

GAUSSIAN STOCHASTIC PROCESSES AND THEIR PROJECTION COEFFICIENTS

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The problem of linear prediction in stochastic processes exhibiting memory presents considerable analytical challenges. We focus on resolving the projection problem for the Gaussian-Volterra process with a linear kernel, defined by the stochastic integral

$$X_t = \int_0^t (t-s) dW_s, \quad (1)$$

where W is a Wiener process.

More precisely, in [1] we find the predictor for the first increment, $\Delta_1 = X_1 - X_0$, based on n subsequent, non-overlapping increments, $\Delta_i = X_i - X_{i-1}$ for $2 \leq i \leq n+1$. For this Gaussian process it is equivalent to the conditional expectation:

$$\mathbb{E}(\Delta_1 | \Delta_2, \dots, \Delta_{n+1}) = \sum_{k=2}^{n+1} c_n^{(k)} \Delta_k.$$

The primary goal was to calculate the projection coefficients $\{c_n^{(k)}\}$ and try to understand how the properties of the process affect the properties of the coefficients. Unexpectedly, we established that the coefficients $\{c_n^{(k)}\}$ change their sign. For example,

$$c_2^{(2)} = \frac{15}{31} \quad \text{and} \quad c_2^{(3)} = -\frac{3}{31}.$$

At the same time, increments of the process (1) across non-overlapping intervals are positively correlated. Thus, we can make the conclusion that a positive correlation of increments does not guarantee the positivity of the projection coefficients.

The similar problem for fBm B_t^H was studied in [2]. The following hypothesis was investigated there:

for $H > 1/2$ all projection coefficients are strictly positive.

This hypothesis was checked in [2] numerically and analytically for small n . However, it was not proved analytically. The projection problem is also still unsolved for the general Gaussian-Volterra process.

The two-sided (bilateral) projections of the fractional Gaussian noise $\Delta_k = B_k^H - B_{k-1}^H$, $k \geq 1$, are studied in [3]. In particular, the coefficients of this

projection are investigated numerically and analytically. We made a rather unexpected observation: while all the coefficients of one-sided projection remain positive (see [4]), with a two-sided projection one of the coefficients steadily becomes negative, but quite small in absolute value.

REFERENCES

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