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NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
“IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE”

MECHANICS

WORKSHOP

Kyiv
Igor Sikorsky Kyiv Polytechnic Institute
2021

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ УКРАЇНИ
„КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ ІМ. ІГОРЯ СІКОРСЬКОГО”

МЕХАНІКА

ПРАКТИКУМ

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як навчальний посібник для здобувачів ступеня бакалавра
за освітньою програмою «Медична інженерія»
спеціальності 163 «Біомедична інженерія»*

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Анотація: Практикум містить практичні завдання з прикладами їх розв’язування по всім розділам дисципліни «Механіка» згідно освітньої програми 163 «Біомедична інженерія». У посібнику розкриваються методичні та технологічні засади вирішення типових задач, що при успішному засвоєнні матеріалу сприятиме формуванню вмінь застосовувати фізичні та математичні методи в аналізі та моделюванні функціонування біотехнічних систем, набуванню здатностей до застосування теоретичних знань в практичній діяльності.

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MECHANICS
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Abstract: The workshop contains practical tasks with examples of their solution in all sections of the discipline "Mechanics" according to the educational program 163 "Biomedical Engineering". The textbook reveals methodological and technological principles of solving typical problems, which upon successful mastering of the material will promote the formation of skills to apply physical and mathematical methods in the analysis and modeling of biotechnical systems, acquiring the ability to apply theoretical knowledge in practice.

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INTRODUCTION

The discipline "Mechanics" plays a significant role in the preparation of bachelors in the specialty 163 "Biomedical Engineering". The study of the discipline contributes to the development of engineering thinking and allows the use of approaches, methods and knowledge of mechanics in mastering other disciplines of a special profile, lays the foundation for professional competencies.

The discipline studies the basic concepts and laws of theoretical mechanics and their consequences; the motion of material bodies, the interaction between them, as well as the conditions of equilibrium of body systems; methods for determining the kinematic and dynamic characteristics of mechanical systems, solids and individual points of the body; basic concepts and definitions of material resistance; methods of application of the theoretical apparatus of mechanics in solving practical problems of biomedical engineering.

The purpose of practical tasks in the discipline "Mechanics" is to consolidate theoretical knowledge, the formation of skills to apply physical and mathematical methods in the analysis and modeling of biotechnical systems, the acquisition of skills to apply theoretical knowledge in practice.

In preparation for the practical lesson, the student must comprehend the relevant theoretical material, methods of its application for practical tasks.

During workshop, the student analyzes in detail the typical examples in the manual, analyzes the algorithms used to solve problems, looks for analogies, and ideally tries to improve the known methods.

Each practical work contains the following sections:

- brief theoretical information required to perform the work;
- typical examples with solutions;
- practical tasks.

Structure and design of the report to the workshop

The report on the workshop is made out on A4 sheets (or in electronic form – pdf-file). The report is compiled in accordance with the content and should contain the following sections:

- title page,
- the goal of the work,
- description of the stages of work,
- description of the results obtained,
- conclusions on the results of work.

Organization, control of implementation and workshops defense

Students who have the theoretical knowledge necessary to perform this work are allowed to perform the workshop.

The workshop is defended immediately after its completion (or until the next practical session) according to the schedule set by the course work program. Works that are protected with a delay are credited with penalty points. When defending the work, the student demonstrates the results of the work performed and answers the control questions on the topic of the work.

1. INTRODUCTION TO STATICS

The purpose of the lesson: acquiring skills in:

- decomposition of force into components;
- finding the projection of the force on the axis.

Before completing the task you need to read:

- basic concepts and definitions of statics;
- concentrated forces and their characteristics;
- axioms of statics;
- types of connections and their reactions;
- decomposition of force into components;
- projection of force on the axis.

Basic concepts, formulas and symbols

The projection of the force on the axis is a scalar equal to the length of the segment between the projections of the beginning and end of the force, taken with the corresponding sign (Fig. 1.1). The projection has a plus sign if the movement from its beginning to the end is in the positive direction of the axis, and a minus sign – if in the negative.

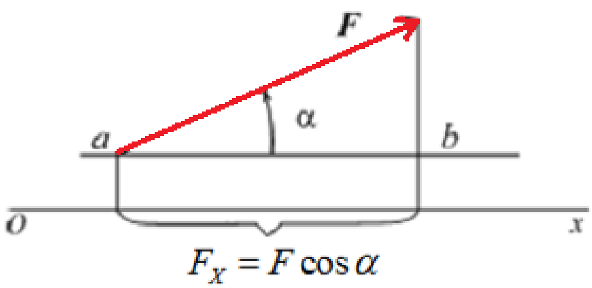


Fig. 1.1

As can be seen from Fig. 1.2, projection of the force on the axis is equal to product of modulus of force on the cosine of angle between positive direction of axis and direction of the force.

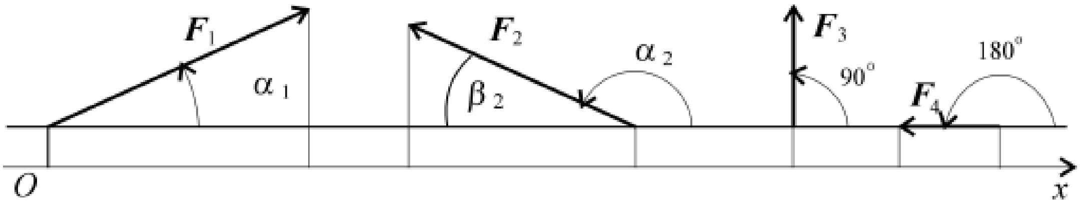
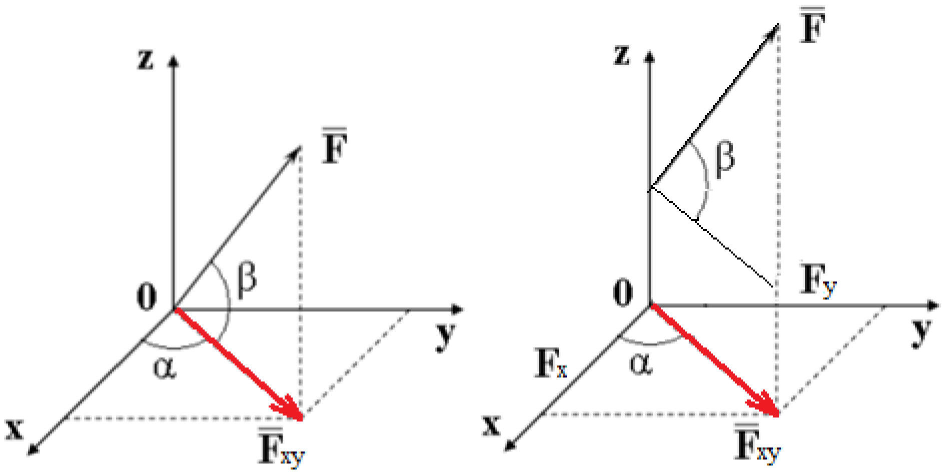


Fig. 1.2

Projection of force on the plane

The projection of force on the plane Oxy is the vector **F**xy, which is equal to interval between projections of beginning and end of the force **F** on this plane (Fig. 1.3). Thus, the projection of force on the plane is a vector, as it is characterized not only by modulus but also by direction.

Vector modulus $F_{XY} = F\cos\beta$, where β is the angle between direction of the force and its projection Fxy.



Analytical method of setting forces

To solve static problems, it is more convenient to set the force through its projections. The force **F** will be given analytically if its projections Fx, Fy, Fz on the axis of the rectangular Cartesian coordinate system are known:

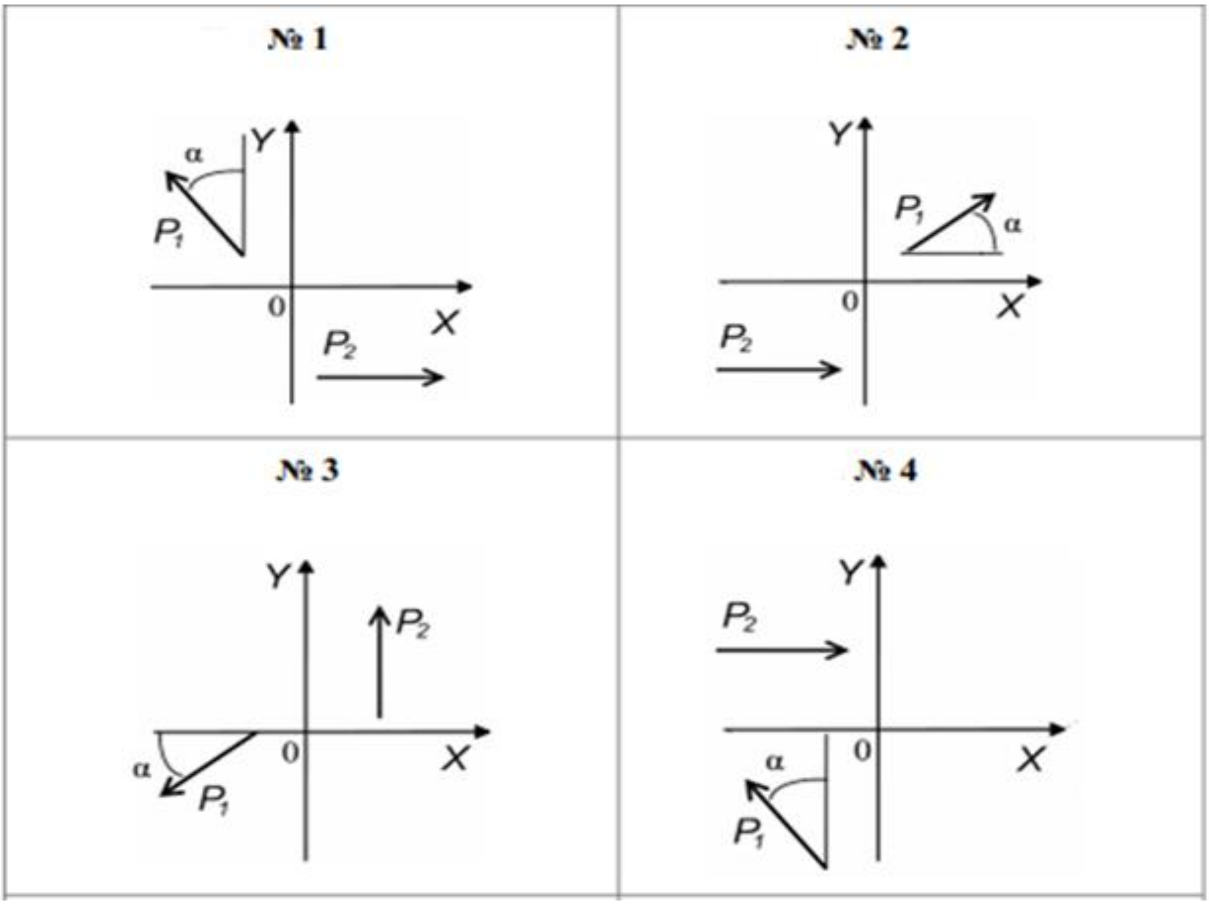
$$F_X = F\cos\theta_X; \qquad F_Y = F\cos\theta_Y; \qquad F_Z = F\cos\theta_Z,$$

Where $\theta_X, \theta_Y, \theta_Z$ – the angles of the force vector **F** with respect to the axes Ox, Oy, Oz, respectively.

TASKS 1

Calculate projections of the force vectors $\vec{P}_1$, $\vec{P}_2$ on the coordinate axes, if P_1 = 2 N, P_2 = 3 N, angle α = 30° and determine the angles between X axis and the force vectors. Summarize results in a table.

Варіант №	P_{1X}	P_{1Y}	P_{2X}	P_{2Y}	Angle between X axis and $\vec{P}_1$	Angle between X axis and $\vec{P}_2$



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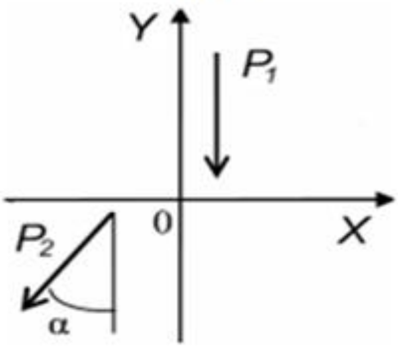

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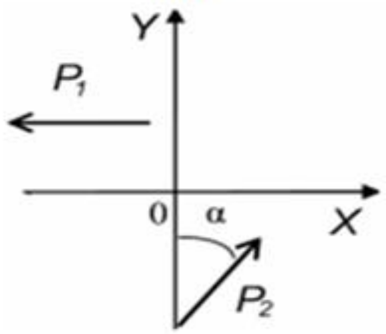

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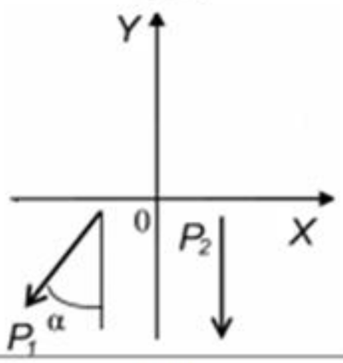

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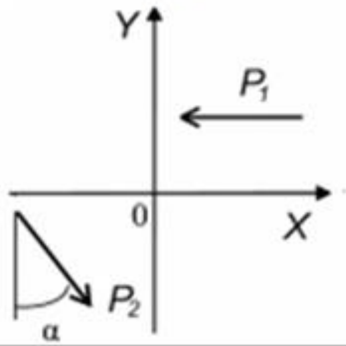

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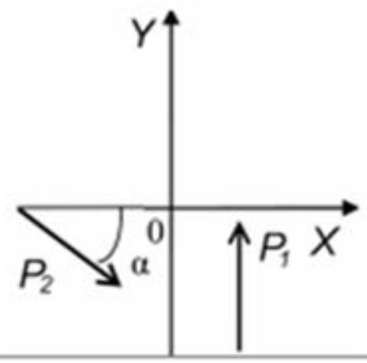

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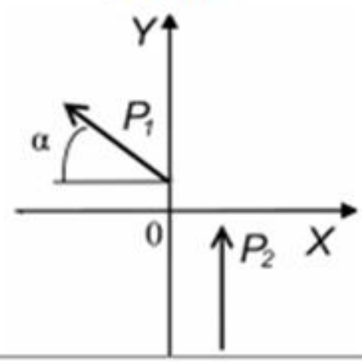

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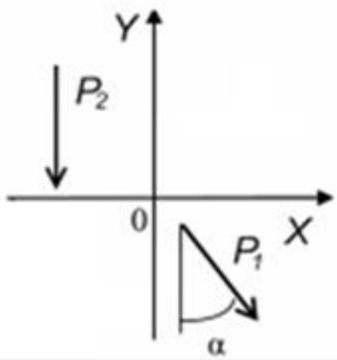

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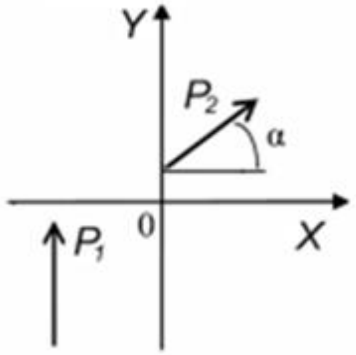

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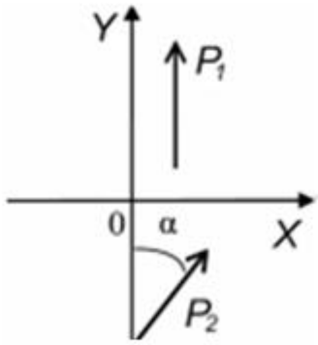

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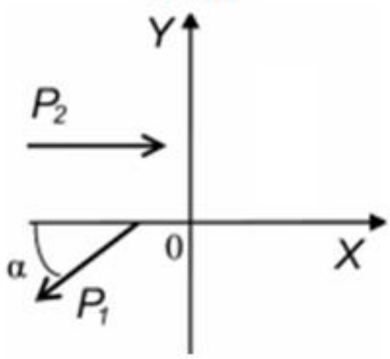

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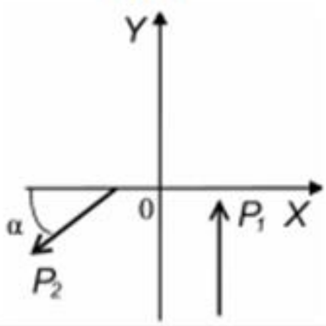

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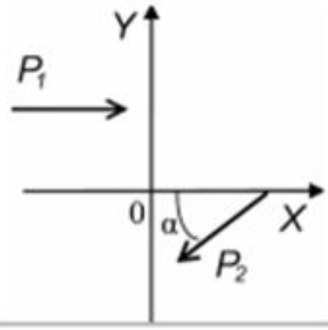

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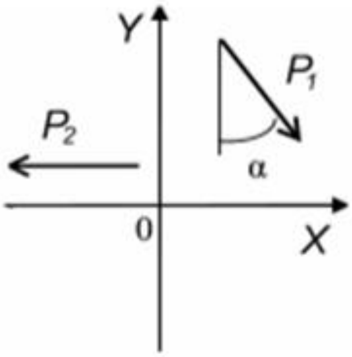

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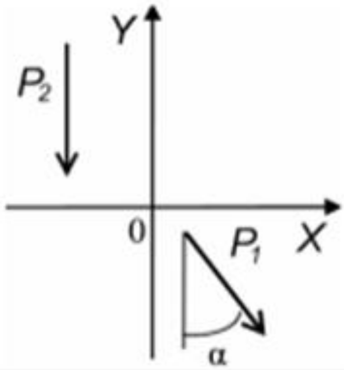

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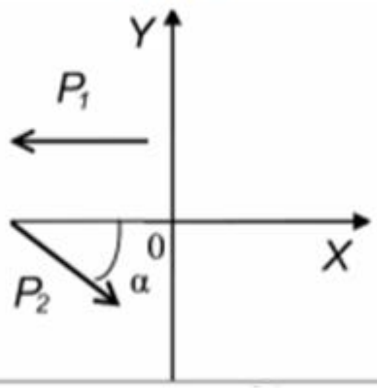

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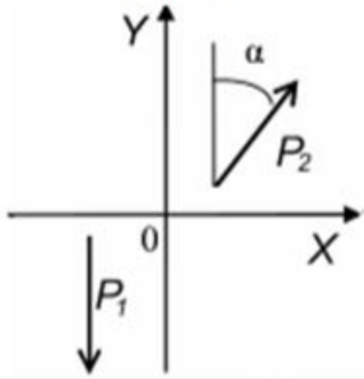

TASKS 2

Name the type of support and show the direction of the support reactions at points A, B, C of the body.

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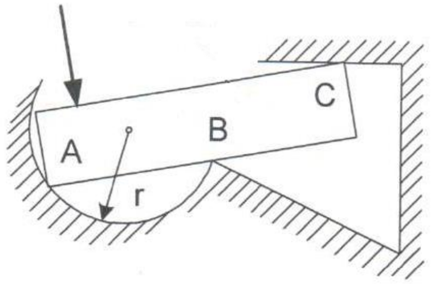	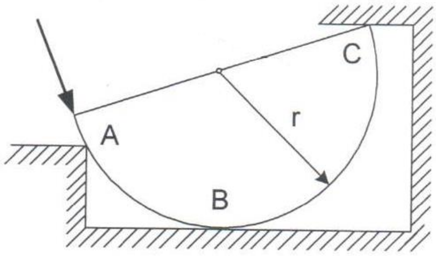
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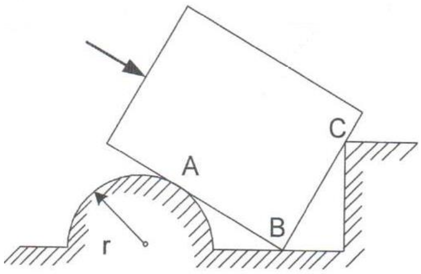	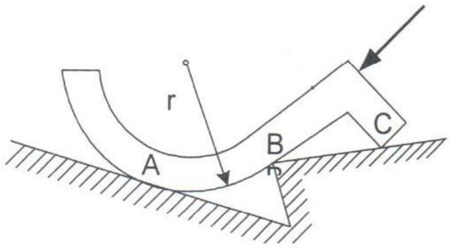
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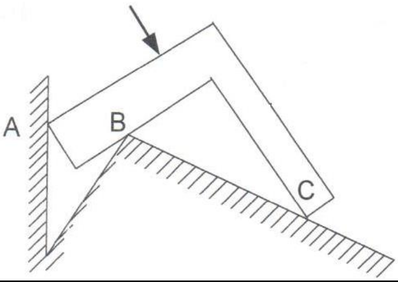	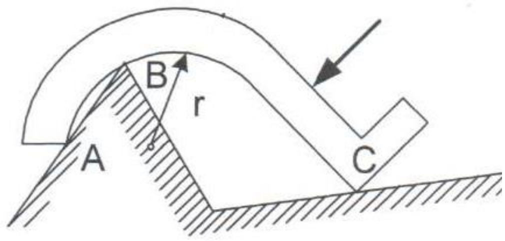
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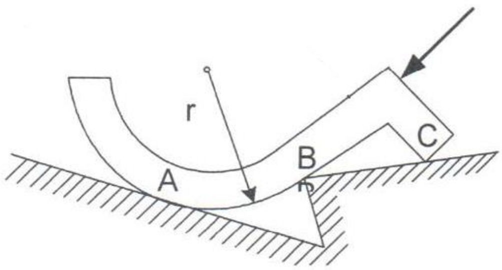	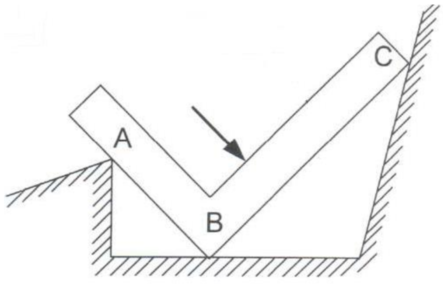

TASKS 3

Name the type of support and show the direction of the support reactions at points A, B, C of the body

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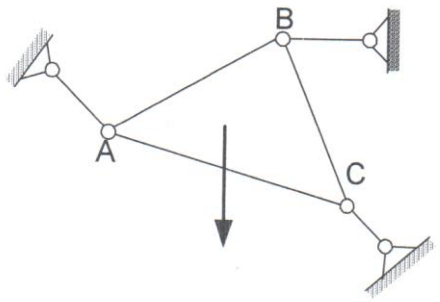	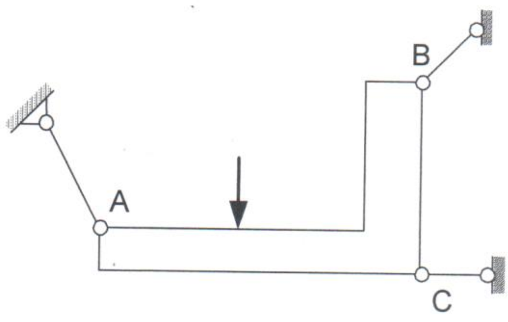
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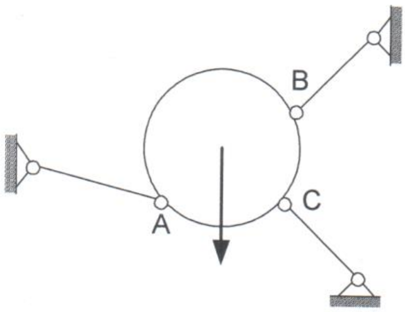	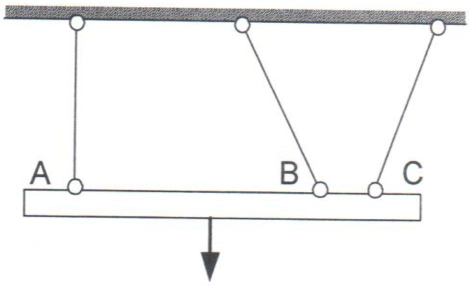
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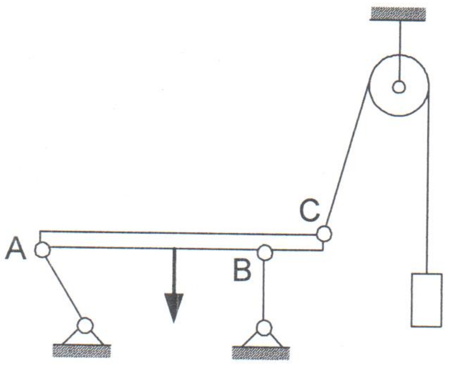	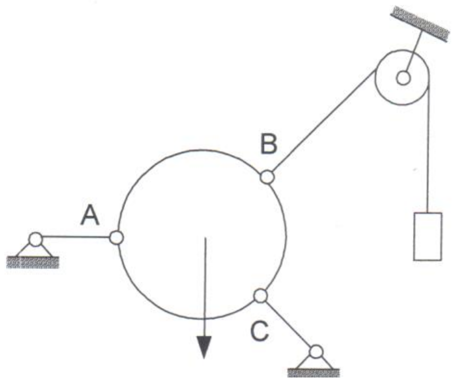
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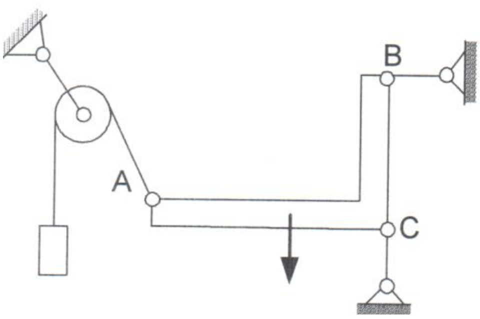	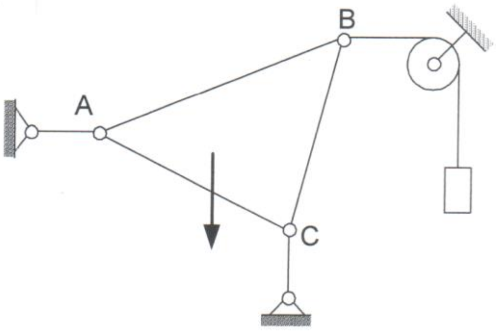

TASKS 4

Name the type of support and show the direction of the support reactions at points A, B of the body

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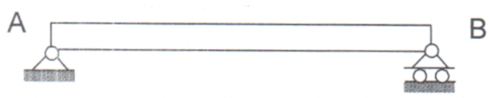
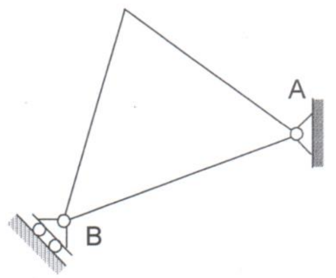
<tr><td></td><td></td></tr>
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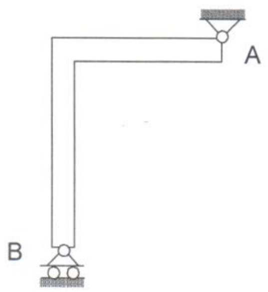
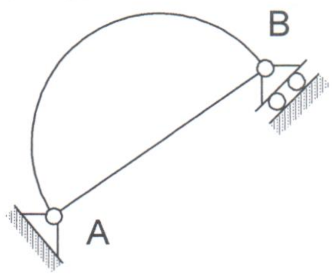
<tr><td></td><td></td></tr>
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TASKS 5

Name the type of supports and show beam AB support reactions

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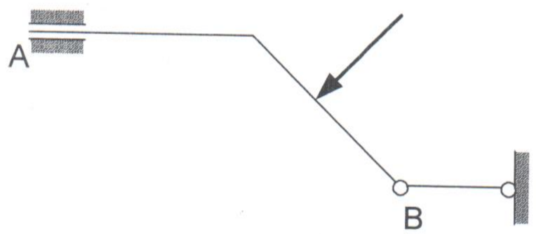
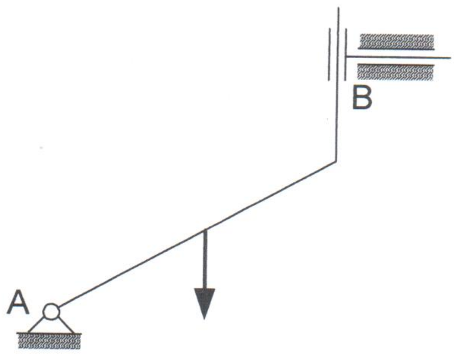
<tr><td></td><td></td></tr>
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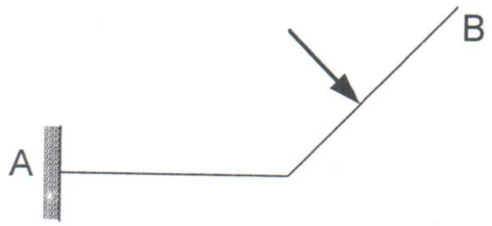

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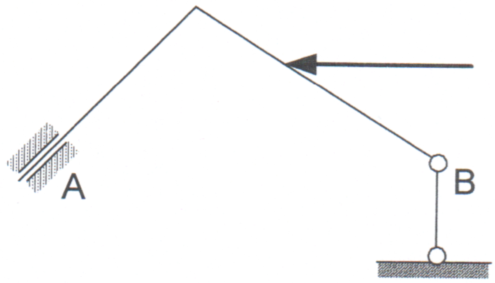

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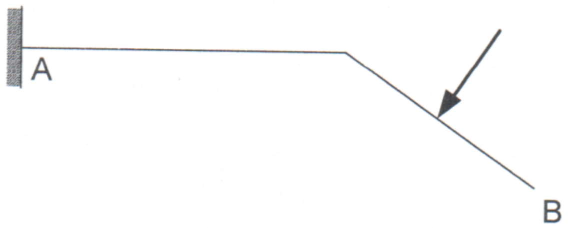

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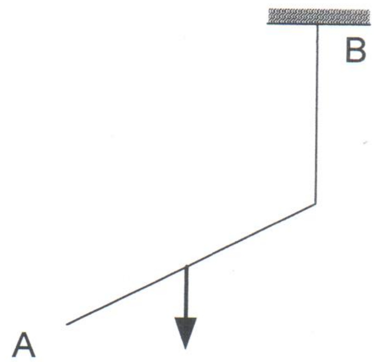

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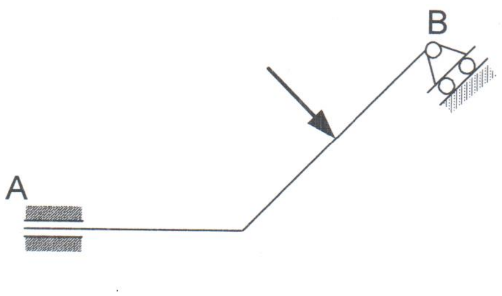

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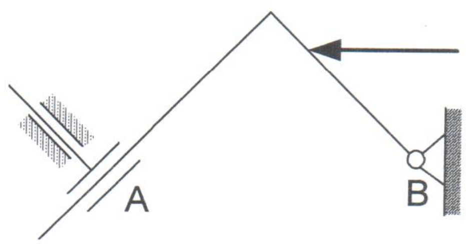
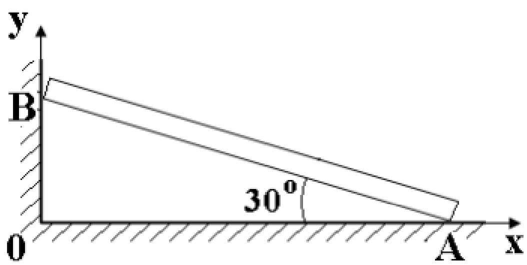
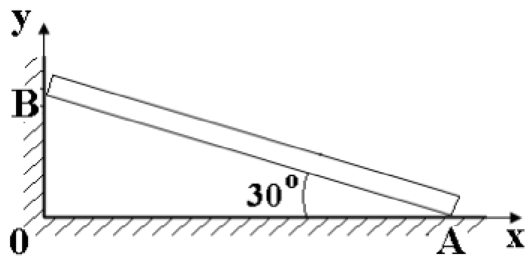


TASKS 6

1. Determine the angle between axis Oy and reaction of smooth wall (p. B)



2. Determine the angle between axis Ox and reaction of smooth wall (p. B)



2. COPLANAR CONCURRENT FORCES

The purpose of the lesson: acquiring skills:

- Finding force projections on the coordinate axis.
- Decomposition of force into components.
- Determining the moments of forces.
- Drawing up analytical conditions for the equilibrium of a system of concurrent forces.

Before completing the tasks you need to read:

- Basic concepts and definitions of statics.
- Concentrated forces and their characteristics.
- Axioms of statics.
- Types of supports and their reactions.
- Projection of force on the axis.
- Decomposition of force into components.
- The properties of the moment of force relative to the point (center).
- Geometric and analytical conditions of equilibrium of a coplanar system of concurrent forces.

TOPIC 1. PROJECTIONS OF FORCE ON THE AXIS

Find the projections of the force vector on the coordinate axes

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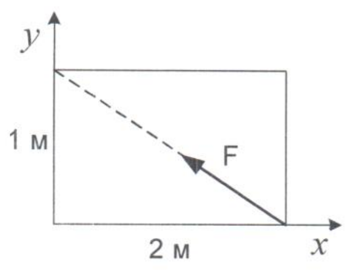	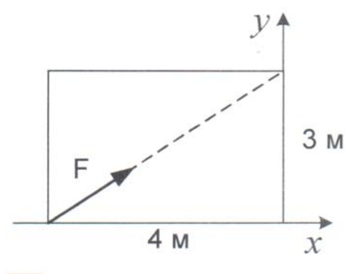

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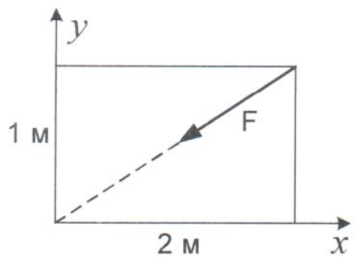
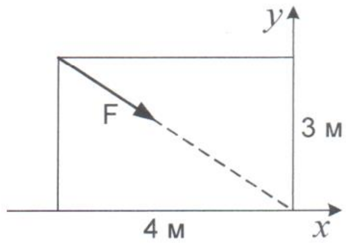
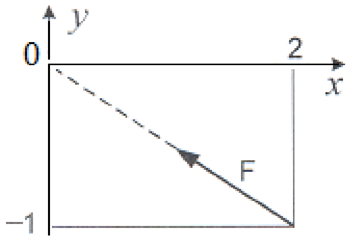
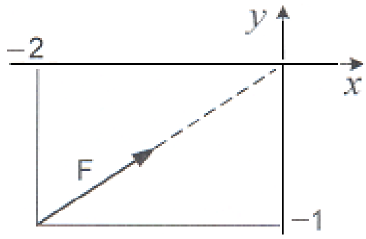
	
5	**6**
	

TOPIC 2. MOMENT OF A FORCE RELATING TO A POINT

The rotational effect of force is characterized by its moment.

The moment of force is considered positive when the force rotates the body around a point counterclockwise.

The algebraic moment of force F relative to a point is denoted by a symbol $M_O(F)$ and is determined by the formula:

$$M_O(F) = \pm F\,h$$

where *h* - a perpendicular drawn from a point on the line of force.

Properties of the moment of force relative to an arbitrary point (arbitrary center):

1. The moment of force does not depend on the transfer of force along the line of its action.

2. The moment of force relative to the center is zero when the line of action of the force passes through the center or when the force is zero;

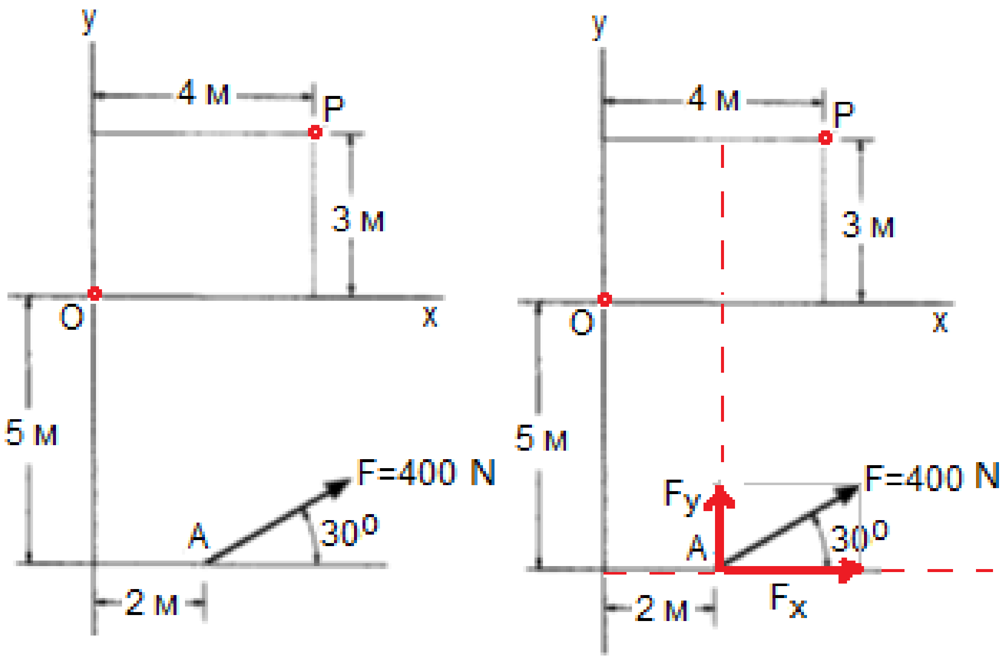
3. The moment of force relative to the center is a bound vector.

EXAMPLE

Calculate the moments of force = 400 N relative to the origin O and relative to the point P.

Solution

- We decompose the force F into components Fx, Fy.
- Find the shoulders for the components Fx, Fy.
- Find the moment of force F relative to the points O and P as the sum of the moments of the components Fx, Fy, taking into account their signs.



The moment of force F = 400 N relative to the origin O is equal to:

$$M_O(F) = M_O(F_X) + M_O(F_Y) = 5F_X + 2F_Y$$

$$M_O(F) = 5 \cdot 400\, cos\, 3\,0° + 2 \cdot 400\, sin\, 3\,0°$$

$$M_O(F) = 1732 + 400 = 2132 \text{ Nм}$$

The moment of force F = 400 N relative to point P is equal to:

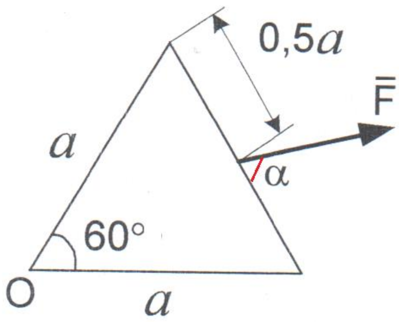
$$M_{\text{P}}(F) = M_{\text{P}}(F_X) + M_{\text{P}}(F_Y) = (5+3)F_X - (4-2)F_Y$$

$$M_{\text{P}}(F) = 8 \cdot 400\, cos\, 3\,0° - 2 \cdot 400\, sin\, 3\,0°$$

$$M_{\text{P}}(F) = 2771 - 400 = 2371 \text{ Nм}$$

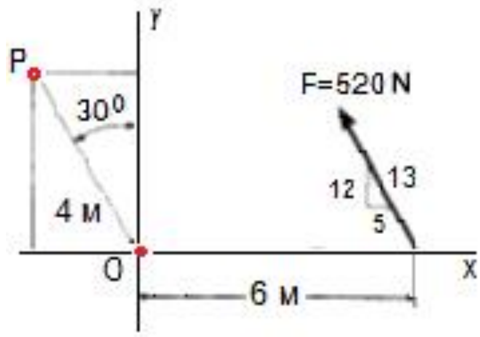
Answer: $M_O(F) = 2132$ Nм; $M_P(F) = 2371$ Nм

1. Calculate the moment of force F relative to the point O

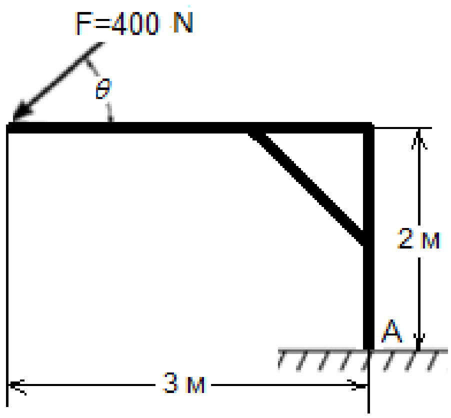


2. Calculate the moments of every forces relative to the point O

3. Calculate the moment of force F =520 N relative to the point P



4. Calculate the angle θ $(0 \le \theta \le \pi)$ of force **F**, at which the force creates: a) the maximum moment relative to point A; b) the minimum moment relative to point A. Calculate these moments.

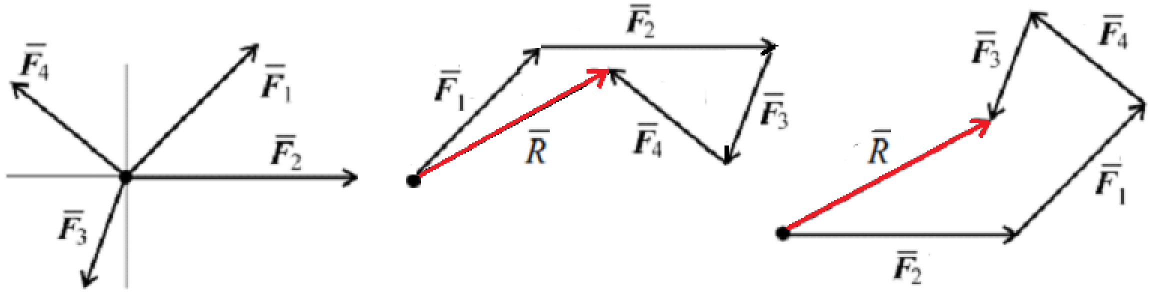


TOPIC 3. A SYSTEM OF CONCURRENT FORCES

A system of forces whose lines of action intersect at one point is called a system of concurrent forces.

The system of concurrent forces has an equivalent, which is equal to the geometric sum of forces of the system and passes through the point of intersection of their lines of action.

With geometric method of equivalent determining, the force vectors can be added in any order - the magnitude and direction of equivalent will not change. This means that forces adding is a unique operation.



$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{F}_2 + \vec{F}_1 + \vec{F}_4 + \vec{F}_3$$

Fig. 2.1

The projection of the sum vector of forces on any axis is equal to the sum of projections of its terms on this axis.

Since the system of concurrent forces is reduced to an equivalent force, it is necessary and sufficient for its equilibrium that this equivalent force be equal to zero.

Geometric conditions of equilibrium: The system of concurrent forces will be in equilibrium if and only if the force polygon is closed.

Analytical conditions of equilibrium: For the equilibrium of a system of concurrent forces it is necessary and sufficient that the sum of projections of all forces of the system on each of three coordinate axes is zero.

In the case of a flat system of concurrentt forces, the analytical equilibrium conditions are as follows:

$$\sum_{k=1}^{n} F_{kX} = 0; \qquad \sum_{k=1}^{n} F_{kY} = 0$$

That is, a system of concurrent forces acting in one plane is in equilibrium if sums of projections of forces of the system on two mutually perpendicular Cartesian axes are equal to zero.

If a free solid is in equilibrium under the action of three nonparallel forces lying in the same plane, then the lines of action of these forces intersect at one point.

Solve of tasks geometrically

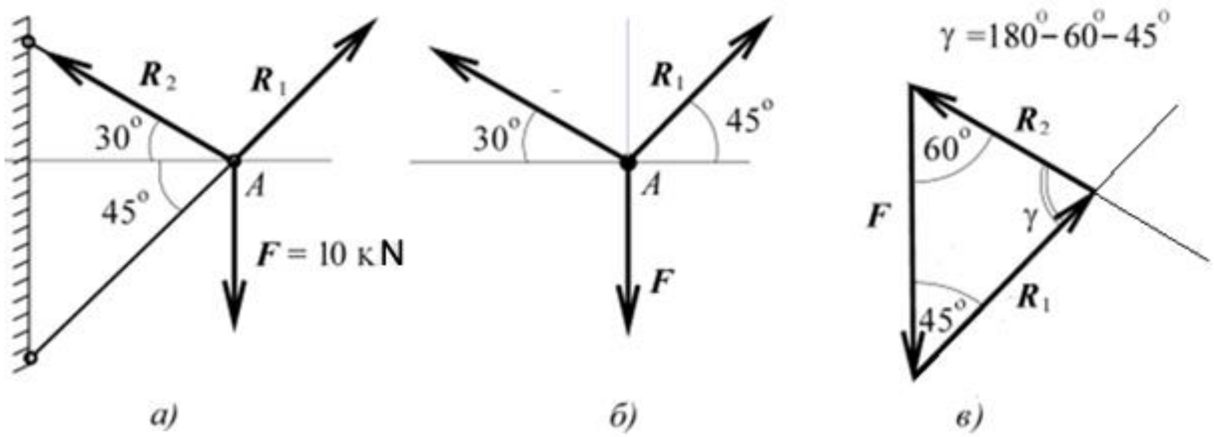


Fig. 2.2

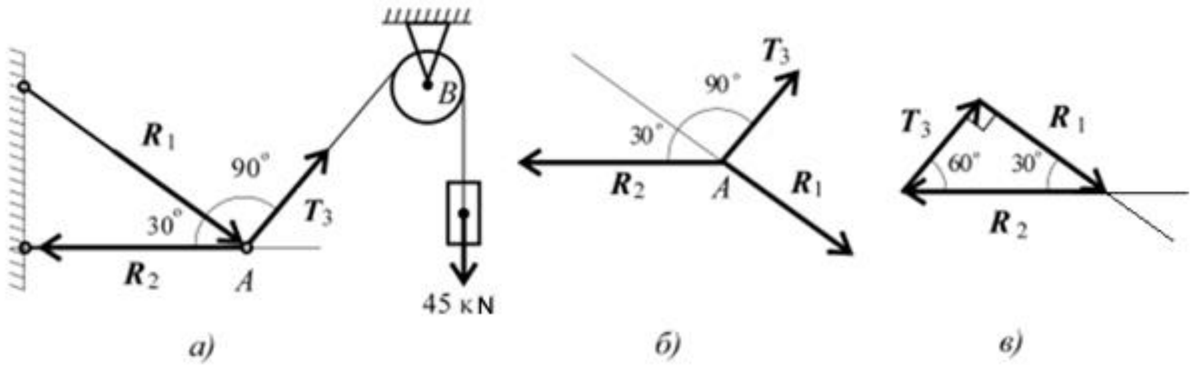


Fig. 2.3

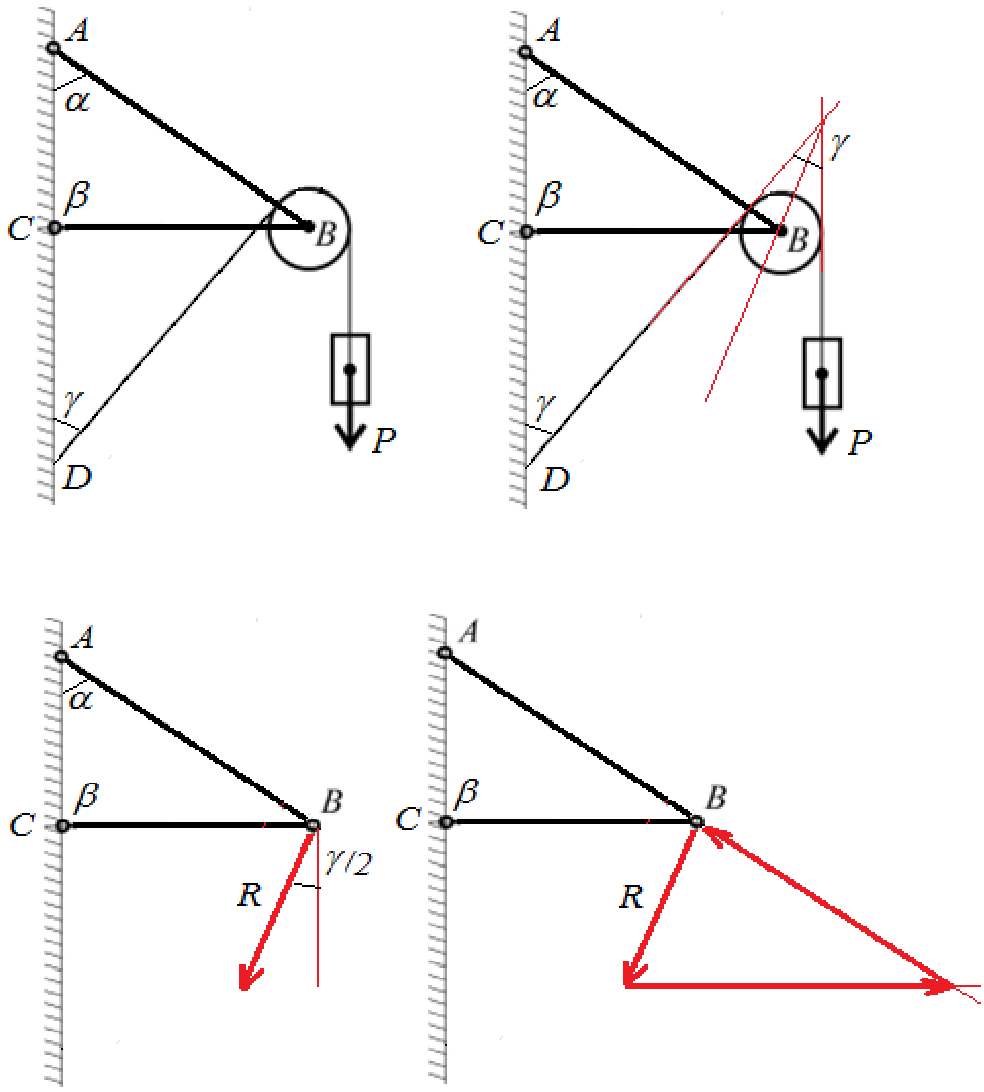
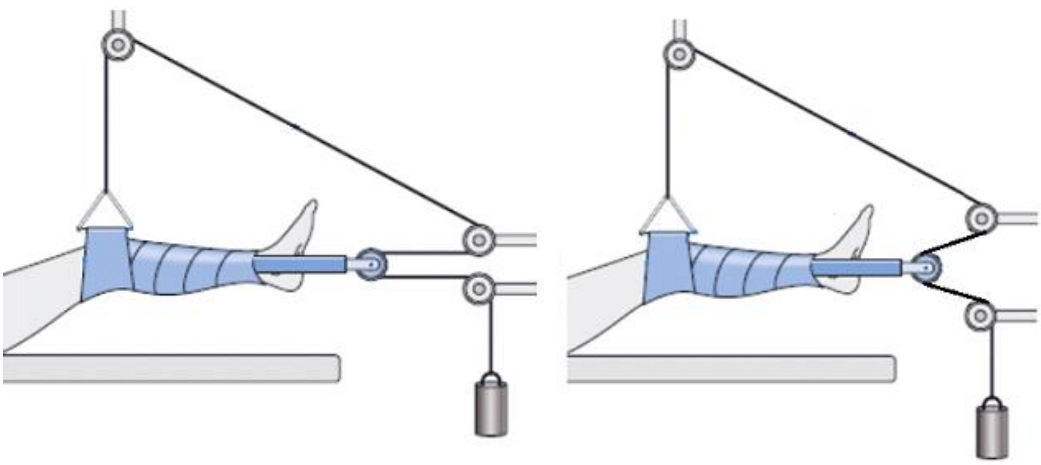
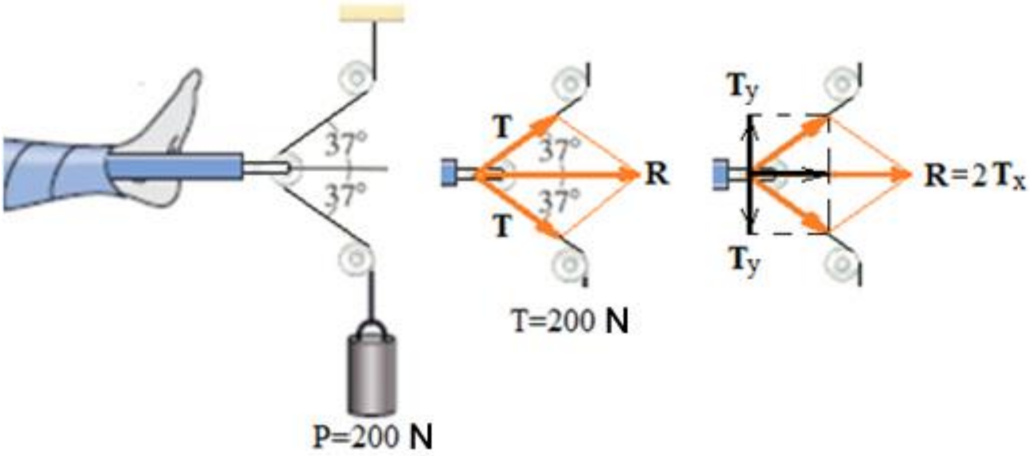


Fig. 2.4





$$R = 2T_X = 2T\cos 37^\circ = 2 \cdot 200\cos 37^\circ = 319\ N$$

Fig. 2.5

INDIVIDUAL TASKS FOR INDEPENDENT WORKING

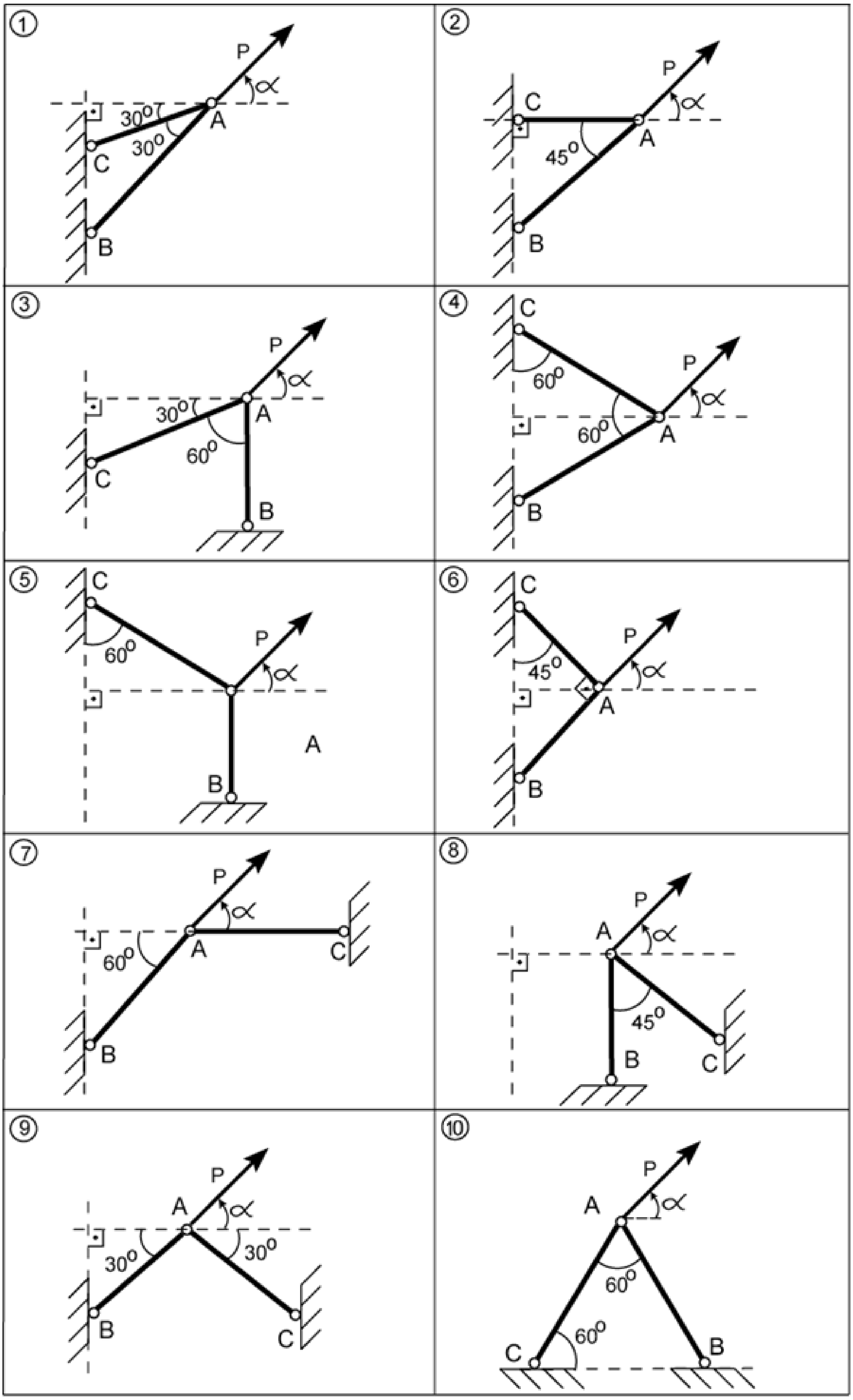
Calculate by geometric and analytical methods the forces in the weightless articulated rods AB and AC, if the point A is applied to the force P = 100 N at an angle α to the horizon. The data required for calculations are given in table 2.1 and table 2.2. Schemes of structures are presented in Figure 2.6.

Table 2.1-Options 1-10

Option N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
scheme N	1	2	3	4	5	6	7	8	9	10
α, degree	120	15	-15	75	-75	105	-105	165	-165	195

Table 2.2-Options 11-20

Option N	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
scheme N	1	2	3	4	5	6	7	8	9	10
α, degree	-30	60	-225	-100	45	-60	110	20	100	-30



1
P
α
30°
30°
A
C
B
2
P
α
C
A
45°
B
3
P
α
30°
A
60°
C
B
4
C
60°
P
α
60°
A
B
5
C
60°
P
α
A
B
6
C
P
45°
α
A
B
7
P
α
60°
A
C
B
8
P
A
α
45°
B
C
9
P
A
α
30°
30°
B
C
10
P
A
α
60°
60°
C
B

Fig. 2.6

3. SYSTEM OF PARALLEL FORCES

The purpose of the lesson: acquiring skills:

- Finding an equivalent distributed load.
- Determining the moments of forces.
- Drawing up analytical conditions for the equilibrium of a system of parallel forces.

Before completing the task you need to read:

- Concentrated forces and their characteristics.
- Axioms of statics.
- Types of supports and their reactions.
- Decomposition of force into components.
- The theory of a pair of forces.
- Analytical conditions of equilibrium of a flat system of parallel forces.

Brief theoretical information

In engineering calculations, along with the concentrated forces that are applied to a solid body at some point, there are forces whose action is distributed over the volume of the body, its surface or line.

Since all axioms and theorems of statics are formulated for concentrations of forces, it is necessary to consider ways of transition from distributed loading to concentrated forces.

Consider some simple cases of distributed load by parallel forces lying in one plane along a line segment.

A flat system of distributed forces is characterized by intensity $q(x)$, ie the magnitude of the force per unit length of the loaded segment. The unit of intensity is Newton divided by a meter (N/m). The intensity can be constant (evenly distributed load) or vary according to linear or arbitrary laws.

The distributed load is replaced by a concentrated - the resulting (equivalent) force applied to the center of gravity. The magnitude of resulting force is equal to the area of plot of the distributed load.

The resulting force Q of the distributed load q (x) on the section of length L is determined by the formula:

$$Q = \int_0^L q(x)\,dx$$

The moment created by the distributed load q (x) relative to the point O is determined by the formula:

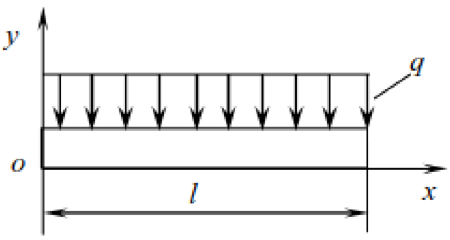
$$M_O = \int_0^L q(x)\,x\,dx$$

The distance of the equivalent concentrated force of the distributed load from the origin is equal to:

$$x_O = \frac{M_O}{Q} = \frac{\int_0^L q(x)\,x\,dx}{\int_0^L q(x)\,dx}$$

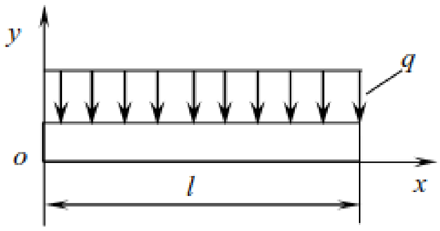
TOPIC 1. DISTRIBUTED LOAD

1. The modulus of equivalent Q of uniformly distributed load q is equal to:



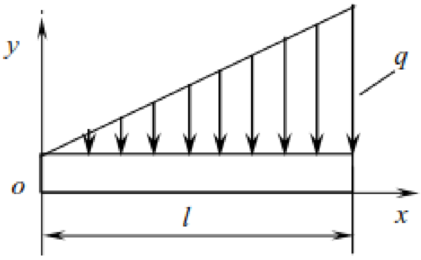
$a)\ Q = q^2 l;$ $b)\ Q = ql;$ $c)\ Q = l/q;$ $d)\ Q = 0{,}5ql$

2. The coordinate x of the point of application of an equivalent Q of uniformly distributed load q is equal to:



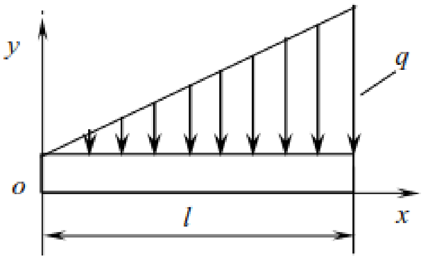
$a)\ x = l;$ $b)\ x = 2l;$ $c)\ x = \frac{2}{3}l;$ $d)\ x = 0,5l$

3. The load, the maximum value of which q, is distributed according to a linear law (the law of a triangle). The equivalent Q of this load is equal to:



$a)\ Q = 2ql;$ $b)\ Q = \frac{1}{2}ql;$ $c)\ Q = \frac{1}{3}ql;$ $d)\ Q = ql$

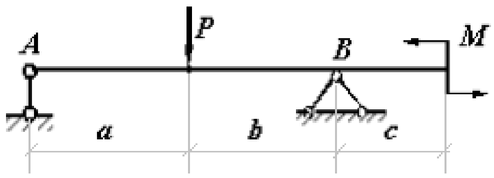
4. The load, the maximum value of which q, is distributed according to a linear law (the law of a triangle). The coordinate x of the point of application of the equivalent Q is equal to:

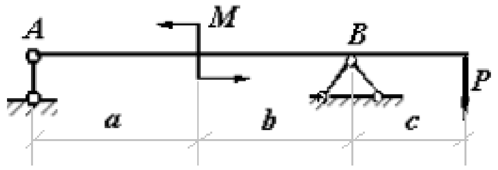


$a)\ .x = 0,5l;$ $b)\ x = \frac{1}{3}l;$ $c)\ x = \frac{2}{3}l;$ $d)\ x = l$

TOPIC 2. DETERMINATION OF REACTIONS OF BEAM SUPPORTS

1. The horizontal beam is mounted on supports A and B, loaded with a force P = 20 kN and a pair of forces with a



moment M = 10 kN m; a = b = c = 1 m. The reaction of support A is equal to: a) 5; b) 10; c) 15; d) 20
2. The horizontal beam is mounted on supports A and B, loaded with a force P = 20 kN and a pair of forces with a moment M = 10 kN m; a = b = c = 1 m. The reaction of support A is equal to:  a) - 5; b) 10; c) -15; d) 20

INDIVIDUAL TASKS FOR INDEPENDENT WORKING

A horizontal beam of length l is mounted on supports B and C, loaded with a concentrated force F, distributed load intensity q, a pair of forces with torque M. Determine the reactions of supports B and C, not taking into account the weight of the beam.

The data required for the calculations are given in table.3.1. Schemes of structures are presented in Figure 3.1.

Table 3.1

Case N	Scheme N	F, кN	q, кN/m	M, кN m	l, m
1.	1	20	20	30	8
2.	2	30	30	25	12
3.	3	25	10	35	10
4.	4	10	20	40	8
5.	5	30	30	25	12
6.	6	35	10	20	10
7.	7	20	20	30	8
8.	8	25	30	35	12
9.	9	15	10	40	6

10.	10	40	10	20	10
11.	1	30	30	25	12
12.	2	35	10	20	10
13.	3	15	10	40	6
14.	4	40	10	20	10
15.	5	20	20	30	8
16.	6	25	30	35	12
17.	7	25	10	35	10
18.	8	10	20	40	8
19.	9	20	20	30	8
20.	10	30	30	25	12

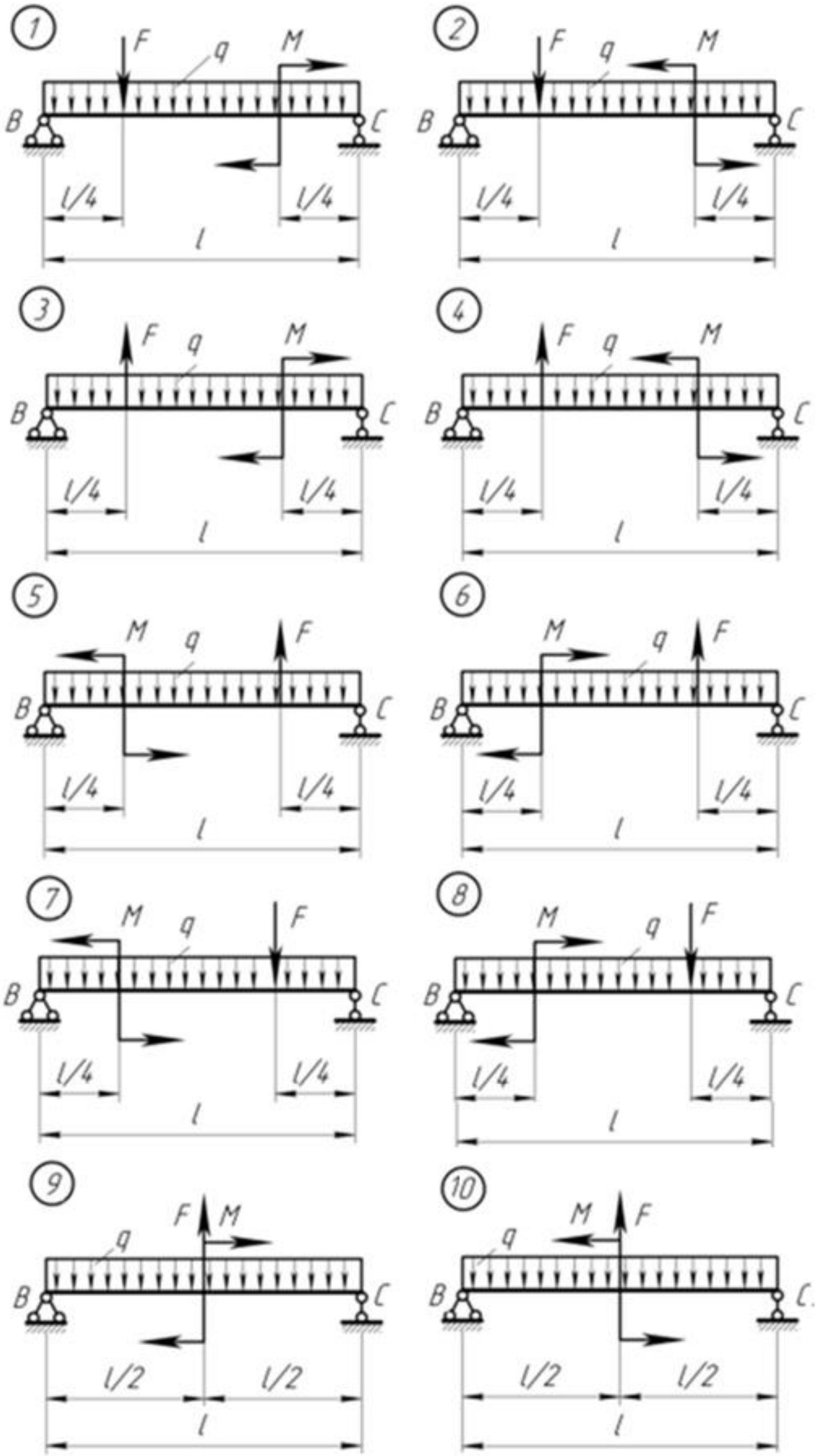

1
F
q
M
B
C
l/4
l/4
l
2
F
q
M
B
C
l/4
l/4
l
3
F
q
M
B
C
l/4
l/4
l
4
F
q
M
B
C
l/4
l/4
l
5
M
q
F
B
C
l/4
l/4
l
6
M
q
F
B
C
l/4
l/4
l
7
M
q
F
B
C
l/4
l/4
l
8
M
q
F
B
C
l/4
l/4
l
9
F
M
q
B
C
l/2
l/2
l
10
M
F
q
B
C
l/2
l/2
l

Fig. 3.1

EXAMPLE

task solution

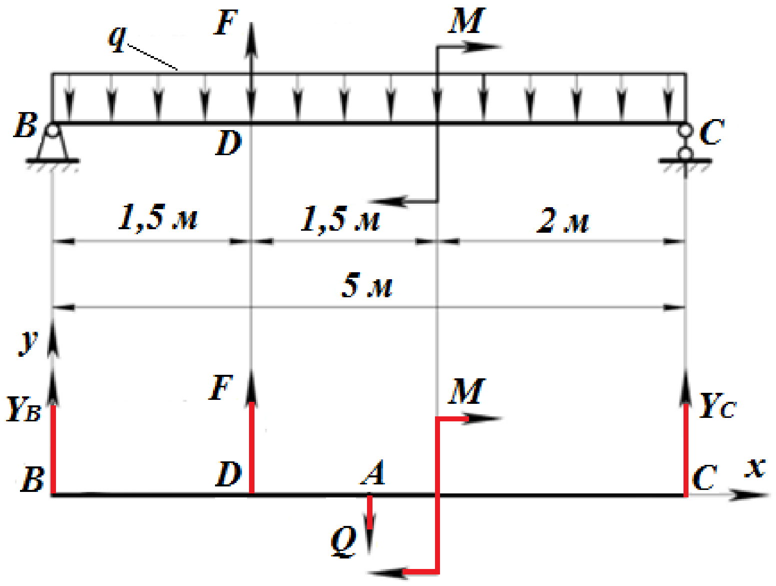
DETERMINATION OF REACTIONS OF BEAM SUPPORTS

Горизонтальна балка довжиною $l = 5\ м$ встановлена на опорах B і C і навантажена розподіленим навантаженням інтенсивністю $q = 20\ кН / м$, зосередженою силою $F = 15\ кН$ і парою сил з моментом $M = 16\ кН\ м$. Визначити реакції опор B і C. Вагою балки знехтувати.

A horizontal beam of length $l = 5\ m$ is mounted on supports B and C and loaded with a distributed load intensity $q = 20\ \kappa N/m$, concentrated force $F = 15\ \kappa N$ and a pair of forces with torque $M = 16\ \kappa Nm$. Determine the reactions of supports B and C. Neglect the weight of the beam

Solution

Rejecting the supports, replace them with reactions. The reactions of the supports are the reactive forces Y_B and Y_C. We will direct the reactive forces vertically, because the active forces acting on the beam do not have horizontal components. The distributed load is replaced by an equivalent one $Q = ql = 20 \cdot 5 = 100\ \kappa N$.



Using the equilibrium condition of a planar system of parallel forces, we write two equilibrium equations for the sum of the moments of all forces relative to point B and relative to point C:

$$\sum_{k=1}^{n} M_B(F) = 0; \qquad F \cdot BD - Q \cdot BA - M + Y_C BC = 0$$

$$\sum_{k=1}^{n} M_C(F) = 0; \qquad -Y_B BC - F \cdot CD + Q \cdot AC - M = 0$$

From the given equilibrium equations we define unknown reactions:

$$Y_C = \frac{Q \cdot BA + M - F \cdot BD}{BC} = \frac{100 \cdot 2{,}5 + 16 - 15 \cdot 1{,}5}{5} = 48{,}7\ \kappa N$$

$$Y_B = \frac{Q \cdot AC - M - F \cdot CD}{BC} = \frac{100 \cdot 2{,}5 - 16 - 15 \cdot 3{,}5}{5} = 36{,}3\ \kappa N$$

Let's check the solution by making the control equation - the sum of projections of all forces on vertical axis:

$$\sum_{k=1}^{n} F_{kY} = 0; \qquad Y_B + F - Q + Y_C = 0; \qquad 36{,}3 + 15 - 100 + 48{,}7 = 0$$

The solution is correct.

Result: $Y_C = 48{,}7\ \kappa N; \quad Y_B = 36{,}3\ \kappa N.$

4. ARBITRARY FORCE SYSTEMS

The purpose of the lesson: acquiring skills from:

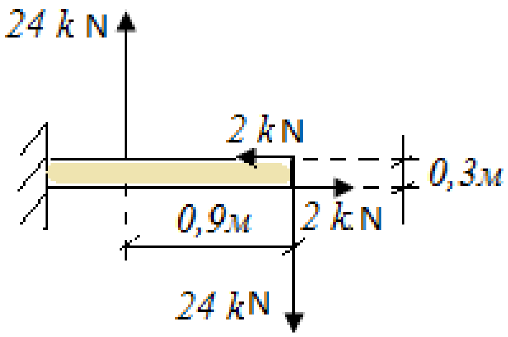
- Finding the moment of an equivalent pair of forces.
- Drawing up analytical conditions for the equilibrium of a planar system of parallel forces.
- Determination of the moment of forces relative to the axis.
- Drawing up analytical conditions for the equilibrium of a plane system of arbitrary forces.

Before completing the task you need to read:

- The theory and properties of a pair of forces.
- Decomposition of force into components.
- Ways to determine the moments of forces relative to the coordinate axes.
- Analytical conditions of equilibrium of a plane system of parallel and arbitrary forces.

THEME 1. PAIR OF FORCES

1. Two pairs of forces act on the rod. The modulus of the moment of the equivalent pair is equal to:



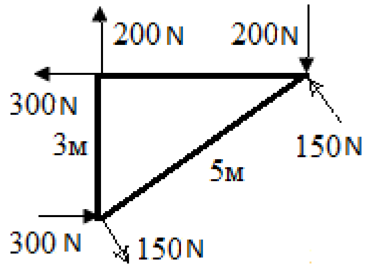
a) 0

b) 14,4 кNм

c) 22,2 кNм

d) 21 кNм

2. A system of pairs of forces acts on a figure in the form of a triangle. The modulus of the moment of the equivalent pair is equal to:



a) 0 Nм
b) 850 Nм
c) 1600 Nм
d) 2450 Nм

TOPICS 2. PLANE PARALLEL SYSTEM OF FORCES

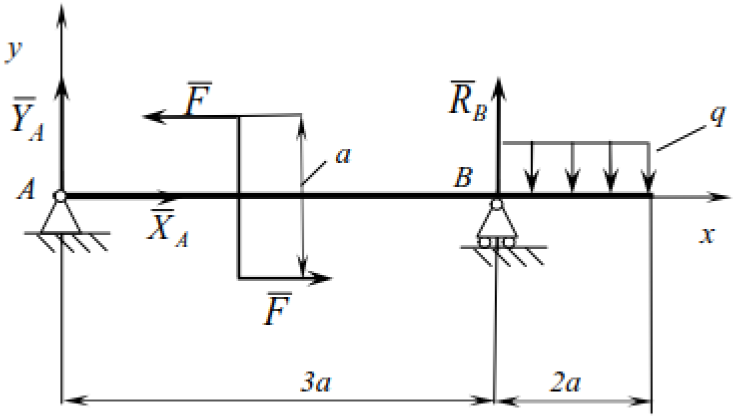
1. The horizontal beam is mounted on supports A, B and loaded with a pair of forces with the arm a and the distributed load q. Specify the correct equation of equilibrium of the beam:

$a)\quad 3R_B a - Fa - 3qa = 0$

$b)\quad X_A - F = 0$

$c)\quad Y_A + R_B - 2qa = 0$

$d)\quad Y_A + R_B + 2qa = 0$



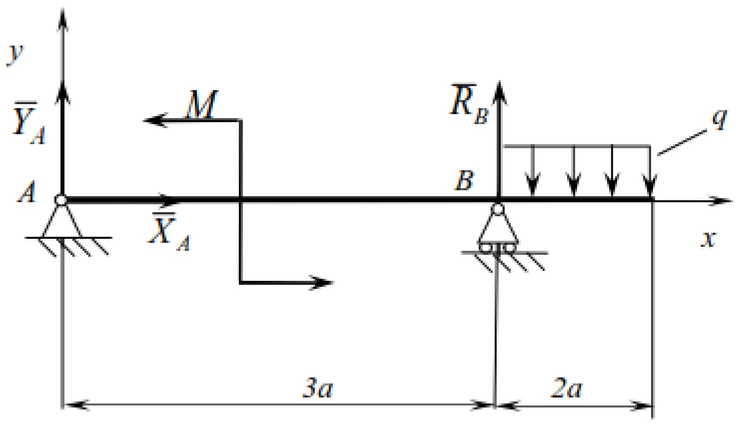
2. The horizontal beam is mounted on supports A, B and loaded with a pair of forces with the torque M and the distributed load q. Specify the correct equation of equilibrium of the beam:

a) $3R_B a - M - 6qa^2 = 0$

b) $3R_B a + M - 8qa^2 = 0$

c) $M + 3R_B a - 4qa = 0$

d) $Y_A + R_B + X_A - 2qa = 0$



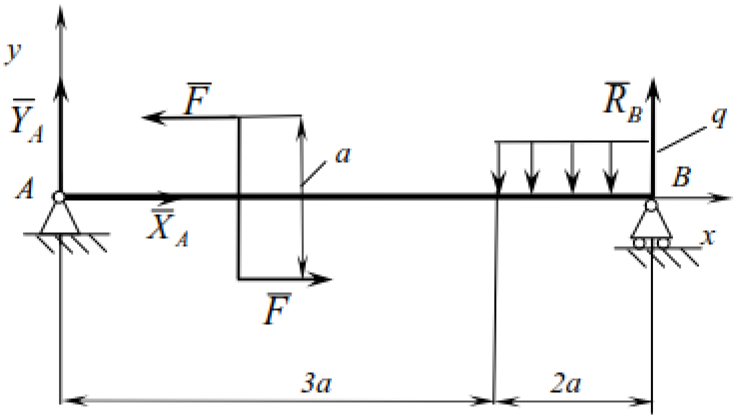
3. The horizontal beam is mounted on supports A, B and loaded with a pair of forces with the arm a and the distributed load q. Specify the correct equation of equilibrium of the beam:

a) $Fa - 5R_B a - 8qa = 0$

b) $Y_A + R_B + 2F - qa = 0$

c) $5R_B a + Fa - 4qa^2 = 0$

d) $5R_B a + Fa - 8qa^2 = 0$



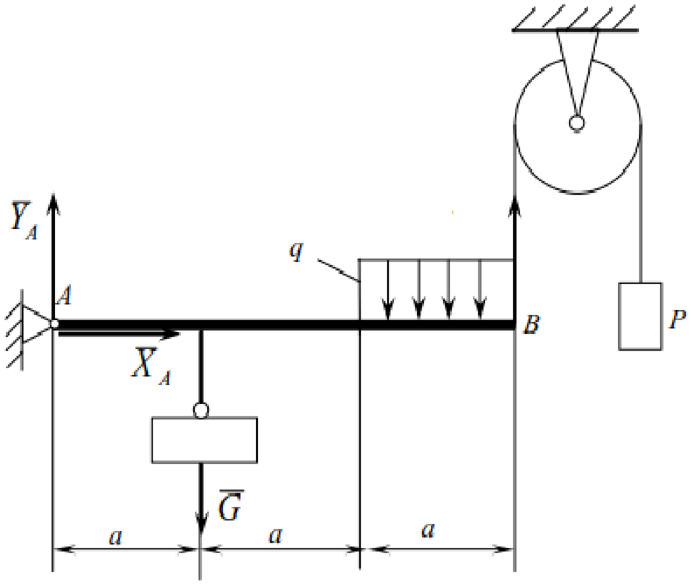
4. The reaction YA of the hinge A of a homogeneous weightless beam AB is equal to:

$a)\quad Y_A = G + P - qa$

$b)\quad Y_A = G - P + 0,5qa$

$c)\quad Y_A = G + qa - P$

$d)\quad Y_A = G + qa^2 - P$



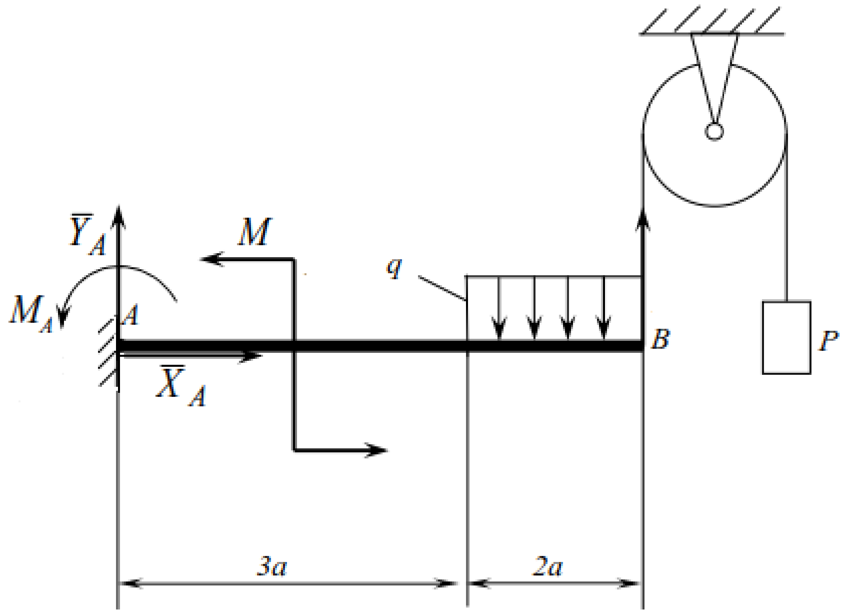
5. The reactive moment MA of rigid fixing weightless beam AB is equal to:

a) $M_A = 4qa^2 - 5Pa - M$

b) $M_A = 8qa^2 - 5Pa - M$

c) $M_A = M - 8qa^2 - 5Pa$

d) $M_A = 8qa^2 - 5Pa$



TOPIC 3. MOMENT OF FORCE RELATING TO THE AXIS

INDIVIDUAL TASKS FOR INDEPENDENT WORKING

Table 4.1 - Options 1-10

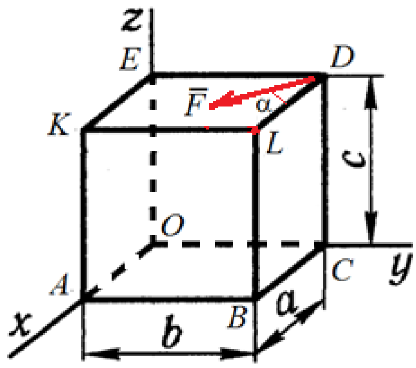
Option N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Scheme N	1	1	1	2	2	2	3	3	3	4
Determine the moment of force relative to the axis:	X	Y	Z	X	Y	Z	X	Y	Z	X

Table 4.2 - Options 11-20

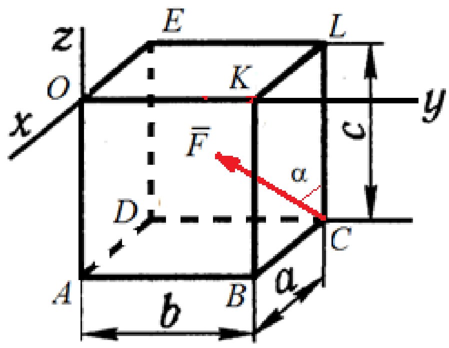
Option N	11	12	13	14	15	16	17	18	19	20
Scheme N	4	4	5	5	5	6	6	6	7	7
Determine the moment of force relative to the axis:	Y	Z	X	Y	Z	X	Y	Z	X	Y

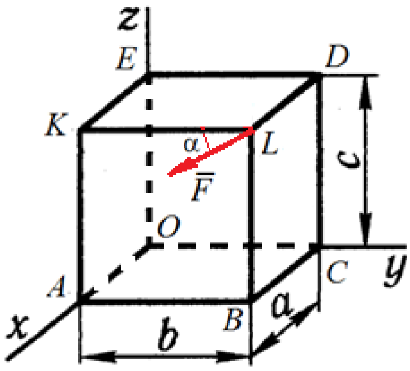
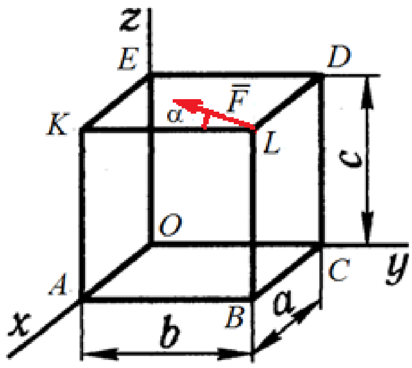
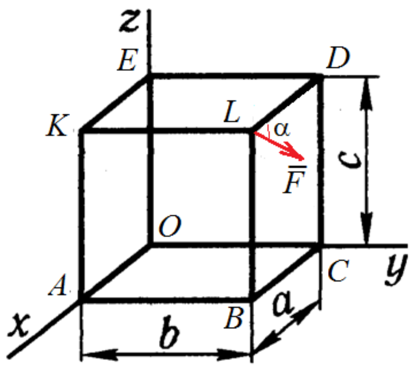
Scheme N

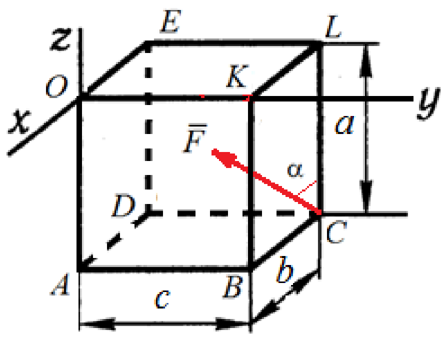
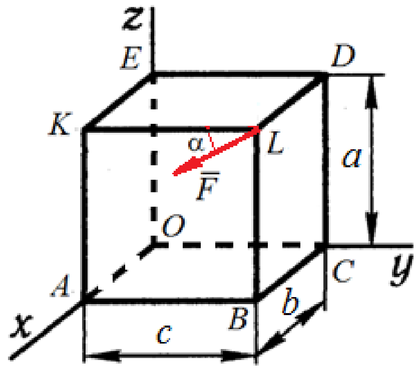
1. The force F lies in the plane DEKL and is applied at point D at an angle α to DL. Determine the moments of force relative to the X, Y, Z axes



2. The force F lies in the plane CDEL and is applied at point C at an angle α to CL. Determine the moments of force relative to the X, Y, Z axes



3. The force F lies in the plane ABLK and is applied at point L at an angle α to LK. Determine the moments of force relative to the X, Y, Z axes	
4. The force F lies in the plane DEKL and is applied at point L at an angle α to LK. Determine the moments of force relative to the X, Y, Z axes	
5. The force F lies in the plane BCDL and is applied at point L at an angle α to LD. Determine the moments of force relative to the X, Y, Z axes	

6. The force F lies in the plane CDEL and is applied at point C at an angle α to CL. Determine the moments of force relative to the X, Y, Z axes	
7. The force F lies in the plane ABLK and is applied at point L at an angle α to LK. Determine the moments of force relative to the X, Y, Z axes	

TOPIC 4. PLANE ARBITRARY SYSTEM OF FORCES

INDIVIDUAL TASKS FOR INDEPENDENT WORKING

The horizontal beam is loaded by the force P, by the moment M and by the distribution load intensity q. Determine the reactions of the supports. Perform a check. In the final calculations accept:

P = 10 кN;

M = 5 кN·m;

q = 2 кN/m;

α = 30°;

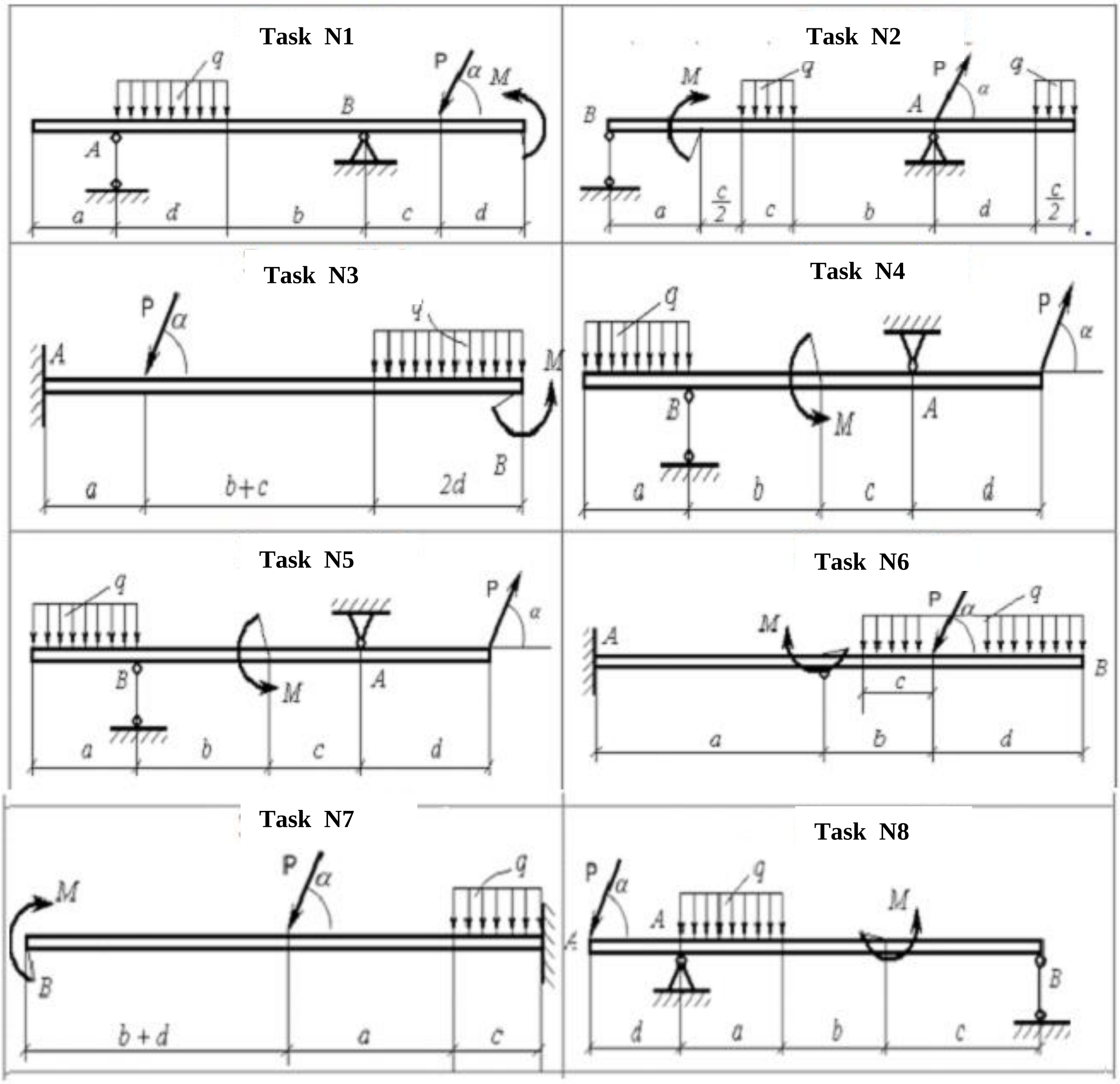
a = 4 m, b = 5 m, c = 2 m, d = 3 m.

Table 4.3 - Options 1-10

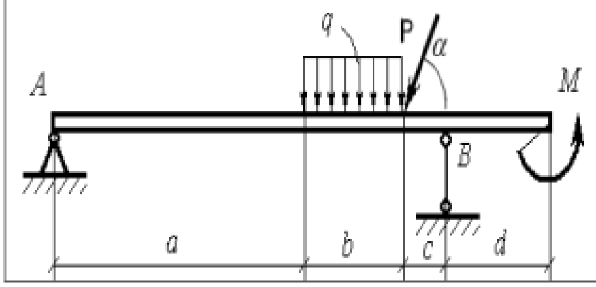
Case N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Task N	1	2	3	4	5	6	7	8	9	10

Table 4.4 - Options 11-20

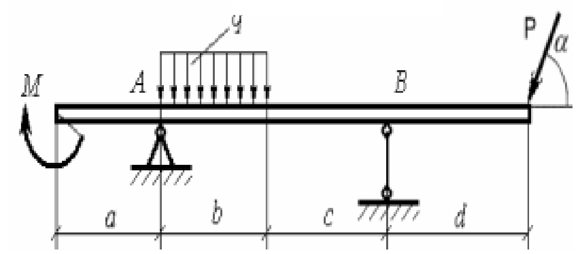
Case N	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Task N	11	12	13	14	15	16	17	18	19	20



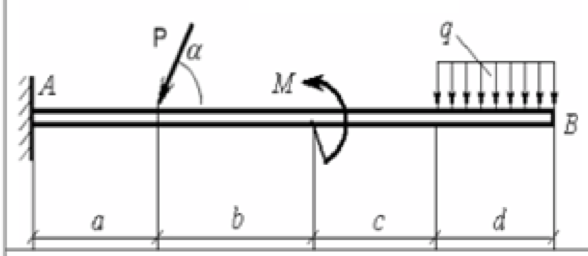
Task N9



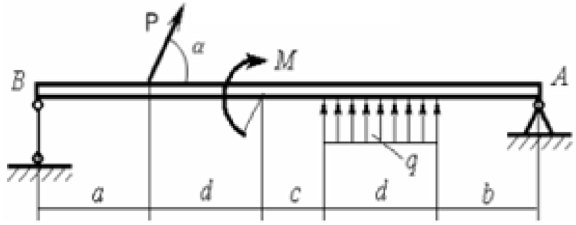
Task N10



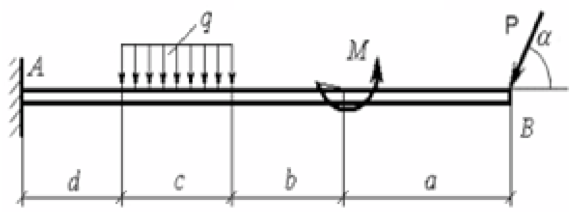
Task N11



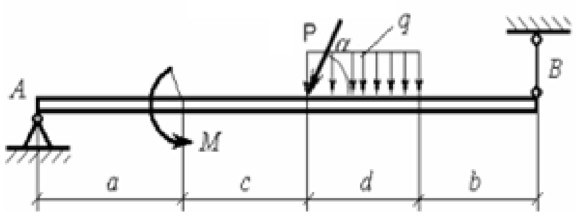
Task N12



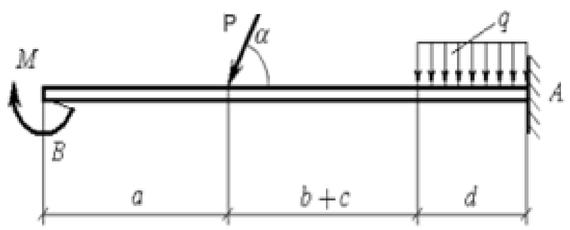
Task N13



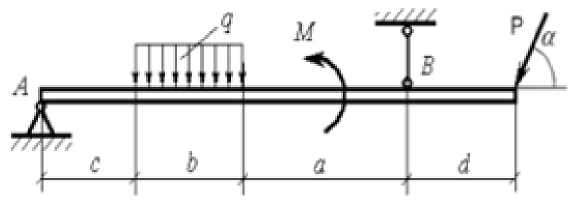
Task N14



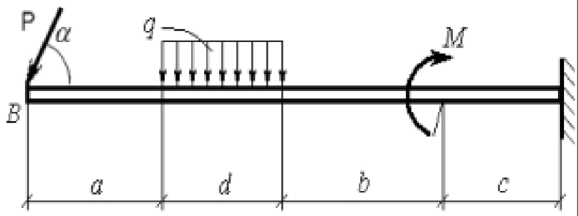
Task N15



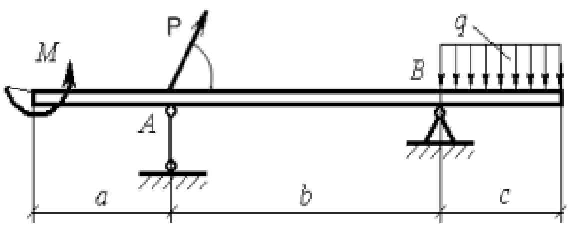
Task N16



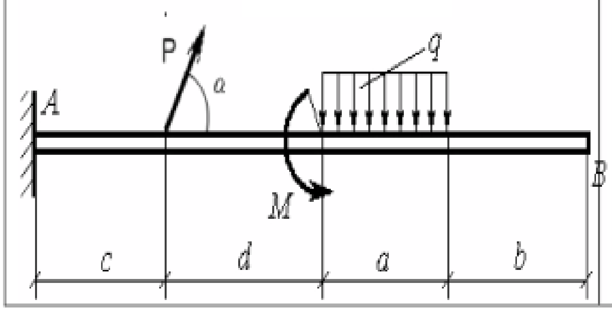
Task N17



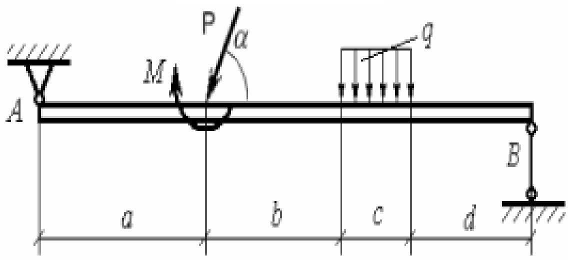
Task N18



Task N19



Task N20



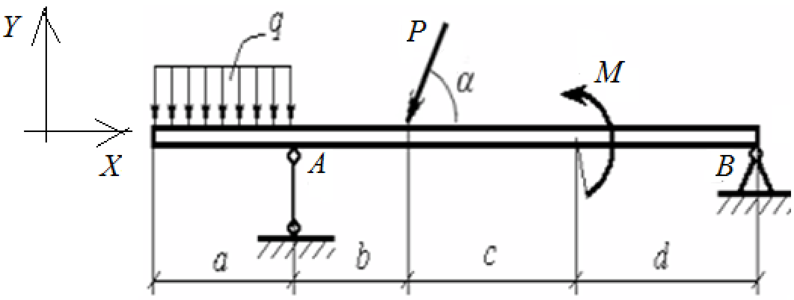
EXAMPLE OF TASK SOLUTION

Determination of reactions of beam supports under the action of an arbitrary system of forces

The horizontal beam is loaded with force P, torque M and distribution load intensity q. Determine the reactions of supports. Run the test.

Take: P = 10 кN; M = 5 кN·m; q = 2 кN/m; α = 30°;

a = 4 m, b = 5 m, c = 2 m, d = 3 m.



Solution

1. We replace the distributed load q with an equivalent force Q:

2. $Q = qa = 2 \cdot 4 = 8$ кN

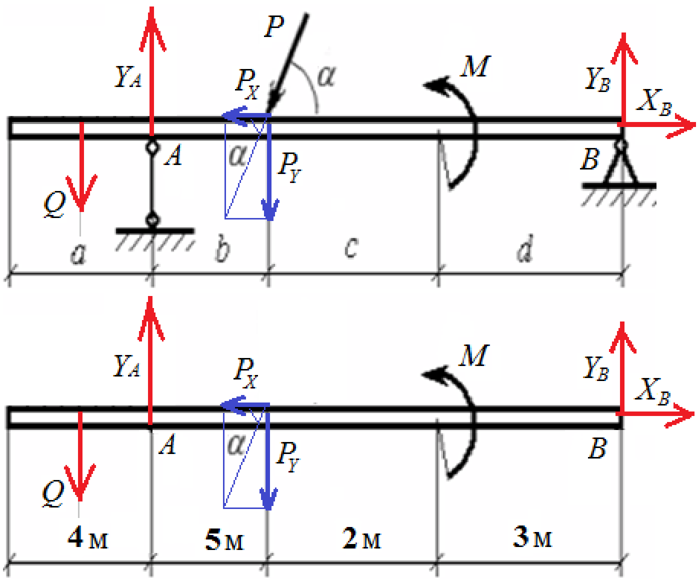
The equivalent force insertion point is at a distance of 0.5a from support A.

3. *Decompose* force P on horizontal and vertical components:

$P_X = -P\cos 30° = -10\cos 30° = -8{,}66$ кN

$P_Y = -P\sin 30° = -10\sin 30° = -5$ кN

4. We replace *supports* with reactions YA, YB and XB, mentally discarding the supports:



5. We make three equilibrium equations using the basic form of equilibrium conditions of a flat system of arbitrary forces:

$$\sum_{k=1}^{n} F_{kX} = 0; \qquad X_B - P_X = 0 \qquad (1)$$

$$\sum_{k=1}^{n} F_{kY} = 0; \qquad Y_A + Y_B - Q - P_Y = 0 \qquad (2)$$

$$\sum_{k=1}^{n} M_A\left(\vec{F}_k\right) = 0; \qquad Q\frac{a}{2} + M + Y_B\left(b + c + d\right) - P_Y b = 0 \qquad (3)$$

From the equation (1) we find the reaction X_B:

$X_B = P_X = 10\cos 30° = 8{,}66$ кN

From the equation (3) we find the reaction Y_B:

$$Y_B = \frac{P_Y b - Q\frac{a}{2} - M}{b + c + d} = \frac{(10\,sin\,30°)5 - 8 \cdot 2 - 5}{5 + 2 + 3} = 0{,}4\ \kappa N$$

From the equation (2) we find the reaction Y_A:

$$Y_A = Q + P_Y - Y_B = 8 + 5 - 0{,}4 = 12{,}6\ \kappa N$$

6. We check the solution by passing the control equation of equilibrium – the sum of the moments of all these forces relative to the point B:

$$\sum_{k=1}^{n} M_B\left(\vec{F}_k\right) = 0; \qquad Q\left(\frac{a}{2} + b + c + d\right) + P_Y\left(c + d\right) + M - Y_A\left(b + c + d\right) = 0$$

$$Y_A = \frac{Q\left(\frac{a}{2} + b + c + d\right) + P_Y(c + d) + M}{b + c + d}$$

$$Y_A = \frac{8(2 + 5 + 2 + 3) + 5(2 + 3) + 5}{5 + 2 + 3} = \frac{8 \cdot 12 + 5 \cdot 5 + 5}{10} = 12{,}6\ \kappa N$$

Solution is correct

Answer: $Y_A = 12{,}76\ \kappa N;\quad Y_B = 0{,}4\ \kappa N,\quad X_B = 8{,}66\ \kappa N$

5. ARBITRARY FORCE SYSTEMS-2

The purpose of the lesson is to acquire skills:

- to determine the vector of moment of forces relative to the point and axis in 3D;
- replace force with an equivalent system;
- use the Poinso theorem in determining the main vector and the main moment of the arbitrary force system;
- reduce the force factor system to a dynamic screw.

Before performing the task, you must familiarize yourself with:

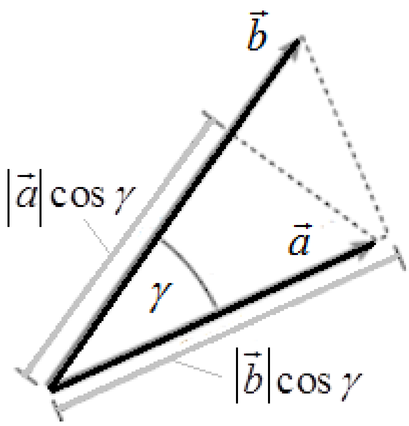
- the rules of scalar, vector and mixed vectors product;
- the rule of finding ort vectors;
- the theorem of parallel transfer of force and the main theorem of statics;
- the main cases of reduce of force system.

BRIEF THEORETICAL INFORMATION

Scalar product (dot product, inner product) of two vectors

$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}; \qquad k\vec{a}\cdot\vec{b}=\vec{a}\cdot k\vec{b}$$

$$\vec{a}\cdot\vec{b}=a_X b_X + a_Y b_Y + a_Z b_Z$$



$$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\gamma$$

$$\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}=|\vec{a}|\cos\gamma; \qquad \vec{a}\cdot\vec{e}_B\cdot=|\vec{a}|\cos\gamma$$

$$\frac{\vec{a}}{|\vec{a}|}\cdot\vec{b}=|\vec{b}|\cos\gamma; \qquad \vec{e}_A\cdot\vec{b}=|\vec{b}|\cos\gamma$$

$$\frac{\vec{a}}{|\vec{a}|}\cdot\frac{\vec{b}}{|\vec{b}|}=\cos\gamma; \qquad \cos\gamma=\vec{e}_A\cdot\vec{e}_B$$

Vector product (cross product, oueter product) of vectors

$$\vec{A}\times\vec{B}=\left(|\vec{A}||\vec{B}|\sin\theta_{AB}\right)\vec{n}; \qquad |\vec{n}|=1; \quad \vec{n}\perp\vec{A}; \quad \vec{n}\perp\vec{B}$$

$$|\vec{A}\times\vec{B}|=|\vec{A}||\vec{B}|\sin\theta_{AB}$$

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

$$m\left(\vec{A}\times\vec{B}\right)=\left(m\vec{A}\right)\times\vec{B}=\vec{A}\times\left(m\vec{B}\right)$$

$$\vec{A}\times\left(\vec{B}+\vec{C}\right)=\left(\vec{A}\times\vec{B}\right)+\left(\vec{A}\times\vec{C}\right)$$

$$\vec{A}\times\vec{B}=\begin{vmatrix} i & j & k \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

$$\vec{A}\times\vec{B}=\left(A_Y B_Z - A_Z B_Y\right)\vec{i}++\left(A_Z B_X - A_X B_Z\right)\vec{j}+\left(A_X B_Y - A_Y B_X\right)\vec{k}$$

Mixed product (triple scalar) of vectors

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\times\vec{B}\right)\cdot\vec{C}$$

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix} A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \\ C_X & C_Y & C_Z \end{vmatrix}$$

Theorem on parallel force transfer (Poinso’s lemma)

Without changing the state of the solid, the force applied to this body can be transferred in parallel to itself to any point of the body, while adding a couple of forces, the moment of which is equal to the moment of a given force relative to the new point of application.

Basic statics theorem (Poinso's theorem)

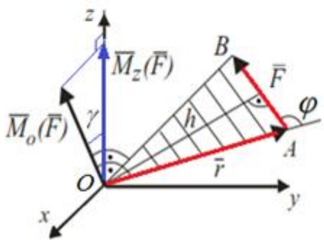
An arbitrary system of forces can be reduced to any center O and replaced by main vector $\vec{R}$ and main moment $\overrightarrow{M_O}$, with the main vector $\vec{R}$ equal to the geometric sum of all forces and the main moment $\overrightarrow{M_O}$ equal to the geometric sum of the moments of all forces relative to the reduction center O.

The moment of force about the axis

The moment of force relative to, for example, z-axis is equal to projection of the force-moment vector on z-axis:

$$M_Z = \left|\vec{M}_O\left(\vec{F}\right)\right| cos\gamma,$$

where γ is the angle between force moment vector and z axis.



The moments of force relative to the axes of the Cartesian coordinate system M_X, M_Y, M_Z are determined from the formula

$$\vec{M}_O\left(\vec{F}\right)=\vec{r}\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ x & y & z\\ F_X & F_Y & F_Z\end{vmatrix}=$$

$$=\left(yF_Z-zF_Y\right)\vec{i}+\left(zF_X-xF_Z\right)\vec{j}+\left(xF_Y-yF_X\right)\vec{k},$$

$$M_X=yF_Z-zF_Y; \qquad M_Y=zF_X-xF_Z; \qquad M_Z=xF_Y-yF_X$$

$$\vec{M}_O\left(\vec{F}\right)=M_X\vec{i}+M_Y\vec{j}+M_Z\vec{k}; \qquad \left|\vec{M}_O\left(\vec{F}\right)\right|=\sqrt{M_X^2+M_Y^2+M_Z^2}$$

$$\frac{M_X}{\left|\vec{M}_O\left(\vec{F}\right)\right|}=\cos\alpha; \qquad \frac{M_Y}{\left|\vec{M}_O\left(\vec{F}\right)\right|}=\cos\beta; \qquad \frac{M_Z}{\left|\vec{M}_O\left(\vec{F}\right)\right|}=\cos\gamma$$

$$\vec{M}_O\left(\vec{F}\right)=\left|\vec{M}_O\left(\vec{F}\right)\right|\left(\vec{i}\cos\alpha+\vec{j}\cos\beta+\vec{k}\cos\gamma\right)$$

$$\frac{\vec{M}_O\left(\vec{F}\right)}{\left|\vec{M}_O\left(\vec{F}\right)\right|}=\vec{i}\cos\alpha+\vec{j}\cos\beta+\vec{k}\cos\gamma$$

where α, β, γ - the angles between force moment vector and corresponding axes x, y, z.

EXAMPLES

EXAMPLE 1

A force $\vec{F}$ with a modulus of 10 N acts along the vector $\overrightarrow{AB}=\vec{i}-6\vec{j}-3\vec{k}$ and has the same direction. Determine the force vector $\vec{F}$

Solution

Find the orth of the vector $\overrightarrow{AB}$, which is also the orth of the force vector $\vec{F}$:

$$\vec{e}_{AB} = \vec{e}_F = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{i-6j-3k}{\sqrt{1+36+9}} = \frac{i-6j-3k}{\sqrt{46}}$$

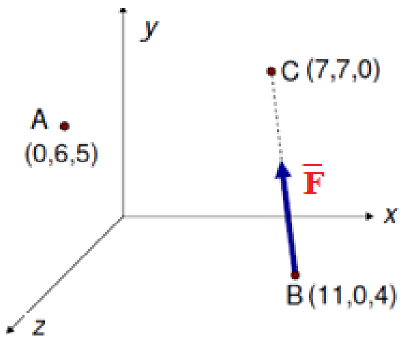
A force vector $\vec{F}$ is equal to:

$$\vec{F} = \left|\vec{F}\right| \vec{e}_F = \frac{10}{\sqrt{46}}(i-6j-3k)$$

Answer: $\vec{F} = \frac{10}{\sqrt{46}}\left(\vec{i} - 6\vec{j} - 3\vec{k}\right)$

EXAMPLE 2

The line of action of the force $\vec{F}$, the modulus of which is equal to 90 N, passes through points B and C. Determine the moment of force relative to point A.

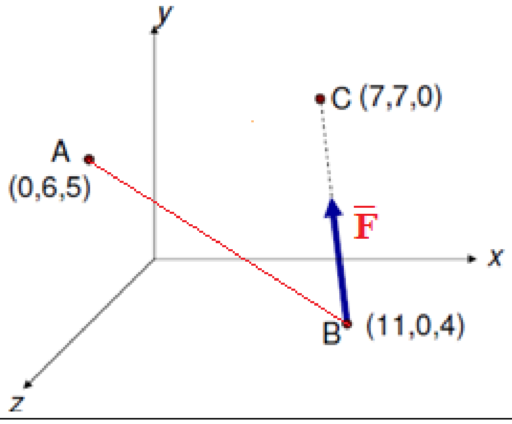


Solution

The moment of force relative to point A is determined from the equation:

$$\overrightarrow{M_A} = \vec{r}_{AB} \times \vec{F},$$

where $\vec{r}_{AB} = (11i - 6j - k)\, m$



The vector F is determined from the equation:

$$\vec{F} = |\vec{F}|\vec{e}_{BC} = |\vec{F}|\frac{\overrightarrow{BC}}{|\overrightarrow{BC}|} = 90\frac{-4i + 7j - 4k}{\sqrt{16 + 49 + 16}} = (-40i + 70j - 40k)N$$

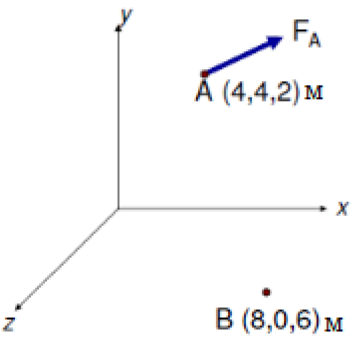
Moment of force relative to point A is equal to:

$$\overrightarrow{M_A} = \vec{r}_{AB} \times \vec{F} = \begin{vmatrix} i & j & k \\ 11 & -6 & -1 \\ -40 & 70 & -40 \end{vmatrix} = (240 + 70)i + (40 + 440)j + (770 + 240)k$$

Answer: $\vec{M}_A = (310\vec{\imath} + 480\vec{\jmath} + 530\vec{k})Nm$

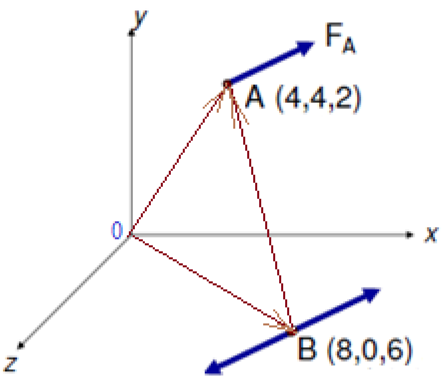
EXAMPLE 3

Replace the force $\overrightarrow{F_A}$ applied at point A with an equivalent system in the form of a pair of forces and the force applied at point B. $\overrightarrow{F_A} = (10\vec{i} + 4\vec{j} - 3\vec{k})\,H.$



Solution

The force applied to a body, according to the theorem on parallel force transfer (Poinso's lemma), can be transferred parallel to itself to any point of the body, adding a pair of forces, the moment of which is equal to the moment of a given force relative to the new point of application.



The new point of application is point B. At this point we place two balanced forces, parallel to force $\overrightarrow{F_A}$ and equal their in magnitude. As a result, we obtain the force applied at point B and a pair of forces that create a moment

$\overrightarrow{M}_B$ relative to the new point of application B:

$$\overrightarrow{F_B} = \overrightarrow{F_A} = \left(10\vec{i} + 4\vec{j} - 3\vec{k}\right)$$

$$\overrightarrow{M}_B = \overrightarrow{r_{BA}} \times \overrightarrow{F_A} = \overrightarrow{BA} \times \overrightarrow{F_A}$$

Consider the vector triangle OBA, in which:

$$\overrightarrow{OB} + \overrightarrow{BA} = \overrightarrow{OA}$$

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \left(-4\vec{i} + 4\vec{j} - 4\vec{k}\right)$$

Determine the moment that creates a pair of forces relative to the new point of application B:

$$\overrightarrow{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 4 & -4 \\ 10 & 4 & -3 \end{vmatrix} = (-12+16)\vec{i} + (-40-12)\vec{j} + (-16-40)\vec{k}$$

Answer: $\overrightarrow{F_B} = \left(10\vec{i} + 4\vec{j} - 3\vec{k}\right)H$; $\vec{M}_B = \left(4\vec{i} - 52\vec{j} - 56\vec{k}\right)Nm$

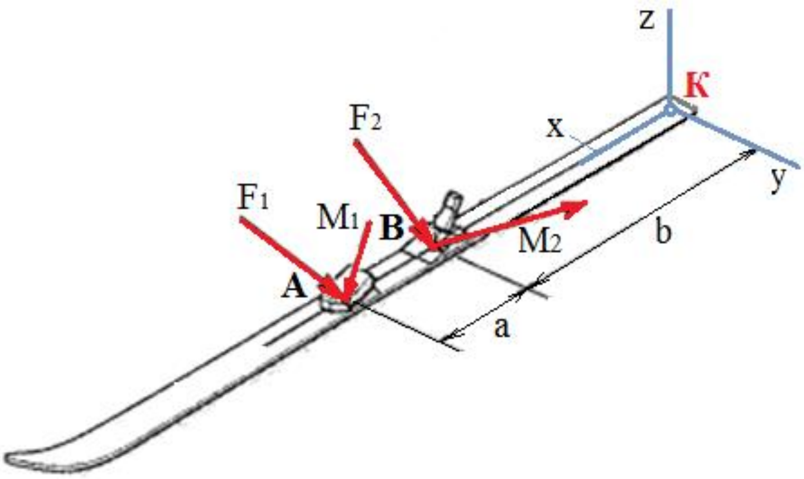
EXAMPLE 4

The sock and the heel of ski (points A, B) are affected by forces and moments of forces, which are respectively equal to:

$$\begin{cases} F_1 = (-50i + 80j - 158k)N \\ M_1 = (-6i + 4j + 2k)Nm \end{cases}$$

$$\begin{cases} F_2 = (-20i + 60j - 250k)N \\ M_2 = (-20i + 8j + 3k)Nm \end{cases}$$

Replace this system with the main vector and the main moment with respect to the new reduction center K. The result present in vector form. Take a = 0,12 m; b = 0,8 m.



Solution

An arbitrary system of forces, according to the basic theorem of statics (Poinso's theorem), can be reduced to any center O and replaced by the main vector $\vec{R}$ and main moment $\vec{M}_O$, with the main vector $\vec{R}$ equal to the geometric sum of all forces and the main moment $\vec{M}_O$ – to the geometric sum of moments of all forces relative to the center O of reduction.

In this case, the center of reduction is the point K.

Let’s define the main vector $\vec{R}$:

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = (-50i + 80j - 158k) + (-20i + 60j - 250k)$$

$$\vec{R} = (-70i + 140j - 408k)N$$

the main moment is equal to:

$$\vec{M}_K = \left(\vec{r}_1 \times \vec{F}_1\right) + \left(\vec{r}_2 \times \vec{F}_2\right) + \vec{M}_1 + \vec{M}_2,$$

where $\vec{r}_1 = (a + b)\vec{\imath} = (0{,}12 + 0{,}8)\vec{\imath} = 0{,}92\vec{\imath}$; $\vec{r}_2 = b\vec{\imath} = 0{,}8\vec{\imath}$

$$M_K = \begin{vmatrix} i & j & k \\ 0{,}92 & 0 & 0 \\ -50 & 80 & -158 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0{,}8 & 0 & 0 \\ -20 & 60 & -250 \end{vmatrix} + (-6i + 4j + 2k) + (-20i + 8j + 3k)$$

$$M_K = (-26i + 357j + 127k)Nm$$

Answer:

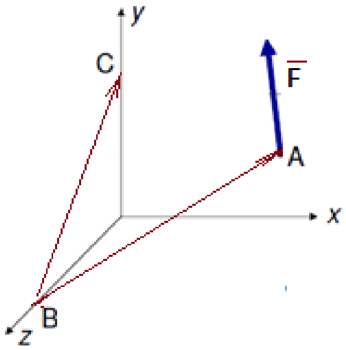
$\vec{R} = (-70i + 140j - 408k)N$; $M_K = (-26i + 357j + 127k)HNm$

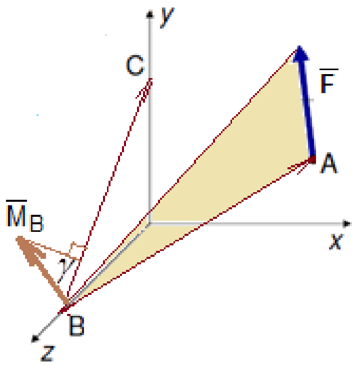
EXAMPLE 5

Determine the magnitude and vector of the moment of force F relative to the axis BC under the following conditions:

$\vec{F} = (-2i + 6j + 3k)\text{к}N$;   A (4; 2; 2) m; B (0; 0; 3) m; C (0; 4; 0) m.

Solution





Moments of force F relative to the point B:

$$\overrightarrow{M}_B = \vec{r}_{BA} \times \overrightarrow{F}; \qquad \vec{r}_{BA} = 4i + 2j - 1k$$

$$\overrightarrow{M}_B = \vec{r}_{BA} \times \overrightarrow{F} = \begin{vmatrix} i & j & k \\ 4 & 2 & -1 \\ -2 & 6 & 3 \end{vmatrix} = (6+6)i + (2-12)j + (24+4)k$$

$\vec{M}_B = (12i - 10j + 28k)\text{к}N$

To determine the moment of a given force relative to the axis BC we use the equation of the scalar product, according to which:

$$\vec{a} \cdot \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right| \cos\gamma$$

We will assume that $\vec{a} = \vec{M}_B$; $\vec{b} = \overrightarrow{BC}$

$$\overrightarrow{M}_B \cdot \overrightarrow{BC} = \left|\overrightarrow{M}_B\right|\left|\overrightarrow{BC}\right| \cos\gamma; \qquad \overrightarrow{M}_B \cdot \frac{\overrightarrow{BC}}{\left|\overrightarrow{BC}\right|} = \left|\overrightarrow{M}_B\right| \cos\gamma$$

$$\vec{M}_B \cdot \vec{e}_{BC} = \left|\vec{M}_B\right| cos\,\gamma \qquad (1)$$

The moment of force relative to the axis BC is equal to projection of the moment of force $\overrightarrow{M}_B$ on the axis BC:

$$M_{BC} = \left|\vec{M}_B\right| cos\,\gamma \qquad (2)$$

As a result of comparing formulas (1) and (2) we have:

$$M_{BC} = \vec{M}_B \cdot \vec{e}_{BC} \qquad (3)$$

$$\vec{M}_{BC} = M_{BC}\vec{e}_{BC} \qquad (4)$$

The obtained equation (3) proves that the magnitude of moment of force relative to axis is equal to the scalar product of vectors $\left(\vec{M}_B \cdot \vec{e}_{BC}\right)$.

$$\vec{e}_{BC} = \frac{\overrightarrow{BC}}{\left|\overrightarrow{BC}\right|} = \frac{4j - 3k}{\sqrt{16+9}} = \frac{4j - 3k}{5} = 0{,}8j - 0{,}6k$$

$$M_{BC} = \vec{e}_{BC} \cdot \vec{M}_B = (0{,}8j - 0{,}6k)(12i - 10j + 28k) = -24{,}8[\text{к}Nm]$$

The force moment vector relative to the axis BC is equal to:

$$\vec{M}_{BC} = M_{BC}\vec{e}_{BC} = -24{,}8(0{,}8j - 0{,}6k) = (-19{,}84j + 14{,}88k)\text{к}Nm$$

Answer: $M_{BC} = -24{,}8\ \text{к}Nm$; $\vec{M}_{BC} = (-19{,}84j + 14{,}88k)\text{к}Nm$

EXAMPLE 6

Determine the coordinates of the point P on the xoz plane at which the system of force factors

$\vec{F} = (3i + 6j + 2k)N, \quad \vec{M} = (12i + 4j + 6k)Nm$, applied at point O, is equivalent to the dynamic screw applied at point P.

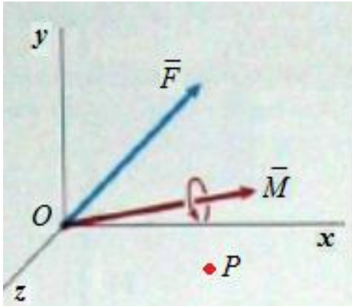


Fig.5.1

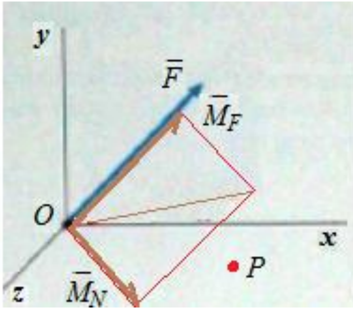


Fig.5.2

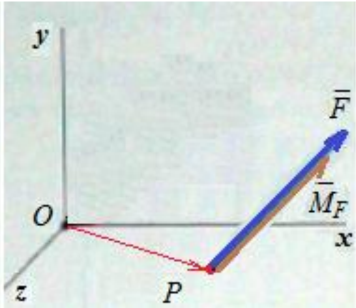


Fig.5.3

We decompose the vector $\overrightarrow{M}$ into two components $\overrightarrow{M}_F$, $\overrightarrow{M}_N$ - respectively along and perpendicular to the force vector $\overrightarrow{F}$.

Since the vectors $\overrightarrow{M}_F$ and $\overrightarrow{F}$ coincide in direction, they have the same unit vector:

$$\vec{e}_F = \frac{\overrightarrow{F}}{\left|\overrightarrow{F}\right|} = \frac{3i + 6j + 2k}{\sqrt{9 + 36 + 4}} = \frac{3i + 6j + 2k}{7}$$

The value M_F is equal to the scalar product of the unit vector and moment (*formula (3) of example 5*):

$$M_F = \vec{e}_F \cdot \overrightarrow{M} = \left(\frac{3i + 6j + 2k}{7}\right)(12i + 4j + 6k) = \frac{36 + 24 + 12}{7} = \frac{72}{7}.$$

Vector $\overrightarrow{M}_F$ is equal to (*formula (4) of example 5*):

$$\overrightarrow{M_F} = M_F \vec{e}_F = \frac{72}{7}\left(\frac{3i+6j+2k}{7}\right) = 4,4i + 8,8j + 2,9k$$

Vector of the normal component $\overrightarrow{M}_N$ of moment $\overrightarrow{M}$ is equal to:

$$\overrightarrow{M_N} = \overrightarrow{M} - \overrightarrow{M_F} = (12i + 4j + 6k) - (4,4i + 8,8j + 2,9k)$$

$$\overrightarrow{M_N} = 7,6i - 4,8j + 3,1k$$

We transfer $\vec{F}$ and $\overrightarrow{M}_F$ to the point P with coordinates x,0,z, at which the condition must be fulfilled:

$$\vec{r}_{OP} \times \vec{F} = \overrightarrow{M}_N$$

$$\vec{r}_{OP} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & 0 & z \\ 3 & 6 & 2 \end{vmatrix} = -6zi + (3z - 2x)j + 6xk = \overrightarrow{M}_N$$

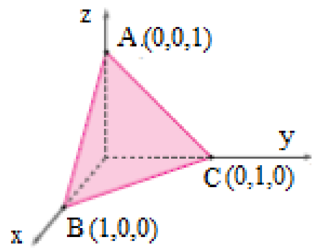
$$-6zi + (3z - 2x)j + 6xk = 7,6i - 4,8j + 3,1k$$

$$\begin{cases} -6z = 7,6; & z = -7,6/6 = -1,27 \\ 3z - 2x = -4,8; & \\ 6x = 3,1; & x = 3,1/6 = 0,52 \end{cases}$$

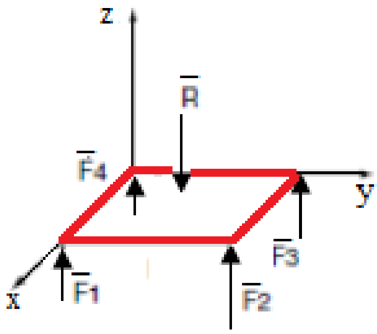
Answer: At the coordinates of the point P (0.52; 0; −1.27) the given system of force factors will be equivalent to the dynamic screw applied at the point P.

TASKS FOR INDEPENDENT DEVELOPMENT

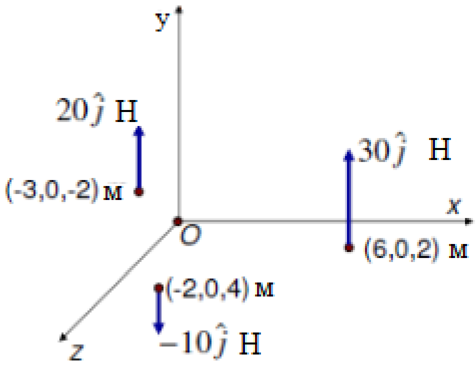
1. Determine the unit vector of the normal to the ABC plane at point A.



2. Homogeneous square plate of the strain gauge with side L rests on four supports located on the tops of plate. Under action of applied force R in supports of strain gauge there are reaction forces F_1, F_2, F_3, F_4. Determine the magnitude and coordinates of the point of application of the balancing force R. The weight of the plate is neglected



3. Replace the system of parallel forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ with one equivalent force $\vec{F}_{EQ}$ and determine coordinates of the point of its application.



6. EQUILIBRIUM IN THE PRESENCE OF FRICTION FORCES

The purpose of the lesson: acquiring skills in:

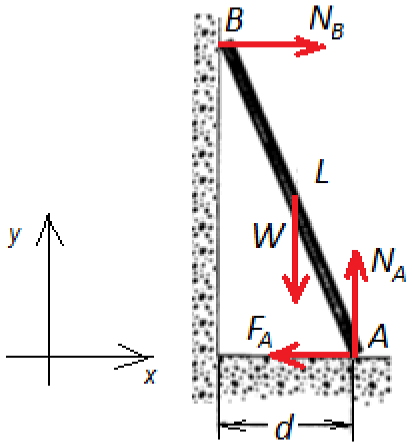
- Drawing up analytical conditions of equilibrium in the presence of friction forces.

Before completing the task you need to read:

- Types of supports and their reactions.
- Decomposition of force into components.
- Determining the moments of forces
- Analytical conditions of equilibrium of a plane system of arbitrary forces.

EXAMPLE

A homogeneous rod of length L and weight W rests on a smooth wall and floor with a coefficient of static friction μ. The distance of the rod to the wall d. Under what conditions will the rod remain stationary?



Solution

Let's make equilibrium conditions and define reactions in supports:

$$\sum_{k=1}^{n} F_{kX} = 0; \qquad N_B - F_A = 0; \qquad N_B = F_A \qquad (1)$$

$$\sum_{k=1}^{n} F_{kY} = 0; \qquad N_A - W = 0; \qquad N_A = W \qquad (2)$$

$$\sum_{k=1}^{n} M_A\left(\vec{F}_k\right) = 0; \qquad -N_B\sqrt{L^2 - d^2} + W\frac{d}{2} = 0 \qquad (3)$$

$$N_B = \frac{0{,}5\,Wd}{\sqrt{L^2 - d^2}}$$

Determine the force of static friction

$$F_T = \mu N_A$$

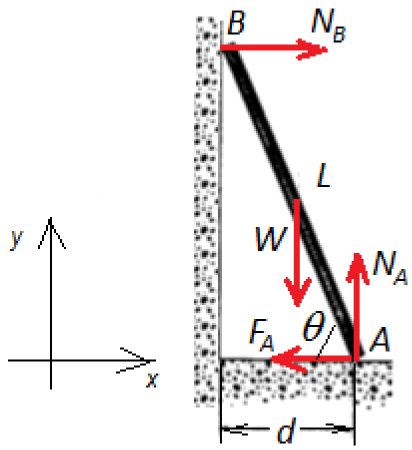
The rod will remain stationary if the maximum friction force is greater than the horizontal component of the reaction of the rough surface.

$$F_T > F_A; \qquad \mu\ W > \frac{0{,}5\ Wd}{\sqrt{L^2 - d^2}}; \qquad \mu\ > \frac{0{,}5\ d}{\sqrt{L^2 - d^2}}$$

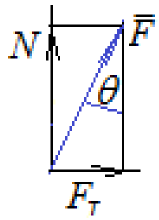
Answer: The rod will remain stationary under the conditions $\mu\ > \frac{0{,}5\ d}{\sqrt{L^2-d^2}}$

TASKS FOR INDEPENDENT DEVELOPMENT

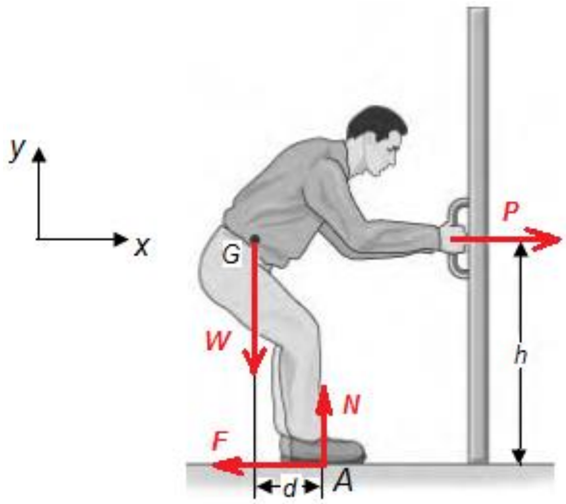
1. A homogeneous rod with length L = 26 ft and weight W = 30 lb rests on a smooth wall and floor with a coefficient of static friction μ = 0.3. Determine the maximum angle of inclination of the rod to the floor and the maximum distance of the rod to the wall d, at which the rod will remain stationary.



2. The patient rests his crutches on the floor with a static friction coefficient of 0.77. Determine the maximum angle θ between the crutch and the normal to the floor, at which the stability of the systems "patient-crutch" is still ensured.

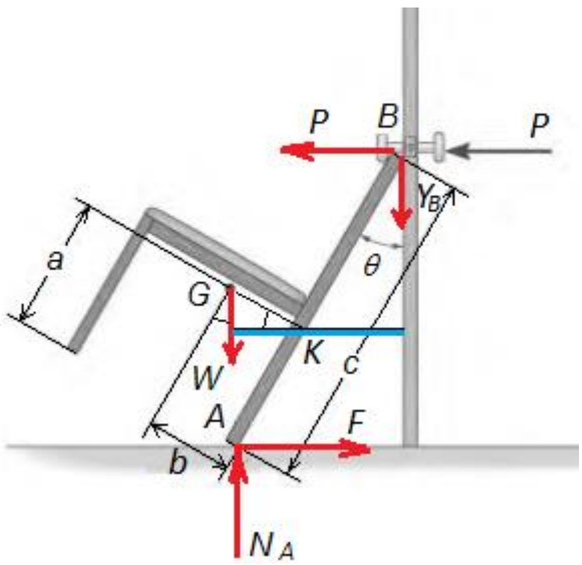


3. A man weighing W = 800 N pulls the door handle. The coefficient of static friction between the shoes and the floor surface μ = 0,5. Determine at what distance d from the center of the footrest (point A) should be its center of gravity G to act on the door with maximum horizontal force. What is the magnitude of this force? The manual door is located at a height of h = 1 m.



4. The chair supports the door as shown. Determine the minimum force P that must be applied to the handle to open the door, and the reactions of the supports NA, YB at a static friction coefficient μ = 0,3. The center of gravity of the chair is at point G.

Given: a = 0.4 m; b = 0.25 m; c = 0.9 m; θ = 30°; W = 4.5 kg.



Solution

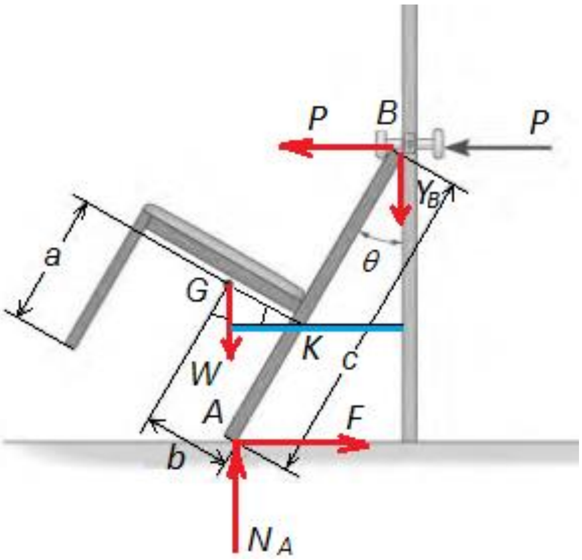
The system is in equilibrium. We make three equilibrium equations, using the basic form of equilibrium conditions of a plane system of arbitrary forces:

$$\sum_{k=1}^{n} F_{kX} = 0; \qquad -P + F = 0 \qquad (1)$$

$$\sum_{k=1}^{n} F_{kY} = 0; \qquad N_A - W - Y_B = 0 \qquad (2)$$

$$\sum_{k=1}^{n} M_B\left(\vec{F}_k\right) = 0;$$

$$Fc\cos\theta - Nc\sin\theta + W\left[b\cos\theta + (c-a)\sin\theta\right] = 0 \qquad (3)$$



Make the solution to the end !!!

7. SOLID WEIGHT/MASS CENTER

The purpose of the lesson - to acquire skills in determining:

- the center of parallel forces;

- coordinates of the centers of gravity of homogeneous plates, rods of different configurations, three-dimensional bodies of rotation;

- the general center of mass of the body system

Before completing the task you need to read:

- Properties of the center of parallel forces.

- Methods of determining the coordinates of the centers of gravity.

THEORETICAL INFORMATION

The center of gravity of a rigid body is point, invariably associated with this body, through which passes the line of action of the equivalent forces of gravity of particles of a given body at any position of the body in space.

Coordinates of a solid body center of gravity:

$$x_C = \frac{\sum_{k=1}^{n} p_k x_k}{\sum_{k=1}^{n} p_k}, \qquad y_C = \frac{\sum_{k=1}^{n} p_k y_k}{\sum_{k=1}^{n} p_k}, \qquad z_C = \frac{\sum_{k=1}^{n} p_k z_k}{\sum_{k=1}^{n} p_k} \qquad (1)$$

where $\sum_{k=1}^{n} p_k = P$ – body weight;

x_k, y_k, z_k – coordinates of points of application of gravity to body parts.

Coordinates of the center of gravity for a homogeneous body:

$$x_C = \frac{\sum_{k=1}^{n} V_k x_k}{\sum_{k=1}^{n} V_k}, \qquad y_C = \frac{\sum_{k=1}^{n} V_k y_k}{\sum_{k=1}^{n} V_k}, \qquad z_C = \frac{\sum_{k=1}^{n} V_k z_k}{\sum_{k=1}^{n} V_k} \tag{2}$$

where V_k – body part volums.

For homogeneous plates

$$x_C = \frac{\sum_{k=1}^{n} A_k x_k}{\sum_{k=1}^{n} A_k}, \qquad y_C = \frac{\sum_{k=1}^{n} A_k y_k}{\sum_{k=1}^{n} A_k}, \qquad z_C = \frac{\sum_{k=1}^{n} A_k z_k}{\sum_{k=1}^{n} A_k} \tag{3}$$

where $\sum_{k=1}^{n} A_k = A$ – the area of entire plate.

For a homogeneous line that has the same cross section:

$$x_C = \frac{\sum_{k=1}^{n} l_k x_k}{\sum_{k=1}^{n} l_k}, \qquad y_C = \frac{\sum_{k=1}^{n} l_k y_k}{\sum_{k=1}^{n} l_k}, \qquad z_C = \frac{\sum_{k=1}^{n} l_k z_k}{\sum_{k=1}^{n} l_k} \tag{4}$$

where $\sum_{k=1}^{n} l_k = l$ – the length of entire line.

If the body cannot be divided into a finite number of parts with known positions of the centers of gravity, then method of integration is used and formulas (2) - (4) take the form:

coordinates of center of gravity for the volume V

$$x_C = \frac{1}{V}\int_V \tilde{x}dv, \qquad y_C = \frac{1}{V}\int_V \tilde{y}dv, \qquad z_C = \frac{1}{V}\int_V \tilde{z}dv \tag{5}$$

Coordinates of the center of gravity for the surface with area A

$$x_C = \frac{1}{A}\int_A \tilde{x}dA, \qquad y_C = \frac{1}{A}\int_A \tilde{y}dA, \qquad z_C = \frac{1}{A}\int_A \tilde{z}dA \tag{6}$$

Coordinates of the center of gravity for the line with length l:

$$x_C = \frac{1}{l}\int_l \tilde{x}dl, \qquad y_C = \frac{1}{l}\int_l \tilde{y}dl, \qquad z_C = \frac{1}{l}\int_l \tilde{z}dl \tag{7}$$

where $\tilde{x}, \tilde{y}, \tilde{z}$ – coordinates of the centers of gravity for elementary elements (volumes, surfaces, lines).

Radius vector of the center of mass of body, consisting of particles with mass $m_1, m_2, m_3, \ldots$ and corresponding radius vectors, is determined by formulas

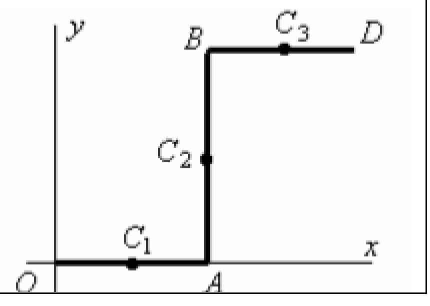
$$\vec{r}_C = \frac{\sum_{k=1}^{n} m_k \vec{r}_k}{\sum_{k=1}^{n} m_k} \tag{8}$$

$$x_C = \frac{\sum_{k=1}^{n} m_k x_k}{\sum_{k=1}^{n} m_k}, \qquad y_C = \frac{\sum_{k=1}^{n} m_k y_k}{\sum_{k=1}^{n} m_k}, \qquad z_C = \frac{\sum_{k=1}^{n} m_k z_k}{\sum_{k=1}^{n} m_k} \tag{9}$$

EXAMPLES

Example 1

Determine the coordinates of center of gravity of the rod curved at right angles, in which: OA = 30 cm, AB = 50 cm, BD = 20 cm.



Solution

The center of gravity of individual sections of the rod are in the middle. Denote their centers of gravity by points C1, C2, C3. The coordinates of center of gravity of the curved rod are determined by formulas (4):

$$x_C = \frac{\sum_{k=1}^{n} l_k x_k}{\sum_{k=1}^{n} l_k} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3} = \frac{30 \cdot 15 + 50 \cdot 30 + 20 \cdot 40}{30 + 50 + 20} = \frac{2750}{100} = 27{,}5\ cm$$

$$y_C = \frac{\sum_{k=1}^{n} l_k y_k}{\sum_{k=1}^{n} l_k} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3} = \frac{30 \cdot 0 + 50 \cdot 25 + 20 \cdot 50}{30 + 50 + 20} = \frac{2250}{100} = 22{,}5\ cm$$

Відповідь: $\boldsymbol{x_C = 27,5}\ cm$; $\boldsymbol{y_C = 22,5}\ cm$

Example 2

Find the coordinates of center of gravity of a homogeneous arc of a circle of radius R with a central angle $\pi/2$.

Solution

Select element on the arc with length $dl = Rd\theta$, the position of which is determined by the angle θ.

Element dl center of gravity coordinates:

$\tilde{x} = R\cos\theta$

$\tilde{y} = R\sin\theta$

Coordinates of the center of gravity of the arc with the central angle $\pi/2$ determine by the formula (7):

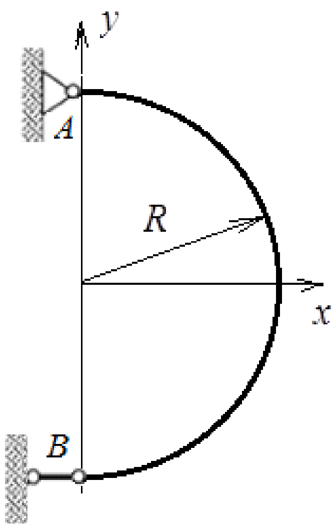
$$x_C = \frac{1}{l}\int_l \tilde{x}dl = \frac{1}{\frac{\pi R}{2}}\int_0^{\pi/2} R\cos\theta\left(Rd\theta\right) = \frac{2R}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right) = \frac{2R}{\pi}$$

$$y_C = \frac{1}{l}\int_l \tilde{y}dl = \frac{1}{\frac{\pi R}{2}}\int_0^{\pi/2} R\sin\theta\left(Rd\theta\right) = -\frac{2R}{\pi}\left(\cos\frac{\pi}{2} - \cos 0\right) = \frac{2R}{\pi}$$

Answer: $x_C = y_C = \dfrac{2R}{\pi}$

Example 3

Find the coordinate of center of gravity of a homogeneous semicircular arc, the linear weight of which is equal to γ, and determine the reactions in supports A and B. Take γ = 0.5 N/m, R = 2 m.



Solution

$y_C = 0$. Find the coordinate of center of gravity of the arc by the formula (7):

$$x_C = \frac{1}{L}\int_L \tilde{x}dl; \qquad L = \pi R$$

$$x_C = 2\frac{1}{\pi R}\int_0^{\pi/2} R\,cos\,\alpha\,(Rd\alpha) = \frac{2R}{\pi}sin\frac{\pi}{2} = \frac{2R}{\pi}$$

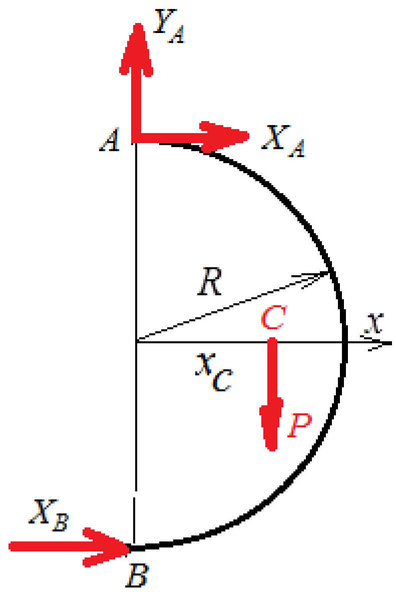
$$x_C = \frac{2\cdot 2}{\pi} = 1{,}273\ m$$

Determine the weight P of the arc and apply it to the center of gravity - to the point C.

$P = \gamma L = \gamma\pi R = 0{,}5\pi \cdot 2 = 3{,}14\ N$

Let's discard supports and replace them with reactions.

For the equilibrium of a planar system of forces acting on a solid, it is necessary and sufficient that the sum of the projections of these forces on each of the two rectangular coordinate axes located in the plane of forces be zero and the sum of the moments of forces relative to any point in the plane of action of forces, was also equal to zero



$$\sum_{k=1}^{n} F_{kX} = 0; \qquad \sum_{k=1}^{n} F_{kY} = 0; \qquad \sum_{k=1}^{n} M_A\left(\vec{F}_k\right) = 0$$

From the third equation we define X_B, from the first X_A, from the second Y_A.

$$X_B \cdot AB - P \cdot OC = 0: \qquad X_B 2R - P x_C = 0$$

$$X_B = \frac{P x_C}{2R} = \frac{3{,}14 \cdot 1{,}273}{2 \cdot 2} = 1\ N$$

$$X_A + X_B = 0; \qquad X_A = -X_B = -1\ N$$

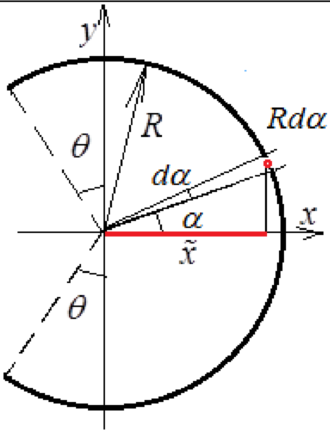
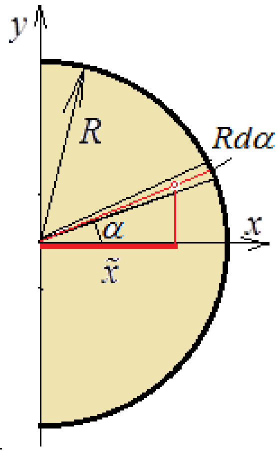
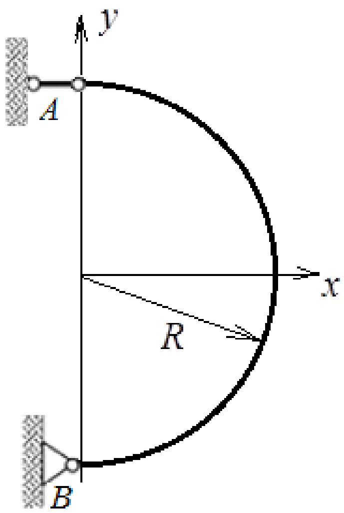
$$Y_A - P = 0; \qquad Y_A = P = 3{,}14\ N$$

Answer:

$x_C = 1{,}273\ m; \quad Y_A = 3{,}14\ N; \quad X_A = -1\ N; \quad X_B = 1\ N$

The X_A reaction is directed in the opposite to the selected direction.

TASKS FOR INDEPENDENT DEVELOPMENT

1. Determine the coordinates of the center of gravity of a homogeneous rod curved in the form of a circular arc. Given R = 0.3 m; θ = 30º.	
2. Find the coordinates of the center of gravity of a homogeneous semicircular plate	
3. Find the coordinate of the center of gravity of a homogeneous semicircular arc, the linear weight of which is equal to γ, and determine the reactions in the supports A and B. Take γ = 0.5 H / m, R = 2 m	

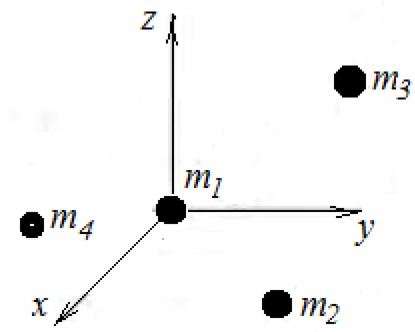
4. Determine the coordinates of the center of mass of a system consisting of four material points with given coordinates

   m_1 = 2 кg, [0 0 0] m;

   m_2 = 3 кg, [2 3 -1] m;

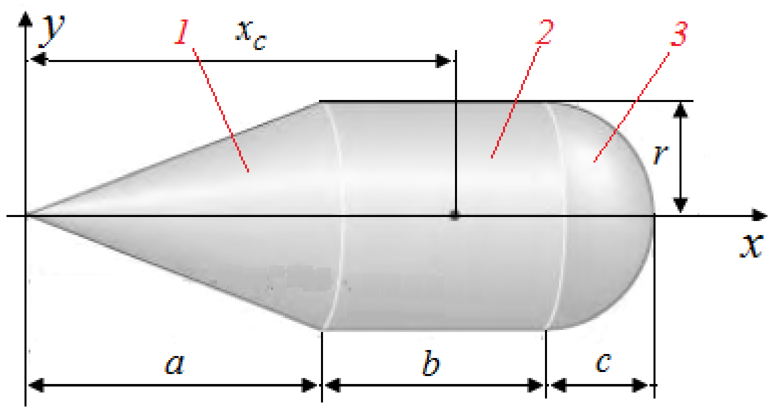
   m_3 = 1 кg, [1 4 4] m;

   m_4 = 1 кg, [2 -2 2] m.



5. Find the coordinate of the center of gravity of a rigid body of rotation, consisting of a cone 1, a cylinder 2 and a hemisphere 3.

   Given: a = 80 mm, b = 60 mm, c = 30 mm, r = 30 mm



8. KINEMATICS OF A POINT. THE SIMPLE MOVEMENTS OF A SOLID BODY

The purpose of the lesson: acquiring skills in:

- finding the velocities and accelerations of points of a rigid body rotating around a fixed axis;
- drawing up of a point motion equations in coordinate form.

Before completing the task you need to read:

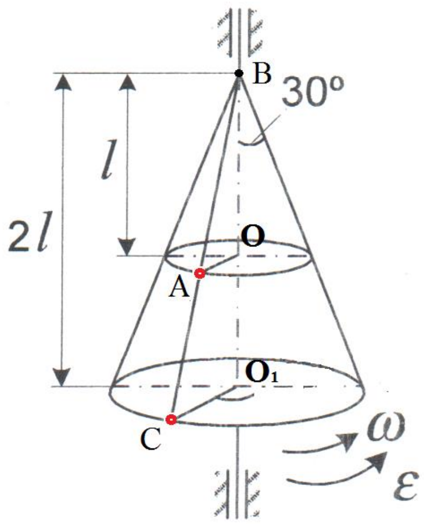
- ways to set the motion of a point;
- classification of point motion by acceleration;
- formulas for determining speeds and accelerations during rotational motion.

EXAMPLES

Example 1

The body in the form of a cone with a height of $2l$ and an angle at the apex of 60° rotates around its axis. Find the velocity and acceleration of points A and C under the following conditions:

$$\omega = 1\ s^{-1}, \quad \varepsilon = \sqrt{3}\ s^{-2}, \quad l = 1\ m.$$



Solution

All points of the cone that are not on the axis of rotation move along a trajectory in the form of circles whose planes are perpendicular to the axis of rotation, and the radii are equal to the distances of these points to the axis of rotation.

Consider the triangles AOB and CO_1B, in which there is a relationship:

$$\frac{AO}{BO} = \frac{CO_1}{BO_1} = tg30°$$

Determine the distances of points A and C to the axis of rotation:

$$AO = BO\ tg30° = \frac{l}{\sqrt{3}} = \frac{1}{\sqrt{3}} m; \qquad CO_1 = BO_1\ tg30° = \frac{2l}{\sqrt{3}} = \frac{2}{\sqrt{3}} m$$

Find the velocities of points A and C:

$$\upsilon_A = \omega \cdot AO = \omega \cdot l \cdot tg30° = \frac{1}{\sqrt{3}}\ m/s$$

$$\upsilon_C = \omega \cdot CO_1 = \omega \cdot 2l \cdot tg30° = \frac{2}{\sqrt{3}} m/s$$

The acceleration of the points is determined by the formula

$a = \sqrt{a_T^2 + a_N^2} = R\sqrt{\varepsilon^2 + \omega^4}$:

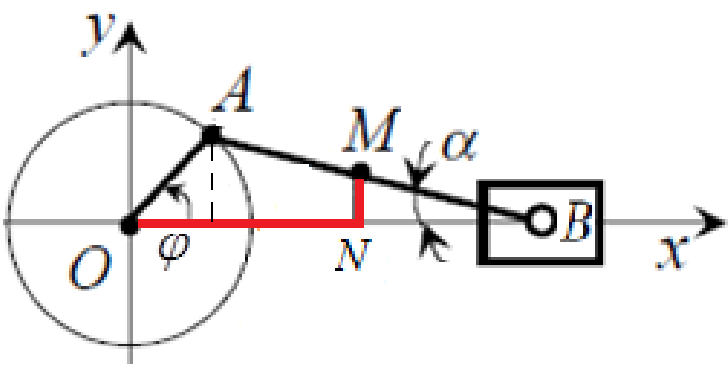
$$a_A = AO\sqrt{\varepsilon^2 + \omega^4} = \frac{1}{\sqrt{3}}\sqrt{3+1} = \frac{2}{\sqrt{3}}\ m/s^2$$

$$a_C = CO_1\sqrt{\varepsilon^2 + \omega^4} = \frac{2}{\sqrt{3}}\sqrt{3+1} = \frac{4}{\sqrt{3}}\ m/s^2$$

Answer: $\upsilon_A = \frac{1}{\sqrt{3}}\ m/s; \quad a_A = \frac{2}{\sqrt{3}}\ m/s^2; \quad \upsilon_C = \frac{2}{\sqrt{3}} m/s; \quad a_C = \frac{4}{\sqrt{3}}\ m/s^2$

Example 2

The figure shows the crank-slider mechanism, which includes: crank OA, connecting rod AB, slider B. For point M find the equation of motion in coordinate form, if: angle $AOB = \varphi = \omega t$, angular velocity $\omega = const$, length of the crank $OA = r$, length of the connecting rod $AB = 2L$; $AM = MB$.



Solution

Choose the coordinate system as shown in the figure. The angle ABO is denoted α

The coordinates of the point M are found from the equations:

$$\begin{cases} x_M = ON = OB - NB = \left(AO\cos\varphi + AB\cos\alpha\right) - MB\cos\alpha = AO\cos\varphi + AM\cos\alpha \\ y_M = MN = MB\sin\alpha \end{cases}$$

Consider the triangle AOB. Using the sine theorem, we find:

$$\frac{AB}{\sin\varphi} = \frac{OA}{\sin\alpha}; \quad \sin\alpha = \frac{OA}{AB}\sin\varphi = \frac{r}{2L}\sin\varphi; \quad \cos\alpha = \sqrt{1 - \left(\frac{r}{2L}\sin\varphi\right)^2}$$

The coordinates of the point M are equal:

$$\begin{cases} x_M = AO\cos\varphi + AM\cos\alpha = r\cos\omega t + L\sqrt{1 - \left(\frac{r}{2L}\sin\omega t\right)^2} \\ y_M = MB\sin\alpha = L\frac{r}{2L}\sin\varphi = \frac{r}{2}\sin\omega t \end{cases}$$

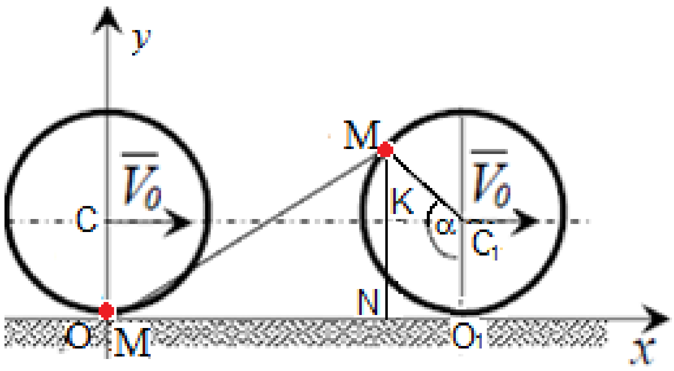
Answer:

Equation of motion of a point M:

$$x_M = r\cos\omega t + L\sqrt{1 - \left(\frac{r}{2L}\sin\omega t\right)^2}; \qquad y_M = \frac{r}{2}\sin\omega t$$

Example 3

Find the equation of motion in coordinate form for the point M of a wheel of radius R that rolls without sliding in a straight line with speed $\vec{\upsilon}_0$.



Solution

The origin is placed in the lower position of the point M.

The angle of rotation of the wheel MC_1O_1 is denoted $\alpha\ [rad]$. Координати точки M знайдемо з рівнянь:

$$\begin{cases} x_M = ON = OO_1 - NO_1 = R\alpha - R\cos\left(\alpha - \dfrac{\pi}{2}\right) = R(\alpha - \sin\alpha) \\ y_M = MN = C_1O_1 + MK = R + R\sin\left(\alpha - \dfrac{\pi}{2}\right) = R(1 - \cos\alpha) \end{cases}$$

When rolling the wheel without sliding in a straight line at a constant speed $\vec{\upsilon}_0$:

$R\alpha = \upsilon_0 t \qquad \alpha = \dfrac{\upsilon_0 t}{R}$, where t is the time of movement.

The equation of motion of the point M of the wheel in the coordinate form takes the form:

$$\begin{cases} x_M = R\left(\dfrac{\upsilon_0 t}{R} - \sin\dfrac{\upsilon_0 t}{R}\right) \\ y_M = R\left(1 - \cos\dfrac{\upsilon_0 t}{R}\right) \end{cases}$$

Answer:

The equation of motion of the point M of the wheel rim is equal to:

$$x_M = R\left(\frac{\upsilon_0 t}{R} - \sin\frac{\upsilon_0 t}{R}\right); \qquad y_M = R\left(1 - \cos\frac{\upsilon_0 t}{R}\right)$$

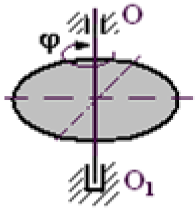
TASKS FOR INDEPENDENT DEVELOPMENT

<table>
<tr>
<td>

1. The body rotates around its axis according to the law

$\varphi = (t-3)^2 - 9\ rad.$

At time t = 3 s the body will rotate …



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<td>

a) slowed down

b) evenly

c) uniformly accelerated

d) uniformly slowed down

e) accelerated

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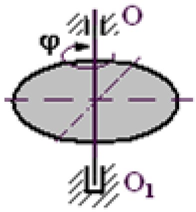
2. The body rotates around its axis according to the law

$\varphi = (t-2)^3$ rad.

At time t = 1 s the body will rotate …



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<td>

a) slowed down

b) evenly

c) uniformly accelerated

d) uniformly slowed down

e) accelerated

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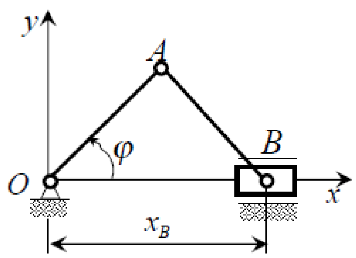
3. The position of the crank is determined by the angle

$\varphi = 0{,}2t\ [rad].$

Find the coordinate X_B of the slider point at time $t = 3\ s$, if the length of the links OA= AB=0,5m.



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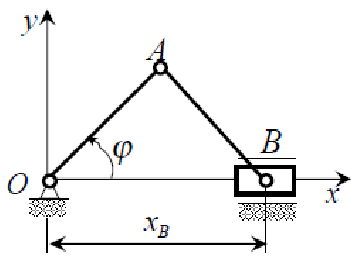
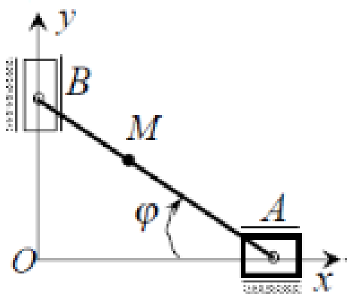
4. Determine the speed of the slider B at the time $t = 4s$ at the position of the crank, which is determined by the angle $\varphi = 0{,}5\ t\ [rad]$ and length of the links OA = AB = 1,5m	
5. Determine the projection of the velocity of the point M on the axis Ox at the time $t = 2s$ in the position of the connecting rod AB, which is determined by the angle $\varphi = 0{,}5\ t\ [rad]$ and length BM = 0.2 m	

INDIVIDUAL TASKS FOR INDEPENDENT WORKING

The body rotates around its axis. Find the speed and acceleration of these two points of the body.

The data required for the calculations are given in table.1.

Schematic representation of the bodies of rotation and № of scheme are presented in table.2

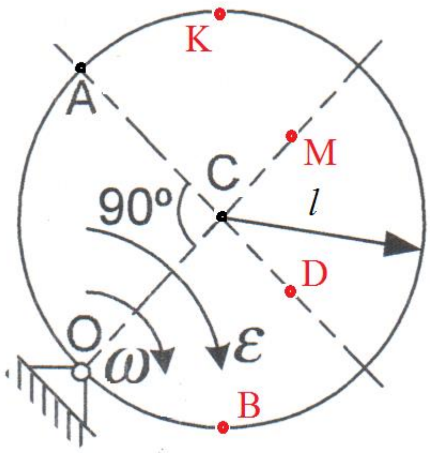
Table 1 - Task options

Case N	Scheme N	Set			Determine the speed and acceleration of points …
		$\omega,\ s^{-1}$	$\varepsilon,\ s^{-2}$	l, m	
1.	1	$2\sqrt{3}$	1	2	A and C
2.	2	1	$\sqrt{3}$	1	B and D
3.	3	1	$\sqrt{3}$	1	A and C
4.	4	2	3	2	C and D
5.	5	2	5	3	A and C
6.	6	1	$\sqrt{3}$	1	B and K
7.	7	2	$\sqrt{5}$	1	A and C

8.	1	1	$\sqrt{3}$	1	B and D
9.	2	2	$\sqrt{5}$	1	A and C
10.	3	3	1	2	K and M
11.	4	1	$\sqrt{3}$	1	A and C
12.	5	3	4	2	K and C
13.	6	2	$\sqrt{5}$	1	A and C
14.	7	1	$\sqrt{3}$	1	C and B
15.	1	2	5	3	K and M
16.	2	$\sqrt{3}$	1	1	A and D
17.	3	3	4	2	K and A
18.	4	2	$\sqrt{5}$	1	D and A
19.	5	1	$\sqrt{3}$	1	C and K
20.	6	$\sqrt{3}$	2	3	D and A

Table 2 - Schematic representation of rotating bodies and N diagrams

1



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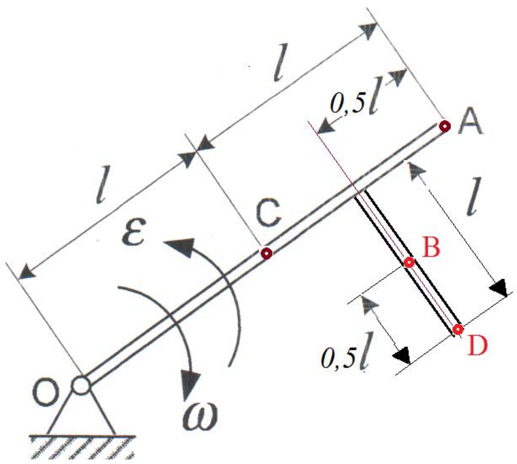
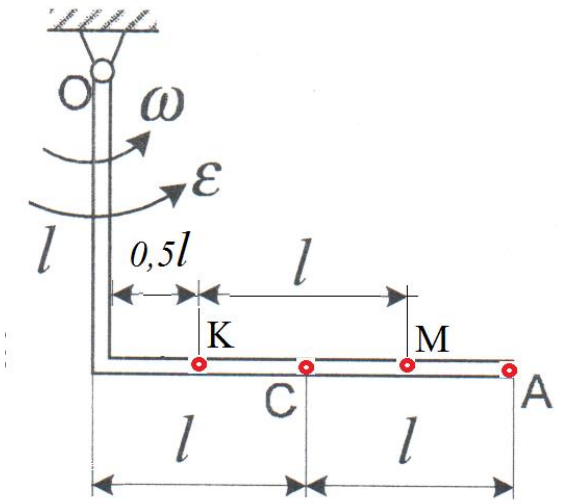
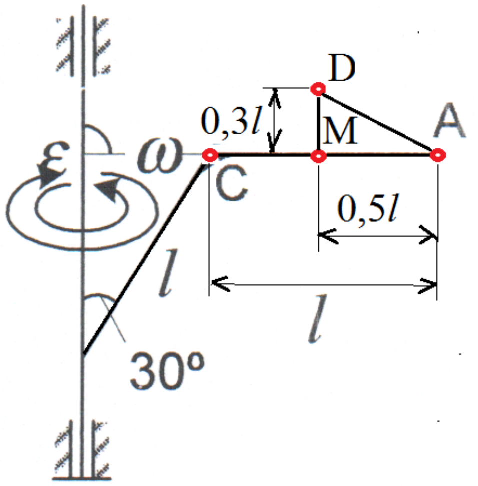
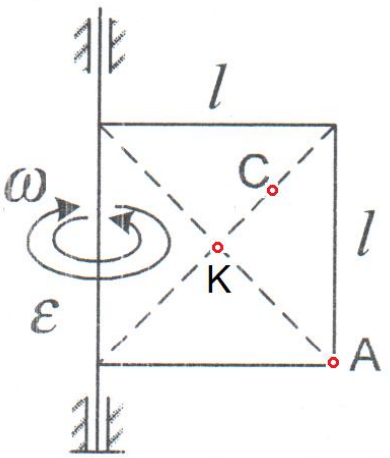
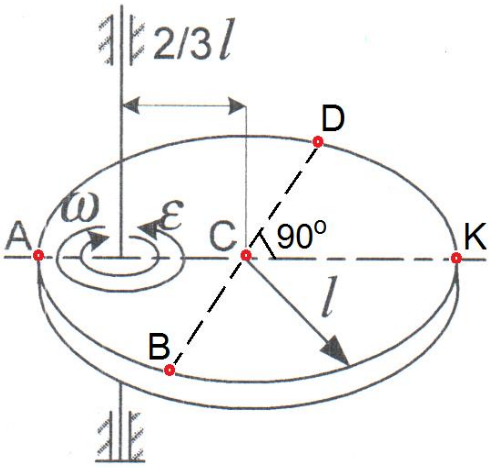


3	4
	
5	**6**
	

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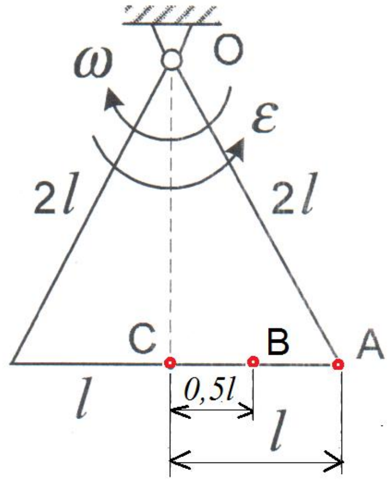
7 	

9. PLANE-PARALLEL MOVEMENT OF THE BODY

The purpose of the lesson: acquiring skills in:

- determination of velocities and accelerations of points of the body, that performing flat motion;
- finding the position of the instantaneous center of velocity;
- study of kinematic characteristics of individual points of the mechanism.

Before completing the task you need to read:

- theoretical material on the topic of practical training;
- formulas for determining speeds and accelerations in plane motion.

Section 1. Determination of velocities and accelerations of points of a body, that performing flat motion

Plane-parallel (flat) is a motion of a rigid body in which all points of the body move in planes parallel to one fixed plane, which is called the main. To study the plane motion of the whole body, it suffices to consider the motion of a flat figure that is parallel to the main plane.

The motion of a flat figure in its plane can be considered as a set of translational motion, which is determined by the motion of an arbitrarily chosen point - the pole, and the rotational motion around this pole. The pole of a flat figure is taken to be a point whose motion is known or can be easily found. The position of a flat figure is uniquely determined by the position of the segment of the line AB, namely: the coordinates of the pole – point $A\left(x_A; y_A\right)$ and the angle φ, set counterclockwise from the positive direction of the x-axis to the segment AB.

Therefore, the equations of plane motion are the following dependences:

$$x_A = x_A\left(t\right); \qquad y_A = y_A\left(t\right); \qquad \varphi = \varphi\left(t\right)$$

The speed of any point of a flat figure is determined by the formula:

$$\vec{\upsilon}_B = \vec{\upsilon}_A + \vec{\upsilon}_{BA}$$

Where $\vec{\upsilon}_A$ is the speed of the pole A; $\vec{\upsilon}_{BA}$ - the speed that the point B acquires when rotating a flat figure with an angular velocity ω around the pole A:

$$\vec{\upsilon}_{BA} = \vec{\omega} \times \overrightarrow{AB}; \quad \upsilon_{BA} = \omega \cdot AB$$

The velocity vector $\vec{\upsilon}_B$ is determined by the diagonal of the parallelogram, built on vectors $\vec{\upsilon}_A$ and $\vec{\upsilon}_{BA}$, as on the sides. The vector $\vec{\upsilon}_{BA}$ is directed perpendicular to the segment AB in the direction of rotation of the flat figure.

The modulus of velocity of point B is determined by the formula:

$$\upsilon_B = \sqrt{\upsilon_A^2 + \upsilon_{BA}^2 + 2\upsilon_A \upsilon_{BA} \cos\gamma}$$

In the case when we know the velocity of the pole A and the direction of the desired velocity of point B, we can use the theorem on the projection of the velocities of two points of a plane figure, according to which the projections of velocities of two points of a plane figure on the line connecting them are equal, so:

$$\upsilon_B \cos\beta = \upsilon_A \cos\alpha$$

The acceleration $\vec{a}_B$ of any point B of a flat figure is equal to the vector sum of the acceleration $\vec{a}_A$ of the pole A and the acceleration $\vec{a}_{BA}$, that the point B acquires when rotating the figure around the pole A:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$$

where

$$\vec{a}_{BA}^T = \vec{\varepsilon} \times \overrightarrow{AB}; \quad a_{BA}^T = \varepsilon \cdot AB$$
$$\vec{a}_{BA}^N = \vec{\omega} \times \vec{\upsilon}_{BA}; \quad a_{BA}^N = \omega^2 \cdot AB$$

The acceleration module $\vec{a}_{BA}$ is equal to:

$$a_{BA} = \sqrt{\left(a_{BA}^{T}\right)^2 + \left(a_{BA}^{N}\right)^2} = AB\sqrt{\varepsilon^2 + \omega^4}$$

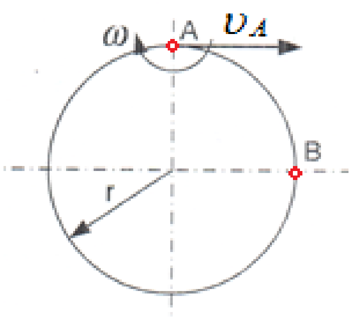
The acceleration module is equal to:

$$a_B = \sqrt{\left(a_A\right)^2 + \left(a_{BA}\right)^2 + 2a_A a_{BA}\cos\theta}$$

EXAMPLES

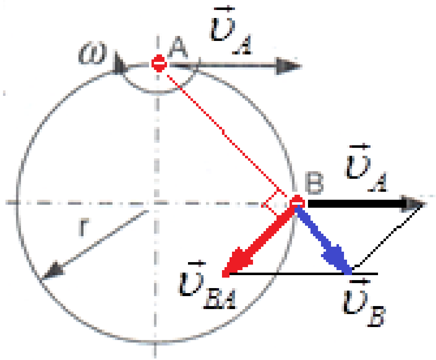
EXAMPLE 1

Find the velocity of point B of a body performing a plane motion under the following conditions: $\upsilon_A = 1\ m \cdot s^{-1}, \quad \omega = 1\ s^{-1}, \quad r = 1\ m.$



Solution

Plane motion is considered as a set of translational motion, which is determined by the motion of an arbitrarily chosen point - the pole, and the rotational motion around this pole. In this case, the pole is point A, which moves translationally with speed υ_A, point B rotates around pole A



The velocity of point B is determined by the formula:

$$\vec{\upsilon}_B = \vec{\upsilon}_A + \vec{\upsilon}_{BA}$$

The vector $\vec{\upsilon}_{BA}$ is directed perpendicular to the segment AB in the direction of rotation of the flat figure.

$$\upsilon_{BA} = \omega \cdot AB; \quad AB = r\sqrt{2} = \sqrt{2}\ m; \quad \upsilon_{BA} = 1 \cdot \sqrt{2} = \sqrt{2}\ m/s$$

The velocity vector $\vec{\upsilon}_B$ is determined by the diagonal of the parallelogram, built on vectors $\vec{\upsilon}_A$ *i* $\vec{\upsilon}_{BA}$, as on the sides.

The modulus of velocity of point B is determined by the formula:

$\upsilon_B = \sqrt{\upsilon_A^2 + \upsilon_{BA}^2 + 2\upsilon_A \upsilon_{BA} \cos\gamma}$, where $\gamma = 135°$ - the angle between the vectors $\vec{\upsilon}_A$ i $\vec{\upsilon}_{BA}$.

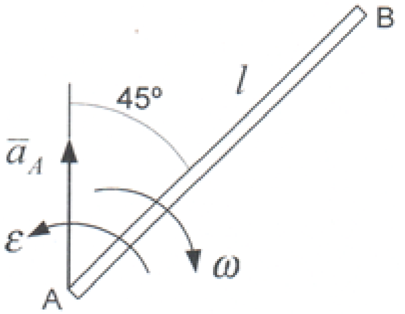
$$\upsilon_B = \sqrt{1 + \left(\sqrt{2}\right)^2 + 2 \cdot 1 \cdot \sqrt{2}\cos 135°} = 1\ m/s$$

Answer: $\upsilon_B = 1\ m/s$.

EXAMPLE 2

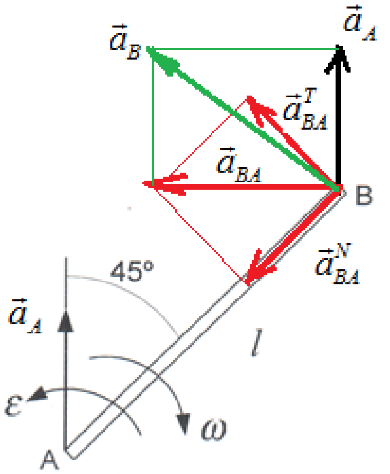
Find the acceleration of point B of a body performing a plane motion under the following conditions:

$a_A = 1\ m \cdot s^{-2}, \omega = 1\ s^{-1},\quad \varepsilon = 1\ s^{-2},\quad l = 1\ m$



Solution

The acceleration of point B of a flat figure is equal to the vector sum of the acceleration of pole A and the acceleration acquired by point B when rotating the figure around pole A.



$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$

$\vec{a}_{BA} = \vec{a}_{BA}^T + \vec{a}_{BA}^N$

The acceleration vector $\vec{a}_{BA}^T$ directed perpendicular to the segment AB in the direction of angular acceleration ε, and the vector $\vec{a}_{BA}^N$ – from point B to pole A.

$$a_{BA}^T = \varepsilon AB = 1\ m \cdot s^{-2}$$

$$a_{BA}^N = \omega^2 AB = 1\ m \cdot s^{-2}$$

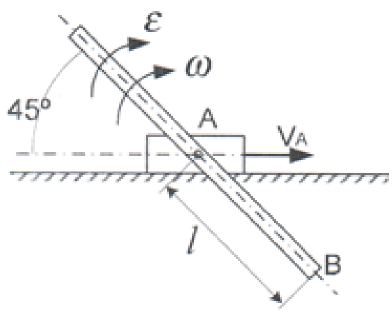
$$a_{BA} = \sqrt{(a_{BA}^T)^2 + (a_{BA}^N)^2} = AB\sqrt{\varepsilon^2 + \omega^4} = \sqrt{1+1} = \sqrt{2}\ m \cdot s^{-2}$$

$$a_B = \sqrt{(a_A)^2 + (a_{BA})^2 + 2a_A a_{BA}\cos\theta} = \sqrt{1 + 2 + 2 \cdot \sqrt{2}\cos 90°} = \sqrt{3}\ m \cdot s^{-2}$$

Answer: $\boldsymbol{a_B} = \sqrt{\mathbf{3}}\ m \cdot s^{-2}$.

EXAMPLE 3

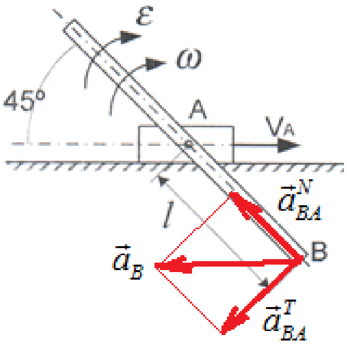
Find the acceleration of point B of a body performing a plane motion under the following conditions:



$\omega = 1\ s^{-1}$, $\quad \varepsilon = 1\ s^{-2}$, $\quad l = 1\ m$, $\quad \upsilon_A = const$

Solution

The acceleration of point B of a flat figure performing a flat motion is determined by the equation:



$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^{T} + \vec{a}_{BA}^{N}$

$\vec{a}_A = 0$, because $\upsilon_A = const$.

The acceleration of point B is equal:

$\vec{a}_B = \vec{a}_{BA}^{T} + \vec{a}_{BA}^{N}$

$\vec{a}_{BA}^{T} = \vec{\varepsilon} \times \overrightarrow{AB}$; $\qquad a_{BA}^{T} = \varepsilon AB = 1\ m \cdot s^{-2}$

$\vec{a}_{BA}^{N} = \vec{\omega} \times \vec{\upsilon}_{BA}$; $\qquad a_{BA}^{N} = \omega^2 AB = 1\ m \cdot s^{-2}$

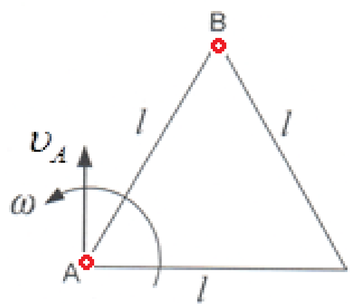
$$a_B = \sqrt{(\vec{a}_{BA}^{T})^2 + (\vec{a}_{BA}^{N})^2} = \sqrt{1+1} = \sqrt{2}\ m \cdot s^{-2}$$

Відповідь: $\sqrt{2}\ m \cdot s^{-2}$

TASK

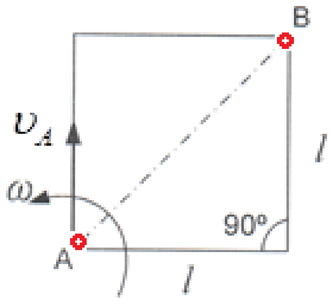
for self-study of section 1

1. Find the velocity of point B of a body performing a plane motion under the following conditions:
   $\upsilon_A = 2\ m \cdot s^{-1}$, $\quad \omega = 1\ s^{-1}$, $\quad l = 1\ m$.



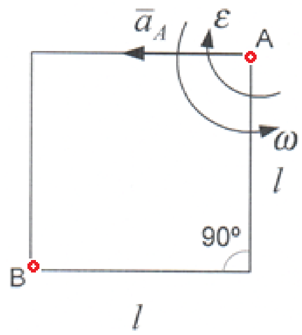
2. Find the velocity of point B of a body performing a plane motion under the following conditions:

$$v_A = 1\ m \cdot s^{-1}, \quad \omega = 1\ s^{-1}, \quad l = 1\ m$$



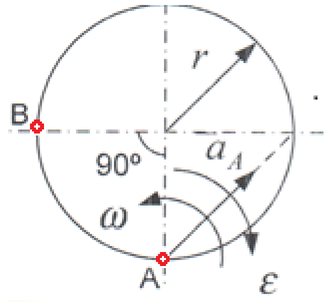
3. Find the acceleration of point B of a body performing a plane motion under the following conditions:

$$a_A = 1\ m \cdot s^{-2}, \quad \omega = 1\ s^{-1}, \quad \varepsilon = 1\ s^{-2}, \quad l = 1\ m$$



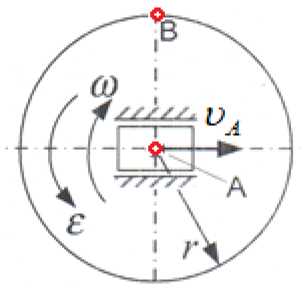
4. Find the acceleration of point B of a body performing a plane motion under the following conditions:

$$a_A = 1\ m \cdot s^{-2}, \quad \omega = 1\ s^{-1}, \quad \varepsilon = 1\ s^{-2}, \quad r = 1\ m$$



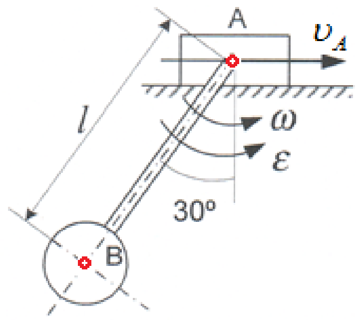
5. Find the acceleration of point B of a body performing a plane motion under the following conditions:

$$\omega = 1\ s^{-1}, \quad \varepsilon = 2\ s^{-2}, \quad r = 1\ m, \quad v_A = const.$$



6. Find the acceleration of point B of a body performing a plane motion under the following conditions:

$\omega = 2\ s^{-1}, \quad \varepsilon = 1\ s^{-2}, \quad l = 1\ m, \quad \upsilon_A = const,$



Section 2. Determination of velocities and accelerations of points of a body performing flat motion

The instantaneous center of velocities (ICV) is the point P of the body, the velocity of which is currently zero, $\upsilon_P = 0$.

The velocity of any point of the body is equal to its velocity in rotational motion around the instantaneous center of velocity P. That is, for a body performing plane-parallel motion, the relations are:

$$\upsilon_M = \omega \cdot PM, \qquad \upsilon_N = \omega \cdot PN, \qquad \upsilon_A = \omega \cdot PA,$$

$$\omega = \frac{\upsilon_A}{PA} = \frac{\upsilon_M}{PM} = \frac{\upsilon_N}{PN}$$

$$\frac{\upsilon_A}{\upsilon_M} = \frac{PA}{PM}; \qquad \frac{\upsilon_A}{\upsilon_N} = \frac{PA}{PN}; \qquad \frac{\upsilon_M}{\upsilon_N} = \frac{PM}{PN}$$

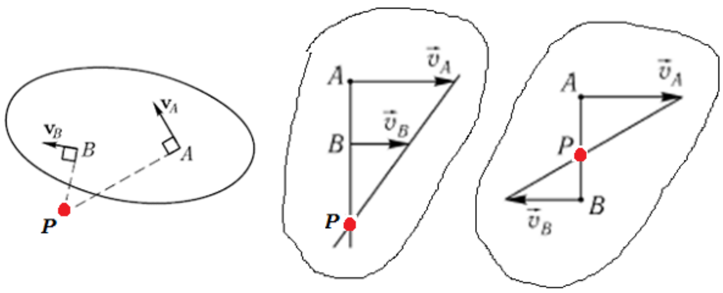
where υ_M, υ_N, υ_A - the velocities of the points M, N, A; ω - angular velocity of the body at a certain point in time relative to ICV P; PM, PN, PA - distances of points M, N, A from the instantaneous center of velocities P.

Ways to find the instantaneous center of velocity

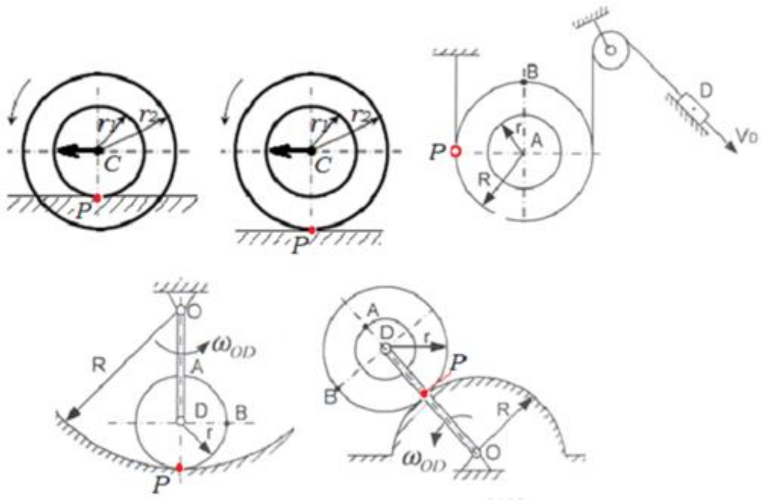
1. If the angular velocity ω of a flat figure and the velocity vector of one of its points, for example A, are known, then the ICV lies on perpendicular to the velocity vector of point A at a distance AP:

$$AP = \frac{\upsilon_A}{\omega}$$

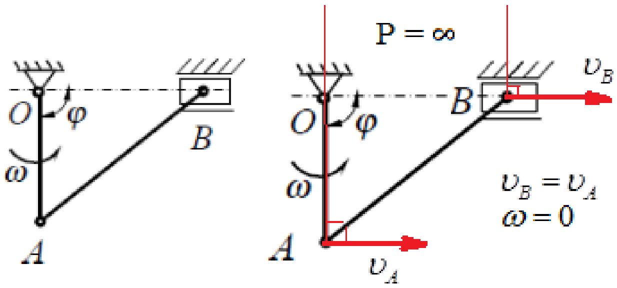
2. If the velocity vectors of two points of a flat figure are known and the vectors of these points are not parallel to each other, then the ICV lies at the intersection of the perpendiculars to the velocity vectors of these points.
3. If the velocities of two points of a flat figure are parallel to each other and perpendicular to the segment connecting these points, and the moduls of these velocities are known and different, then the ICV lies at the intersection of the common perpendicular to the velocity vectors of these points and the line drawn through these vectors.



4. If the plane motion is carried out by rolling without sliding one body on a fixed surface of another, the ICV is at the point of contact of the body with a fixed surface, because in the absence of sliding, the speed of this point of the moving body is zero.

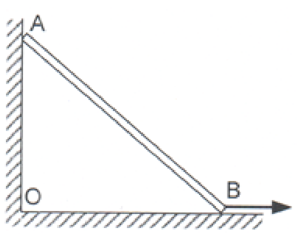


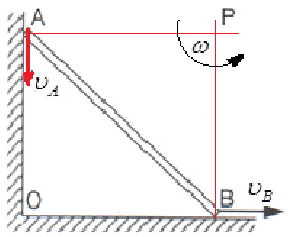
5. If the velocities of two points are parallel to each other, and the line connecting these points is not perpendicular to their velocities, then the ICV lies in infinity. In this case, the body performs an instantaneously translational motion in which the velocities of all points at a given time are equal in magnitude and equally directed, and the angular velocity of a flat figure is zero



EXAMPLES
of finding ICV

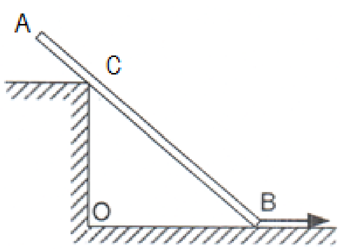
1. Find the instantaneous center of velocity of the body that performs plane-parallel motion

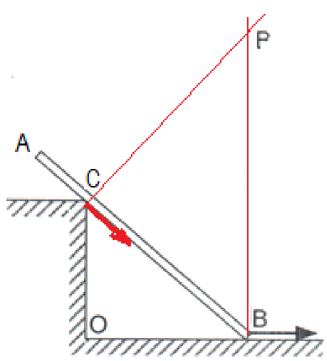




$$\omega = \frac{\upsilon_A}{PA} = \frac{\upsilon_B}{PB}$$

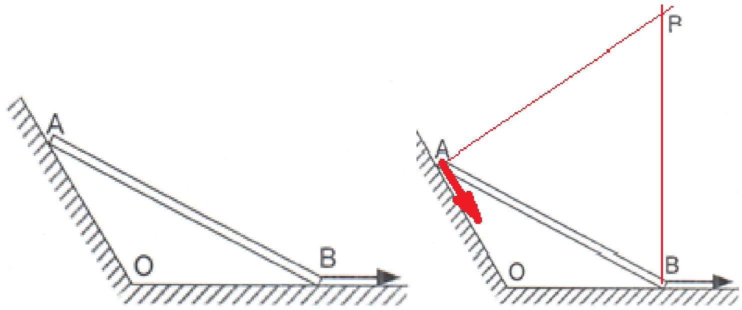
2. Find the instantaneous center of velocity of the body that performs plane-parallel motion





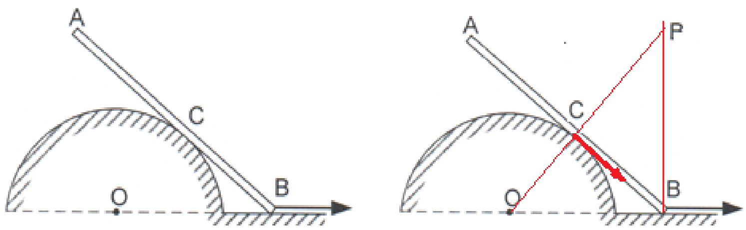
$$\omega = \frac{\upsilon_C}{PC} = \frac{\upsilon_B}{PB}$$

3. Find the instantaneous center of velocity of the body that performs plane-parallel motion



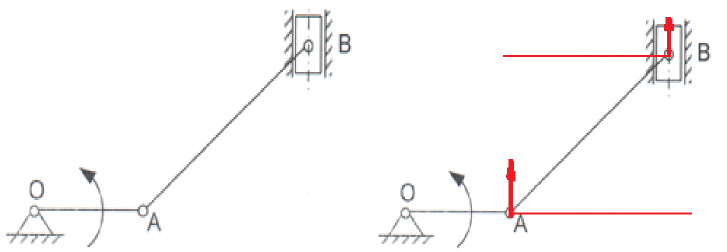
$$\omega = \frac{\upsilon_A}{PA} = \frac{\upsilon_B}{PB}$$

4. Find the instantaneous center of velocity of the body that performs plane-parallel motion



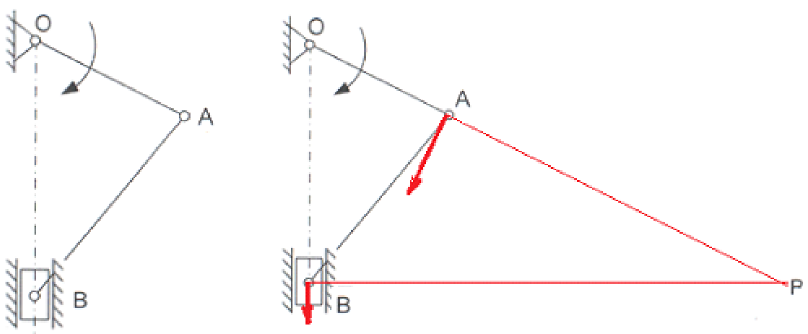
$$\omega = \frac{\upsilon_C}{PC} = \frac{\upsilon_B}{PB}$$

5. Find the instantaneous center of velocity of the mechanism that performs plane-parallel motion

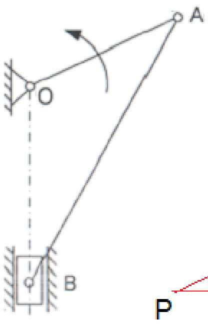


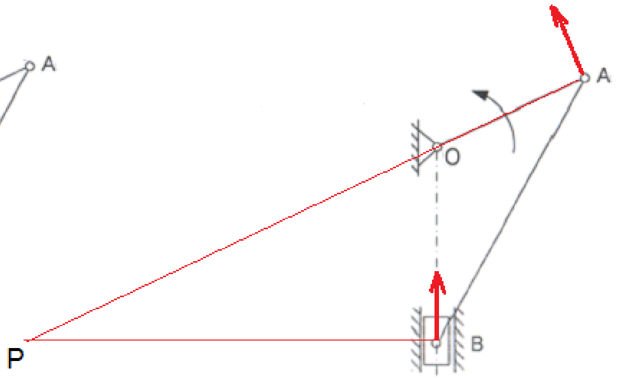
$$\omega = \frac{\upsilon_A}{PA} = \frac{\upsilon_A}{\infty} = 0$$

6. Find the instantaneous center of velocity of the mechanism that performs plane-parallel motion

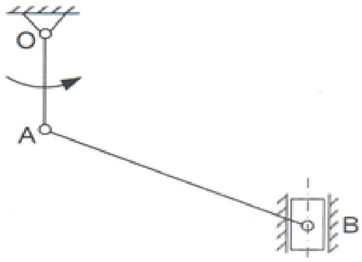


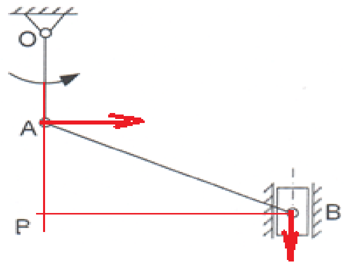
7. Find the instantaneous center of velocity of the mechanism that performs plane-parallel motion



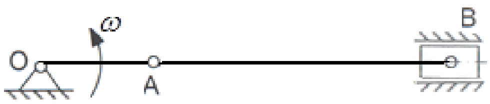


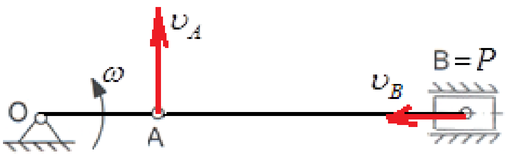
8. Find the instantaneous center of velocity of the mechanism that performs plane-parallel motion



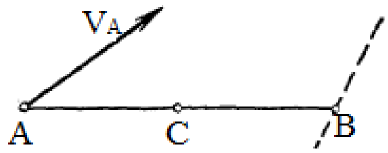


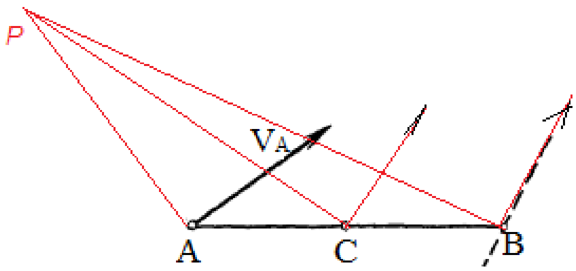
9. The crank OA rotates at a constant angular velocity ω. Determine the distance from point A to the instantaneous center of velocity if the length of the crank and connecting rod, respectively $OA = 8\ cm$, $AB = 20\ cm$.

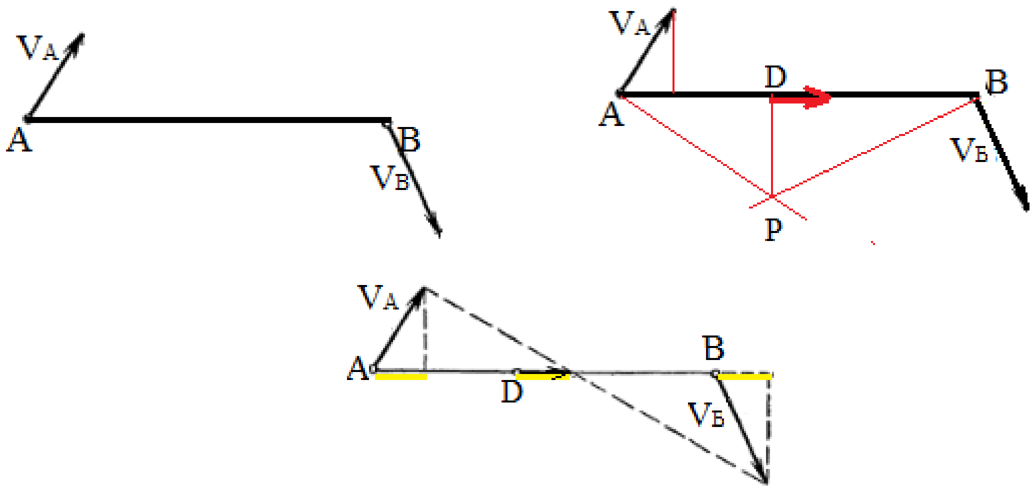
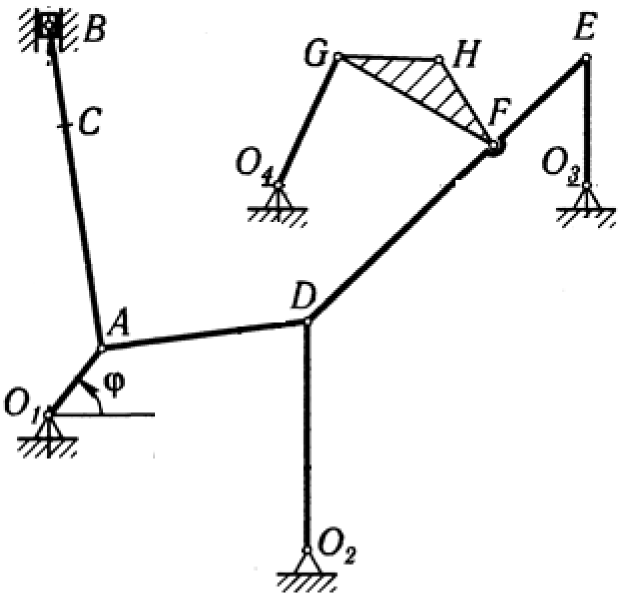


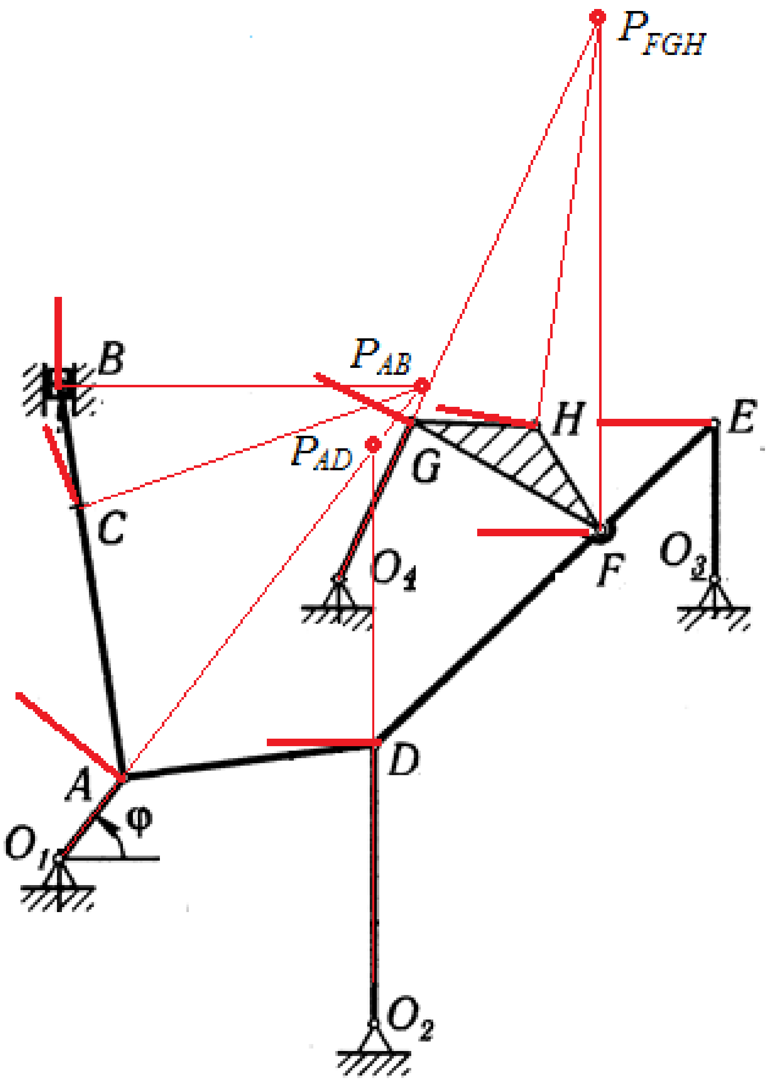


10. The body AB performs plane-parallel motion. Determine graphically the velocities of points B and C, if the speed of point A and the direction of velocity of point B are given.



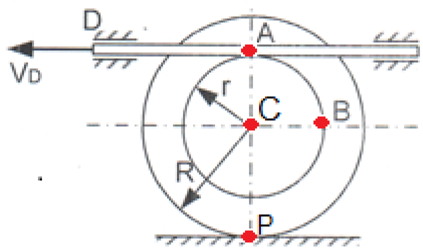


$$\omega = \frac{\upsilon_A}{PA} = \frac{\upsilon_B}{PB} = \frac{\upsilon_C}{PC}$$ $$\upsilon_A \perp PA, \quad \upsilon_B \perp PB, \quad \upsilon_C \perp PC$$
11.The body AB performs plane-parallel motion. Knowing the speed of the ends of the segment AB, determine the point on this segment, the speed of which is currently directed along the segment. 
12.The mechanism performs plane-parallel motion. Find the instantaneous center of velocities for each link of the mechanism. 



13. Find the velocity of points C and B of the presented mechanism if

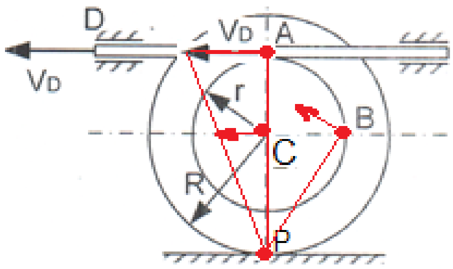
$$\upsilon_D = 1\ m \cdot s^{-1}, \quad R = 2\ m, \quad r = 1\ m$$



Sokving

The instantaneous center of velocities P is at the point of contact of the stepped wheel with the surface. At this point, the speed is zero. The speed of point A of the mechanism is equal to: $\upsilon_A = \upsilon_D$

The velocities of the points C and B of the wheel are found from the following relations:



$$\frac{v_C}{v_D} = \frac{PC}{PA}$$

$$v_C = v_D \frac{PC}{PA} = v_D \frac{R}{R+r} = 1 \cdot \frac{2}{2+1} = \frac{2}{3} \frac{m}{s}$$

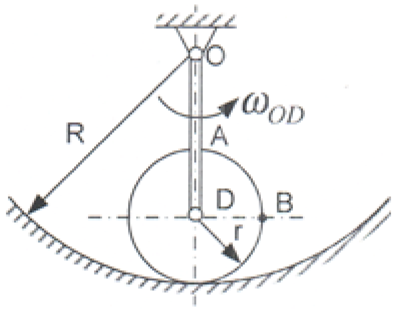
$$\frac{v_C}{v_B} = \frac{PC}{PB}$$

$$v_B = v_C \frac{PB}{PC} = \frac{2}{3} \frac{\sqrt{R^2+r^2}}{R} = \frac{2}{3} \frac{\sqrt{4+1}}{2}$$

$$= \frac{\sqrt{5}}{3}\ m/s$$

Answer: $v_C = \frac{2}{3} m/s; \quad v_B = \frac{\sqrt{5}}{3} m/s$

14. The crank OD rotates around the axis O at an angular velocity $\omega_{OD} = 2\ s^{-1}$ and gives motion to the gear with a radius $r = 1m$ that is freely mounted on its end D. The movable gear rolls without sliding on the inner surface of the stationary gear with a radius $R = 2\ m$. Find the velocities of points A and B and the angular velocity of the movable gear if $\omega_{OD} = 2\ s^{-1}, \quad R = 2\ m, \quad r = 1\ m$



Solution

Determine the speed of the point D, which simultaneously belongs to the crank OD, rotating around the axis O with angular velocity $\omega_{OD} = 2\ s^{-1}$, and the movable gear:

$$v_D = \omega_{OD} OD = \omega_{OD}(R-r) = 2(2-1) = 2\ m/s$$

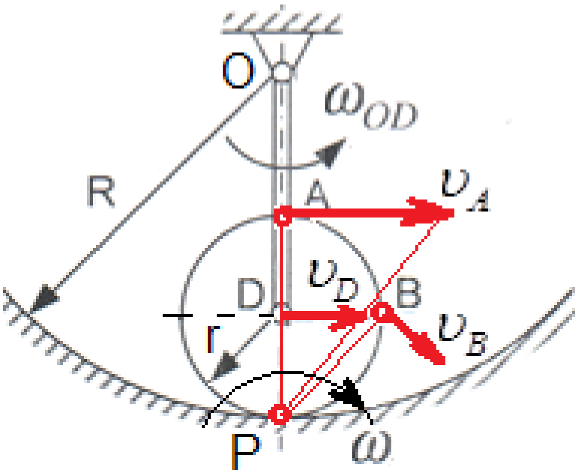
The instantaneous center of velocities P is at the point of contact of the gears. At this point, the speed is zero. The velocities of points A and B of the movable gear are found from the following relations:

$$\frac{\upsilon_A}{\upsilon_D} = \frac{PA}{PD}$$

$$\upsilon_A = \upsilon_D \frac{PA}{PD} = \upsilon_D \frac{2r}{r} = 2 \cdot 2 = 4\ m/s$$

$$\frac{\upsilon_B}{\upsilon_D} = \frac{PB}{PD}$$

$$\upsilon_B = \upsilon_D \frac{PB}{PD} = \upsilon_D \frac{r\sqrt{2}}{r} = 2\sqrt{2}\ m/s$$



The speed of any point of the movable gear is equal to the product of its angular velocity at a distance from the instantaneous center of velocity:

$$\upsilon_D = \omega PD;$$

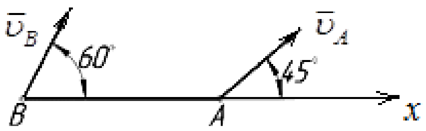
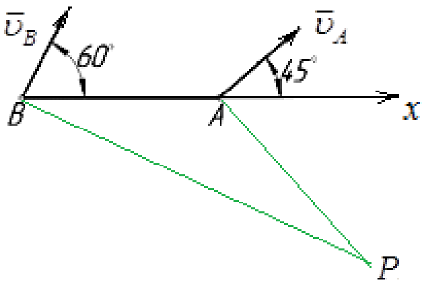
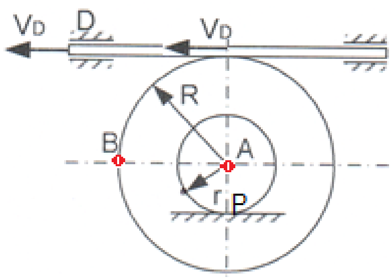
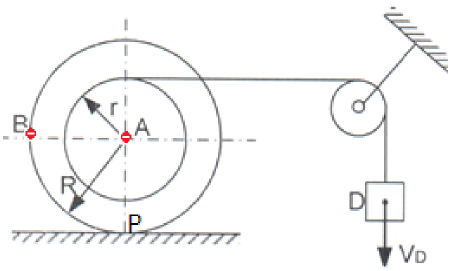
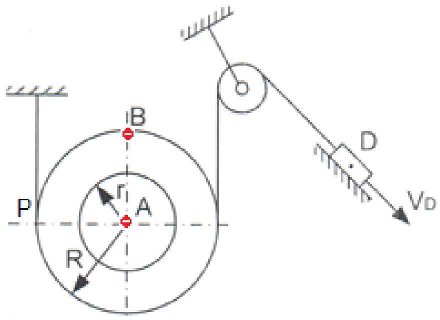
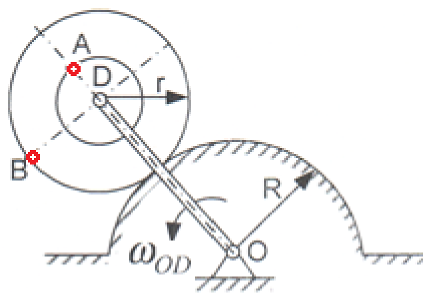
$$\omega = \frac{\upsilon_D}{PD} = \frac{2}{1} = 2\ s^{-1}$$

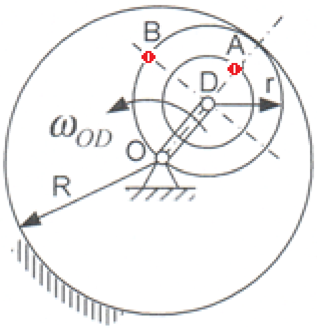
Answer: $\boldsymbol{\upsilon_A} = 4\ m/s;\quad \upsilon_B = 2\sqrt{2}\ m/s;\quad \omega = 2\ s^{-1}.$

TASK

for self-study of section 2

1. The rod AB with a length of 0.5 m moves in the plane of the figure. The velocity of point A is equal to $\upsilon_A = 2m/s$ and forms an angle 45° with the x-axis aligned with the rod AB. The velocity υ_B of point B forms an angle of 60° with the x-axis. Determine the modulus of velocity of point B and the angular velocity of the rod.

 	
2. Find the velocity of points A and B of the presented mechanism if $\upsilon_D = 1\ m \cdot s^{-1}, \quad R = 2\ m, \quad r = 1\ m$	
3. Find the velocity of points A and B of the presented mechanism if $\upsilon_D = 1\ m \cdot s^{-1}, \quad R = 2\ m, \quad r = 1\ m$	
4. Find the velocity of points A and B of the presented mechanism if $\upsilon_D = 1\ m \cdot s^{-1}, \quad R = 2\ m, \quad r = 1\ m$	
5. Find the velocity of points A and B if $\omega_{OD} = 2\ s^{-1}, \quad R = 2\ m,$ $r = 1\ m, \quad AD = 0{,}5\ r$	

6. Find the velocity of points A and B if $\omega_{OD} = 2\ s^{-1}$, $R = 2\ m$, $r = 1\ m$, $AD = 0{,}5\ r$	

10. STUDY OF THE MOTION OF LINKS OF A FLAT MECHANISM

The purpose of the lesson is to learn to determine the acceleration of points by the speed and acceleration of pole.

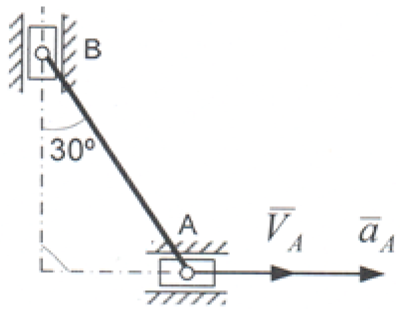
EXAMPLES

EXAMPLE 1

Determine the angular acceleration ε_{BA} of the connecting rod AB if

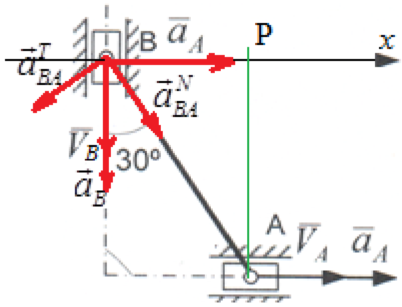
$\upsilon_A = 1\ m \cdot s^{-1}, \quad a_A = 2\ m \cdot s^{-2},$

$AB = 1\ m$



Solution

The acceleration $\vec{a}_B$ of point B of a planar mechanism is equal to the vector sum of the acceleration $\vec{a}_A$ of pole A and the acceleration $\vec{a}_{BA}$ acquired by point B of the connecting rod when it rotates around pole A:



$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$$

To simplify the calculations through point B draw the axis x⊥ $\vec{a}_B$. Find the projection of the vector equation $\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$ on the x-axis, assuming that $\vec{a}_B \perp x$:

$$0 = a_A - a_{BA}^T \cos 30° + a_{BA}^N \cos 60°$$

$$a_{BA}^T = \frac{a_A + a_{BA}^N \cos 60°}{\cos 30°}$$

The angular acceleration of the connecting rod AB is found from the equation:

$$\varepsilon_{BA} = \frac{a_{BA}^T}{BA} = \frac{a_A + a_{BA}^N \cos 60°}{AB \cos 30°}$$

$$a_{BA}^N = \omega_{BA}^2 BA$$

To determine a_{BA}^N the angular velocity ω_{BA} of the connecting rod AB, we find the instantaneous center of velocities P and its distance to point A:

$PA = AB\cos 30°$

$$\omega_{BA} = \frac{v_A}{PA} = \frac{v_A}{AB\cos 30°} = \frac{2}{\sqrt{3}}\ s^{-1}$$

$$a_{BA}^N = \omega_{BA}{}^2 AB = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}\ m\cdot s^{-2}$$

$$\varepsilon_{BA} = \frac{a_A + a_{BA}^N \cos 60°}{AB\cos 30°} = \frac{2+\frac{4}{3}\cdot\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{16}{3\sqrt{3}} = \frac{16\sqrt{3}}{9}s^{-2}$$

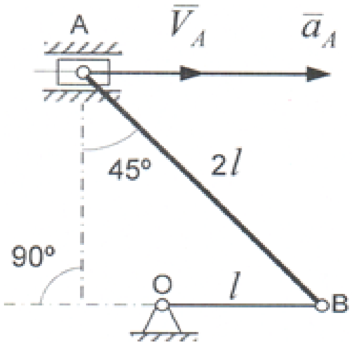
Answer: $\varepsilon_{BA} = \frac{16\sqrt{3}}{9} = 3{,}08\ s^{-2}$

EXAMPLE 2

Determine the angular acceleration ε_{BA} of the connecting rod AB if

$v_A = 1\ m\cdot s^{-1}, \quad a_A = 2\ m\cdot s^{-2},$

$l = 1\ m$



Solution

Find the instantaneous center of velocities P and its distance to points A and B

$$\omega_{AB} = \frac{v_A}{PA} = \frac{1}{\sqrt{2}}\quad rad/s$$

$$v_B = v_A = 1\quad m/s$$

The acceleration $\vec{a}_B$ of point B of a planar mechanism is equal to the vector sum of the acceleration $\vec{a}_A$ of pole A and the acceleration $\vec{a}_{BA}$ that point B acquires when it rotates around pole A:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$$

Since point B moves along a curvilinear trajectory, the equation takes the form:

$$\vec{a}_B^T + \vec{a}_B^N = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$$

Determine the normal components of acceleration:

$$a_B^N = \frac{v_B^2}{OB} = \frac{1^2}{1} = 1\ m\cdot s^{-2}$$

$$a_{BA}^N = \omega_{BA}^2 AB = \left(\frac{1}{\sqrt{2}}\right)^2 2 = 1\ m \cdot s^{-2}$$

To determine the angular acceleration ε_{BA} of the connecting rod AB must be found a_{BA}^T :

$$\varepsilon_{AB} = \frac{a_{BA}^T}{AB}$$

To this end, we make a projection of the vector equation $\vec{a}_B^T + \vec{a}_B^N = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$ on the x-axis, bearing in mind that $\vec{a}_B^T \perp x$:

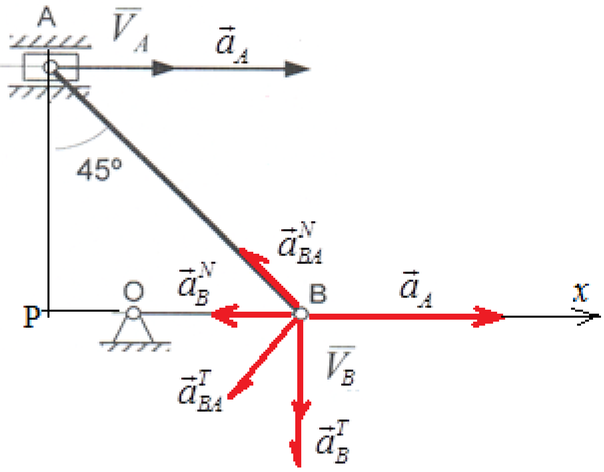
$$0 - a_B^N = a_A - a_{BA}^T \cos 45° - a_{BA}^N \cos 45°$$

$$a_{BA}^T = \frac{a_A + a_B^N - a_{BA}^N \cos 45°}{\cos 45°}$$

$$a_{BA}^T = \frac{2 + 1 - \cos 45°}{\cos 45°} = \frac{3}{\cos 45°} - 1 = 3{,}24 m \cdot s^{-2}$$

the angular acceleration ε_{BA} of the connecting rod AB is equal to:

$$\varepsilon_{AB} = \frac{a_{BA}^T}{AB} = \frac{3{,}24}{2} = 1{,}62 rad/s^2$$



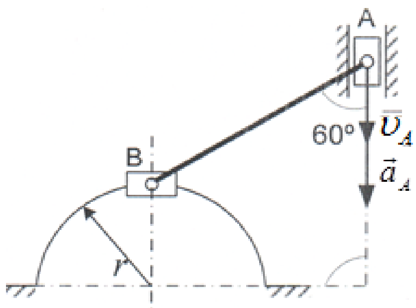
Answer: $\varepsilon_{BA} = 1{,}62 rad/s^2$

EXAMPLE 3

Determine the angular acceleration ε_{BA} of the connecting rod AB if

$v_A = 1\ m \cdot s^{-1}$, $a_A = 2\ m \cdot s^{-2}$,

$AB = 2\ m$, $r = 1\ m$



Solution

The acceleration $\vec{a}_B$ of point B of a planar mechanism is equal to the vector sum of

the acceleration $\vec{a}_A$ of pole A and the acceleration $\vec{a}_{BA}$ acquired by point B of the connecting rod when it rotates around pole A:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$$

Since point B moves along a curvilinear trajectory, the equation takes the form:

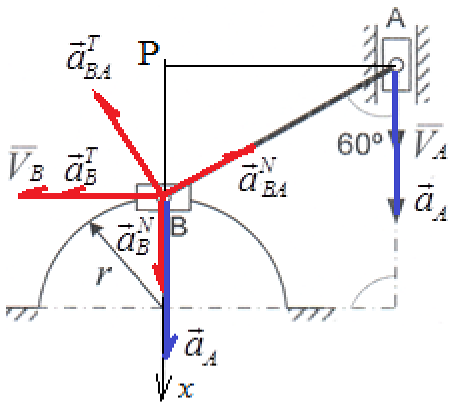
$$\vec{a}_B^T + \vec{a}_B^N = \vec{a}_A + \vec{a}_{BA}^T + \vec{a}_{BA}^N$$

Find the projection of the vector equation on the x-axis, bearing in mind that $\vec{a}_B^T \perp x$. The x-axis is directed vertically down from point B:

$$a_B^N = a_A - a_{BA}^N \cos 60° - a_{BA}^T \cos 30°$$

$$a_{BA}^T = \frac{a_A - a_{BA}^N \cos 60° - a_B^N}{\cos 30°}$$

$$\varepsilon_{BA} = \frac{a_{BA}^T}{AB} = \frac{a_A - a_{BA}^N \cos 60° - a_B^N}{AB \cos 30°}$$



It is necessary to determine a_{BA}^N i a_B^N.

$$a_{BA}^N = \omega_{BA}{}^2 AB; \qquad a_B^N = \frac{\upsilon_B^2}{r}$$

To do this, find the instantaneous center of velocity:

$PB = 1m; \quad PA = \sqrt{3}m$

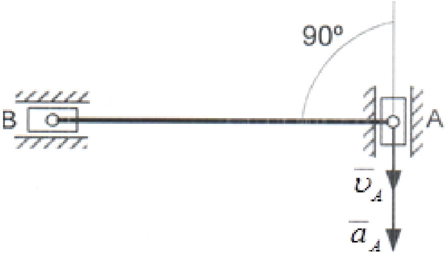
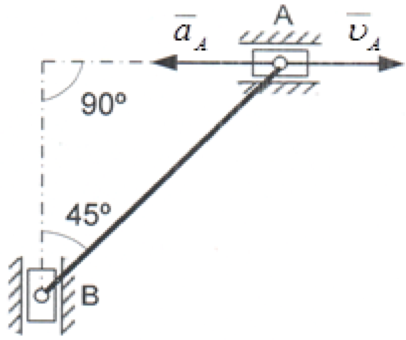
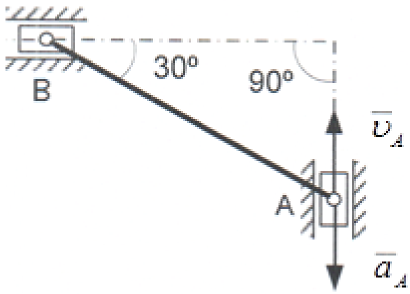
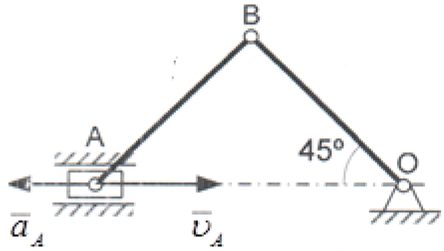
$$\frac{\upsilon_A}{PA} = \frac{\upsilon_B}{PB} \quad \rightarrow \quad \upsilon_B = \upsilon_A \frac{PB}{PA} = \frac{1}{\sqrt{3}} m \quad \rightarrow \quad a_B^N = \frac{\upsilon_B^2}{r} = \frac{1}{3}\ m \cdot s^{-2}$$

$$\omega_{BA} = \frac{\upsilon_A}{PA} = \frac{1}{\sqrt{3}}\ rad/s \qquad a_{BA}^N = \omega_{BA}{}^2 AB = \left(\frac{1}{\sqrt{3}}\right)^2 2 = \frac{2}{3}\ m \cdot s^{-2}$$

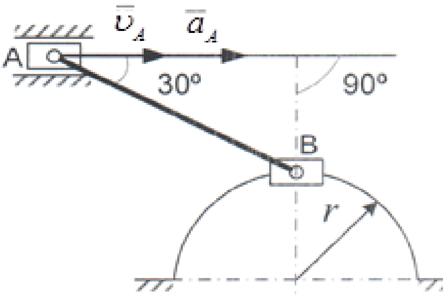
$$\varepsilon_{BA} = \frac{a_A - a_{BA}^N \cos 60° - a_B^N}{AB \cos 30°} = \frac{2 - \frac{2}{3} \cdot \frac{1}{2} - \frac{1}{3}}{2\frac{\sqrt{3}}{2}} = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9} = 0{,}77\ rad \cdot s^{-2}$$

Answer: $\varepsilon_{BA} = \frac{4\sqrt{3}}{9} = 0{,}77 rad \cdot s^{-2}$

TASK FOR INDEPENDENT WORKING

1. Determine the angular acceleration ε_{BA} of the connecting rod AB if $\upsilon_A = 1\ m \cdot s^{-1}$, $a_A = 2\ m \cdot s^{-2}$, $AB = 1\ m$	
2. Determine the angular acceleration ε_{BA} of the connecting rod AB if $\upsilon_A = 1\ m \cdot s^{-1}$, $a_A = 2\ m \cdot s^{-2}$, $AB = 1\ m$	
3. Determine the angular acceleration ε_{BA} of the connecting rod AB if $\upsilon_A = 1\ m \cdot s^{-1}$, $a_A = 2\ m \cdot s^{-2}$, $AB = 1\ m$	
4. Determine the angular acceleration ε_{BA} of the connecting rod AB if $\upsilon_A = 1\ m \cdot s^{-1}$, $a_A = 2\ m \cdot s^{-2}$, $AB = BO = 1\ m$	

5. Determine the angular acceleration ε_{BA} of the connecting rod AB if
$\upsilon_A = 1\ m \cdot s^{-1}$, $a_A = 2\ m \cdot s^{-2}$,
$AB = 2\ m$, $r = 1\ m$



11. COMPLEX MOTION OF A POINT

The purpose of the lesson: acquiring skills from:

- Finding the direction and magnitude of the absolute speed and absolute acceleration of the point in complex motion.
- Determination of the modulus and direction of Coriolis acceleration.

Before completing the task you need to read:

- Methods for solving problems of velocity and acceleration in complex point motion.
- The theorem on the addition of velocities and accelerations in complex motion.
- Rules of vector addition, decomposition and product of vectors.
- The cosine theorem.

Theorem on adding velocities in complex motion

In complex motion, the absolute velocity $\vec{\upsilon}$ of a point is equal to the geometric sum of the transfer $\vec{\upsilon}_E$ and relative $\vec{\upsilon}_R$ velocities:

$$\vec{\upsilon} = \vec{\upsilon}_E + \vec{\upsilon}_R$$

The modulus of absolute velocity, at the angle γ between the vectors $\vec{\upsilon}_R$ *and* $\vec{\upsilon}_E$ is equal to:

$$\upsilon = \sqrt{\upsilon_E^2 + \upsilon_R^2 + 2\upsilon_E \upsilon_R \cos\gamma}$$

Coriolis's theorem on the addition of accelerations

The absolute acceleration $\vec{a}$ of a point in its complex motion is equal to the geometric sum of three accelerations: transfer $\vec{a}_E$, relative $\vec{a}_R$ and Coriolis $\vec{a}_K$:

$$\vec{a} = \vec{a}_R + \vec{a}_E + \vec{a}_K$$

Coriolis acceleration is equal to:

$$\vec{a}_K = 2(\vec{\omega} \times \vec{\upsilon}_R)$$

$$a_K = 2\omega\,\upsilon_R \sin(\vec{\omega}, \vec{\upsilon}_R)$$

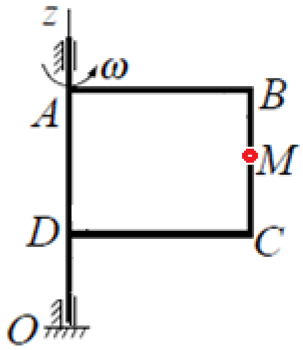
In the case where the transfer and relative motions of the point are carried out along curvilinear trajectories, we have:

$$\vec{a}_E = \vec{a}_E^N + \vec{a}_E^T; \qquad \vec{a}_R = \vec{a}_R^N + \vec{a}_R^T; \qquad \vec{a} = \vec{a}_E^N + \vec{a}_E^T + \vec{a}_R^N + \vec{a}_R^T + \vec{a}_K$$

EXAMPLES

EXAMPLE 1

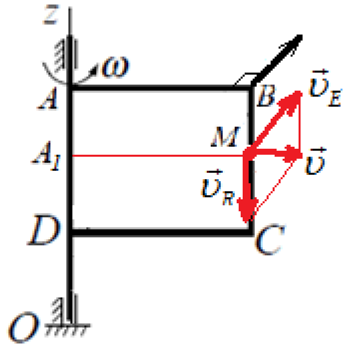
The rectangular plate ABCD rotates around the axis Oz with an angular velocity $\omega = 4t$. On its side BC in the direction from B to C moves the point M with a constant speed of 9 m/s. Determine the modulus of the absolute velocity of the point M at time $t = 3\ s$, if the length AB = 1m.



Solution

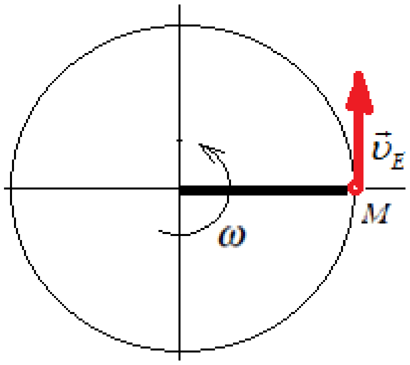
The absolute velocity of the point M is determined by the formula:

$$\vec{\upsilon} = \vec{\upsilon}_E + \vec{\upsilon}_R$$



The transfer velocity $\vec{\upsilon}_E$ of the point M, which belongs to a movable plate rotating relative to the axis Oz with an angular velocity $\omega = 4t$, is

$$\upsilon_E = \omega \cdot A_1M = \omega \cdot AB = 4t \cdot 1 = 4t$$



The transfer velocity $\vec{\upsilon}_E$ is directed perpendicular to the segment A_1M in the course of rotation. At the time $t = 3\ s$:

$$\upsilon_E = 4 \cdot 3 = 12\ m/s$$

The relative velocity $\vec{\upsilon}_R$ of the point M is known from the condition of the task - it is the velocity of the point M along BC:

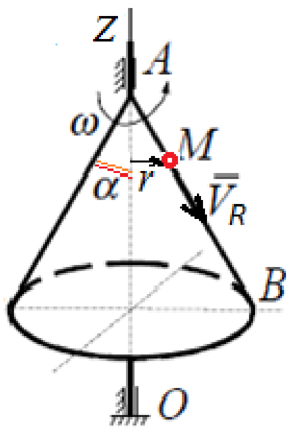
$$\upsilon_R = 9\ m/s$$

The modulus of the absolute velocity of the point M at time $t = 3\ s$ at an angle of 90° between the vectors $\vec{\upsilon}_E$ *and* $\vec{\upsilon}_R$ is determined from the equation

$$\upsilon = \sqrt{\upsilon_E{}^2 + \upsilon_R{}^2 + 2\upsilon_E\upsilon_R \cos 90°} = \sqrt{12^2 + 9^2} = 15\ m/s$$

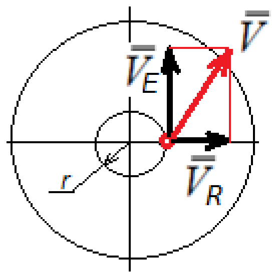
Відповідь: $\upsilon = 15\ m/s$

EXAMPLE 2



The cone rotates around the axis Oz with angular velocity $\omega = 3\ s^{-1}$. On its side AB at a constant speed $\upsilon_R = 4\ m/s$ moves the point M in the direction from A to B. Determine the modulus of the absolute velocity of this point in the position when the distance AM = 2 m, if the angle $\alpha = 30°$

Solution $\quad \vec{\upsilon} = \vec{\upsilon}_E + \vec{\upsilon}_R$



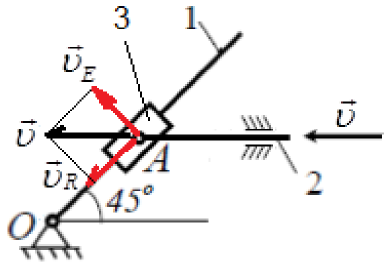
$$\upsilon_E = \omega r = \omega AM \sin\alpha = 3 \cdot 2 \cdot \sin 30° = 3m/s$$

$$\upsilon = \sqrt{\upsilon_E{}^2 + \upsilon_R{}^2 + 2\upsilon_E\upsilon_R \cos 90°} = \sqrt{3^2 + 4^2} = 5m/s$$

Answer: $\upsilon = 5m/s$

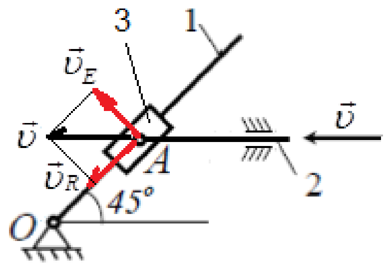
EXAMPLE 3

The rod 2 of the rocker mechanism moves with speed $\upsilon = 1\ m/s$. For a given position of the sliding stone 3 determine the angular velocity of the rocker 1, if the distance OA = 1m.



Solution

The motion of point A of the sliding stone 3 consists of the relative motion along the rocker ($\vec{\upsilon}_R$) and the transfer ($\vec{\upsilon}_E$) - together with the rocker in rotational motion relative to the point O:



$$\vec{\upsilon} = \vec{\upsilon}_E + \vec{\upsilon}_R$$

Relative velocity of point A is equal:

$$\upsilon_R = \upsilon \cos 45° = 0{,}707 m/s$$

Transfer velocity of point A is equal:

$$\upsilon_E = \upsilon \sin 45° = 0{,}707 \text{ m/s}$$

The angular velocity of the rocker 1 relative to the center of rotation O is determined from the equation:

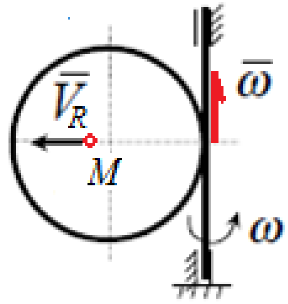
$$\upsilon_E = \omega \cdot OA = 0{,}707 m/s$$

$$\omega = \frac{\upsilon_E}{OA} = \frac{0{,}707}{1} 0{,}707 s^{-1}$$

Answer: $\omega = 0{,}707 s^{-1}$

EXAMPLE 4

The point M moves with relative velocity $\upsilon_R = 4t$ along the diameter of the disk, which rotate around the vertical axis with angular velocity $\omega = 2t$. Determine the modulus and direction of the Coriolis acceleration of the point M at time $t = 2s$.



Solution

Coriolis acceleration is determined by the formula:

$$\vec{a}_K = 2(\vec{\omega} \times \vec{\upsilon}_R)$$

$$a_K = 2\omega\,\upsilon_R \sin(\vec{\omega}, \vec{\upsilon}_R),$$

where $\vec{\omega}$ is the angular velocity of the transfer motion; $\vec{\upsilon}_R$ - speed of relative motion.

The angle between the vectors $\vec{\omega}, \vec{\upsilon}_R$ is 90 degrees, therefore

$$a_K = 2\omega\,\upsilon_R = 2(2t)(4t) = 16t^2$$

If $t = 2\ s$:

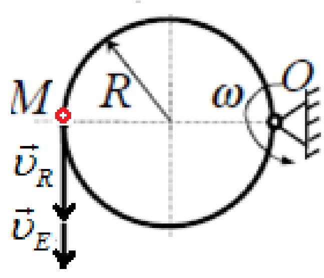
$$a_K = 16(2)^2 = 64\ m/s^2$$

The Coriolis acceleration direction of point M is perpendicular to the figure, is directed at the viewer

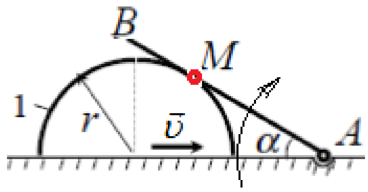
Answer: $a_K = 64\ m/s^2$

TASKS FOR INDEPENDENT DEVELOPMENT

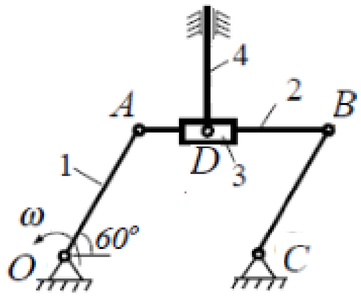
1. Point M moves along the rim of the disk with radius R = 0.06m with velocity $\upsilon_R = 0{,}04\ m/s$. Determine the absolute velocity of point M in the specified position, if the law of disk rotation $\varphi = t$.



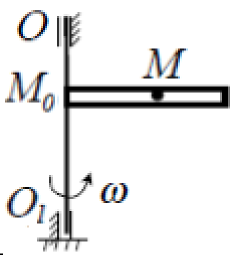
2. The body 1, having the shape of a half-cylinder, when sliding at velocity $\upsilon = 0{,}2\ m/s$ rotates the rod AB, which is hinged at point A. Determine the relative velocity of the point of contact M at the angle of the rod $\alpha = 30°$.



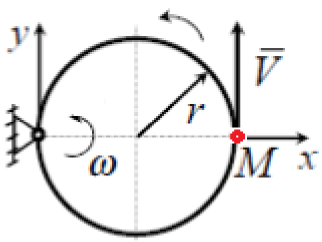
3. The connecting rod 2 of the hinged parallelogram OABC slides on the sleeve 3 and moves along the vertical guide rods 4. For this position of the mechanism determine the velocity of the rod 4, if the velocity of point A of the crank 1 is equal to 2 m/s.



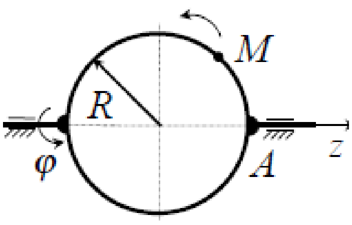
4. The tube rotates around the axis OO_1 with angular velocity $\omega = 1{,}5\ s^{-1}$. The point M moves along the tube according to the law $MM_O = 4t$. Find the modulus and direction of the Coriolis acceleration of the point M.



5. The ring with radius $r = 0{,}5m$ rotates at a constant angular velocity $\omega = 4\ s^{-1}$ in the plane of the figure. The point M moves on the ring at a constant velocity $\upsilon = 2\ m/s$. Determine the modulus of absolute acceleration of the point M at the specified position,



6. The disk rotates around the z axis according to the law $\varphi = 4\sin 0{,}25\pi t$. The point M moves along the rim of the according to the equation $AM = 0{,}25\pi R t^2$. Determine the Coriolis acceleration of the point M at the time $t = 1s$ if the radius $R = 0{,}4\ m$.



12. SPHERICAL AND COMPLEX MOVEMENT OF A SOLID BODY

Brief theoretical information

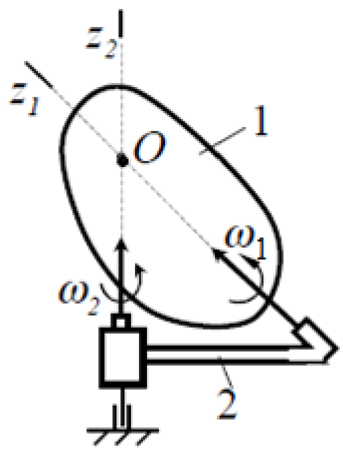
The motion of a body in which one of its points remains stationary is called spherical motion.

Any elementary movement of a body with one fixed point can be obtained by one elementary rotation around some instantaneous axis of rotation that passes through this point.

The motion of a rigid body around a fixed point consists of a set of successive elementary rotations of the body around instantaneous axes of rotation that pass through this fixed point. The position of the instantaneous axis of rotation changes continuously over time, both in the body and in still space.

The angular velocity $\vec{\omega}$ at which the body makes an elementary rotation around the axis of rotation is called the instantaneous angular velocity.

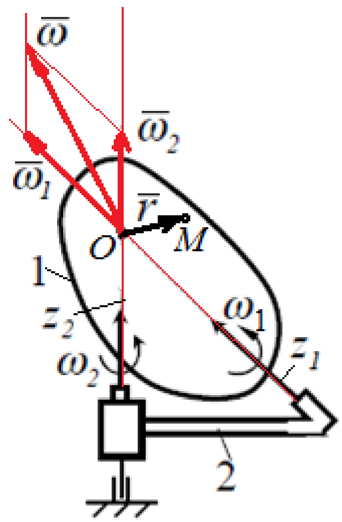
The angular velocity vector $\vec{\omega}$ is directed along the instantaneous axis of rotation in the direction from which the rotation of the body is visible as counterclockwise. The direction of the vector $\vec{\omega}$, as well as the position of the instantaneous axis of rotation, changes over time. The end of the vector $\vec{\omega}$ describes in space the curve - the hodograph of the vector $\vec{\omega}$.



The body 1, shown in the diagram, performs a complex movement. For the relative motion of the body we take its rotation with an angular velocity ω_1 around the axis z_1, which is fixed at the end of the moving part 2. For the transfer motion of the body we take its rotation around the vertical axis z_2, with an angular velocity ω_2. The z_1 and z_2 axes intersect at the O point. The velocity of the O point, which belongs to the two z_1 and z_2 axes at the same time,

will be zero and the resulting body motion (absolute) will be motion around a fixed O point, which is essentially the instantaneous center of velocities. In this case, the body has an angular velocity ω directed along the instantaneous axis of rotation that passes through the point O.

To determine ω we find the velocity of an arbitrary point M of the body, the radius vector of which $\vec{r} = \overrightarrow{OM}$. In relative motion (rotation around the z_1 axis), the point M will have a velocity



$$\vec{\upsilon}_R = \vec{\omega}_1 \times \vec{r}$$

In transfer motion (rotation around the z_2 axis):

$$\vec{\upsilon}_E = \vec{\omega}_2 \times \vec{r}$$

Absolute velocity of the point M:

$$\vec{\upsilon} = \vec{\upsilon}_R + \vec{\upsilon}_E = (\vec{\omega}_1 \times \vec{r}) + (\vec{\omega}_2 \times \vec{r}) = (\vec{\omega}_1 + \vec{\omega}_2) \times \vec{r}$$

On the other hand

$$\vec{\upsilon} = \vec{\omega} \times \vec{r}$$

As a result, we have:

$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2$$

If the body is involved in instantaneous rotations around several axes intersecting at the point O, then:

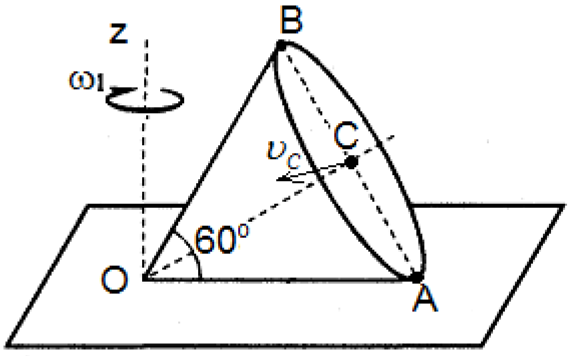
$$\vec{\omega} = \sum \vec{\omega}_i$$

The set of rotations around the intersecting axes is equivalent to one rotation around some instantaneous axis of rotation with an angular velocity equal to the vector sum of the angular velocities of the component rotations.

EXAMPLES

EXAMPLE

A cone with an angle at the vertex $2\alpha = 60^{o}$ and a radius of the base plane of 20 cm rolls on a stationary horizontal plane without slipping. The velocity of the center of the base is constant, $\upsilon_C = 60$ cm / s.



Determine the values:

1) the angular velocity of rotation of the cone $\vec{\omega}$;

2) the angular velocity of rotation of the cone around the z axis $\vec{\omega}_1$;

3) the angular acceleration of the cone $\vec{\varepsilon}$;

4) the velocities of points A and B of the base of the cone υ_A and υ_B;

5) acceleration of points A and B of the base of the cone a_A *and* a_B

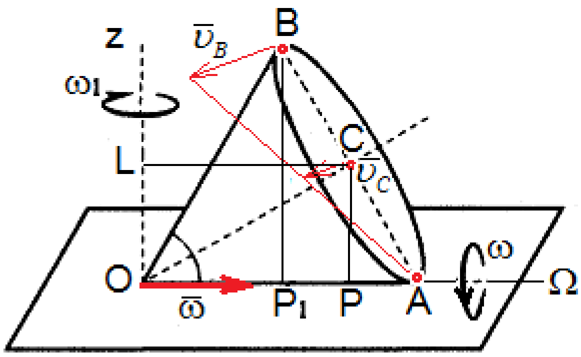
Solution

1) The motion of a cone is spherical because its vertex O remains stationary. Since the cone rolls on a stationary plane, the generating OA, which it touches the plane, is an instantaneous axis, all points of which have zero velocity. The velocity υ_C of point C is the rotational speed around the instantaneous axis. Knowing the velocity of point C, determine the angular velocity of rotation of the cone around the instantaneous axis:

$$\upsilon_C = \omega \cdot CP$$

The distance CP to the instantaneous axis OA found from the triangle CPA:

$$CP = CA \sin 60° = R \sin 60° = 20\frac{\sqrt{3}}{2} = 10\sqrt{3}\ cm$$



The angular velocity of rotation of the cone around the instantaneous axis is equal to:

$$\omega = \frac{v_C}{CP} = \frac{60}{10\sqrt{3}} = 2\sqrt{3}\ rad/s$$

Regarding the direction of the vector $\vec{\upsilon}_C$, we plot the vector $\vec{\omega}$ from the point O along the instantaneous axis so that looking towards it, we see the rotation of the cone counterclockwise.

2) The angular velocity ω_1 of rotation of the cone around the z axis is found as the angular velocity of rotation of the cone axis OC around the z axis. To do this, determine the distance from point C to the z axis:

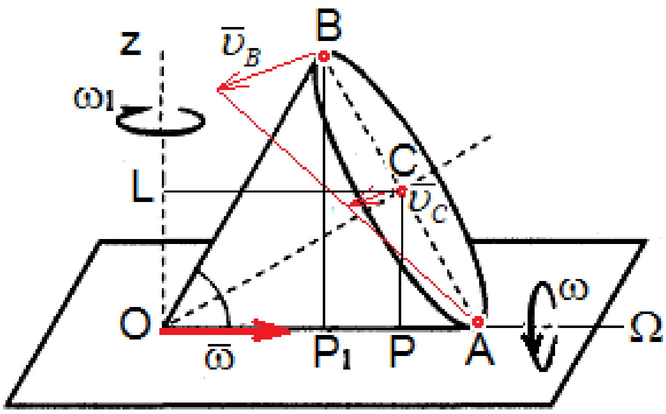
$$CL = OP = OA - PA = 40 - 10 = 30\ cm$$

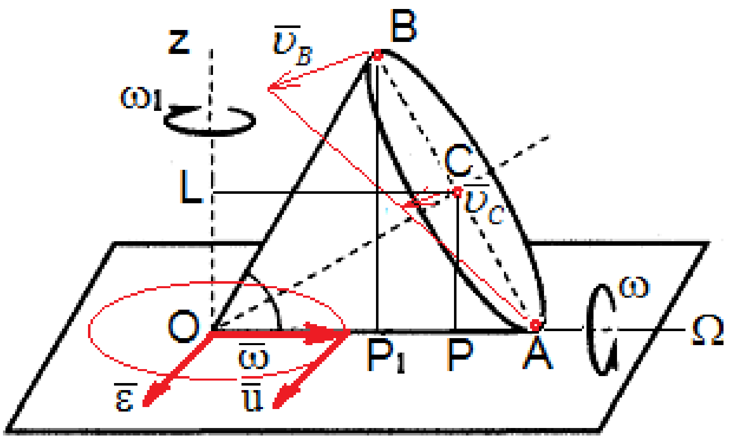
The angular velocity of rotation of the cone around the z axis is equal to:

$$\omega_1 = \frac{v_C}{CL} = \frac{60}{30} = 2\ rad/s$$

3) To determine the angular acceleration $\vec{\varepsilon}$, you need to build a hodograph of the angular velocity $\vec{\omega}$. As the cone moves, the vector $\vec{\omega}$ moves,

rotating around the z axis, while the modulus of angular velocity $\vec{\omega}$ does not change, therefore, the end of the vector describes a circle in the horizontal plane.





The modulus of angular acceleration $\vec{\varepsilon}$ is equal to the rotational velocity $\vec{u}$ of end of the vector $\vec{\omega}$ around z axis:

$$\varepsilon = u = \omega\omega_1 = 2\sqrt{3} \cdot 2 = 4\sqrt{3}\ rad/s^2$$

The angular acceleration vector $\vec{\varepsilon}$ is plotted from point O in the direction of velocity $\vec{u}$ perpendicular to $\vec{\omega}$. The vector $\vec{\varepsilon}$ is perpendicular to the plane of parallelogram of angular velocities.

4) Determine the velocities of points A and B. Point A lies on the instantaneous axis of rotation, so its speed is zero:

$$\upsilon_A = 0$$

The velocity of point B is:

$$\upsilon_B = 2\upsilon_C = 2 \cdot 60 = 120\ cm/s$$

The velocity vector is directed perpendicular to the plane z$O\Omega$.

5) The acceleration $\vec{a}_B$ of point B is equal to the geometric sum of normal acceleration $\vec{a}_B^N$ relative to the axis of angular velocity $\vec{\omega}$ and tangential acceleration $\vec{a}_B^T$ relative to the axis of angular acceleration $\vec{\varepsilon}$:

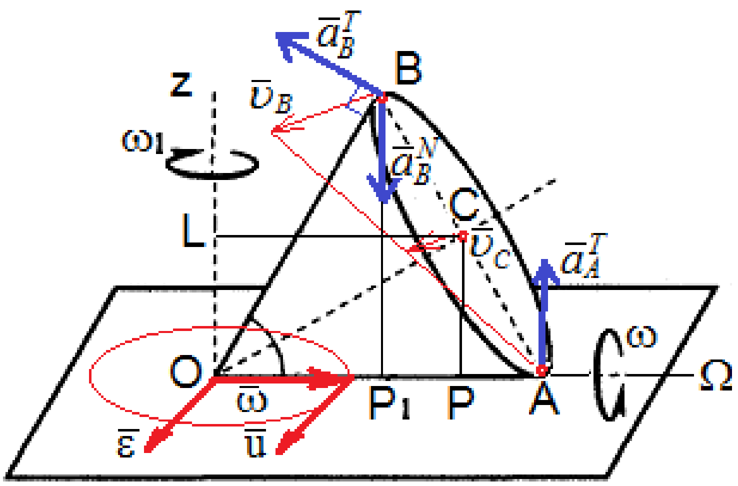
$$\vec{a}_B = \vec{a}_B^N + \vec{a}_B^T$$

The acceleration $\vec{a}_B^N$ is directed along BP_1, ie perpendicular to the axis of angular velocity $\vec{\omega}$:

$$a_B^N = \omega^2 BP_1 = \left(2\sqrt{3}\right)^2 20\sqrt{3} = 240\sqrt{3} = 415{,}7 cm/s$$

The acceleration $\vec{a}_B^T$ is directed perpendicular to BO in a plane perpendicular to the axis of angular acceleration $\vec{\varepsilon}$ so that, looking in the opposite direction of $\vec{\varepsilon}$, you can see the rotation of the cone counterclockwise:

$$a_B^T = \varepsilon OB = 4\sqrt{3} \cdot 40 = 160\sqrt{3} = 277{,}1 cm/s^2$$



Define the modulus $\vec{a}_B$ as the length of the diagonal of the parallelogram:

$$a_B = \sqrt{\left(a_B^N\right)^2 + \left(a_B^T\right)^2 + 2a_B^N a_B^T \cos\left(90+30\right)^\circ}$$

$$a_B = 100\sqrt{\left(2,4\sqrt{3}\right)^2 + \left(1,6\sqrt{3}\right)^2 + 2\left(2,4\sqrt{3}\right)\left(1,6\sqrt{3}\right) cos\, 1\,20^\circ}$$

$$= 366,7 cm/s^2$$

The normal acceleration of point A, which lies on the instantaneous axis of rotation, is zero:

$$a_A^N = 0$$

The tangential acceleration of point A relative to the axis of angular acceleration $\vec{\varepsilon}$ is equal to:

$$a_A^T = \varepsilon OA = 4\sqrt{3} \cdot 40 = 160\sqrt{3} = 277,1 cm/s^2$$

The vector $\vec{a}_A^T$ is directed perpendicular to AO in the plane z$O\Omega$.

$$a_A = a_A^T = 277,1 cm/s^2$$

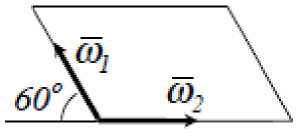
Answer:

	Angular speed of rotation of the cone	$\omega = 2\sqrt{3} = 3,46\ rad/s$
	The angular velocity of rotation of the cone around the z axis	$\omega_1 = 2\ rsd/s$
	Angular acceleration of the cone	$\varepsilon = 4\sqrt{3} = 6,93\ rad/s^2$
	The velocity of point A	$\upsilon_A = 0$
	The velocity of point B	$\upsilon_B = 120\ cm/s$
	Acceleration of point A	$a_A = 277,1\ cm/s^2$
	Acceleration of point B	$\boldsymbol{a_B = 366,7} cm/s^2$

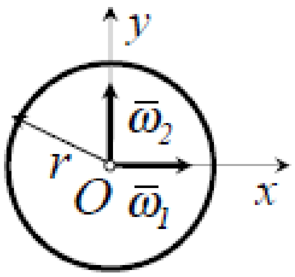
TASKS FOR INDEPENDENT DEVELOPMENT

1. The body simultaneously participates in two rotational motions with angular velocities $\vec{\omega}_1 = 2\vec{i} + 5\vec{j}$ and $\vec{\omega}_2 = 4\vec{i} + 3\vec{j}$. Determine the modulus of the absolute angular velocity of the body

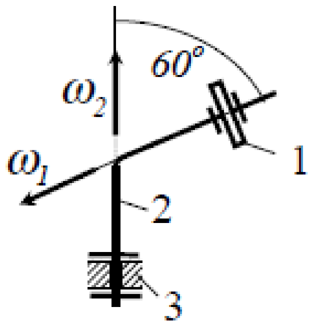
2. The plate is involved in two rotational motions with angular velocities $\omega_1 = 2\ rad/s$ and $\omega_2 = 4\ rad/s$. What angle forms the vector of absolute angular velocity with a vector $\vec{\omega}_2$?



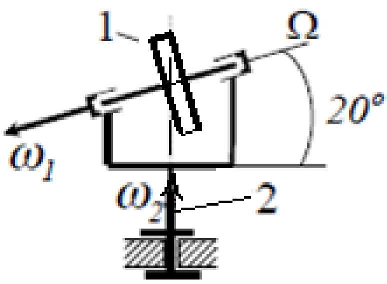
3. The wheel of radius $r = 0{,}5m$ participates simultaneously in two rotational movements with velocities $\omega_1 = \omega_2 = 2\ rad/s$. Determine the modulus of speed of the wheel point for which this module has the maximum value.



4. Wheel 1 rotates on a curved axis 2 with angular velocity $\omega_1 = 4\ rad/s$. The speed of rotation of the axis 2 in the bearing 3 is equal to $\omega_2 = 4 rad/s$. Determine the modulus of the absolute angular velocity of the wheel.



5. Wheel 1 rotates about an axis Ω which, rotates about a vertical axis. Angular velocity $\omega_1 = 20\ rad/s$, $\omega_2 = 10\ rad/s$. Determine the modulus of the absolute angular velocity of the wheel 1.



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