечує свідоме оволодіння навчальним матеріалом, який передбачено робочою програмою з вищої математики.

Практикум може бути рекомендований в якості розрахункової роботи за темою «Ряди» для студентів першого курсу технічних спеціальностей.

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і розповсюджувачів видавничої продукції ДК № 5354 від 25.05.2017 р.