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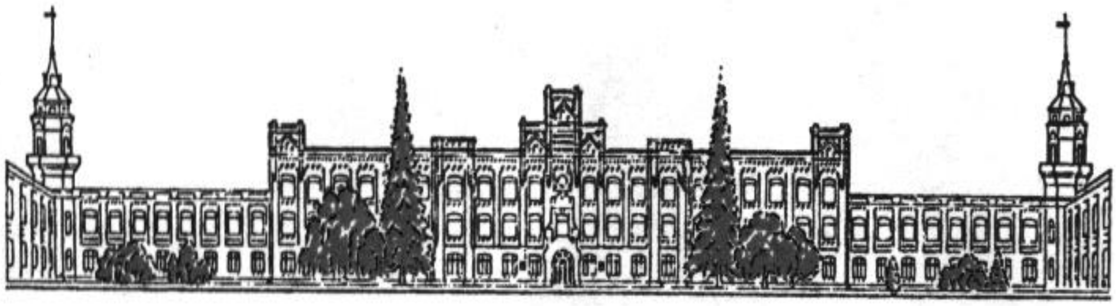

HIGHER MATHEMATICS

ELEMENTS OF LINEAR ALGEBRA

Practice exercises collection

Recommended by the Methodological Council
of the Igor Sikorsky Kyiv Polytechnic Institute
as a study aid for bachelor's degree applicants
on the technical specialties

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The practice book offers additional individual exercises for university students studying Elements of Linear Algebra in the course of Higher Mathematics of Igor Sikorsky KPI. The book contains 30 different variants and each variant consists of 8 exercises (16 tasks). Students master the material being studied and consolidate the acquired knowledge by solving such individual tasks.

The practice book can be recommended as an individual work on Elements of Linear Algebra for first-year students of technical specialties.

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CONTENT

INTRODUCTION

The Elements of Linear Algebra section is included in the course of Higher Mathematics for engineering students of Igor Sikorsky KPI. An important factor in the successful assimilation of the educational material by the students is solving practical tasks on their own.

The practice book offers a systematized set of exercises that students of technical specialties should be able to solve when studying Elements of Linear Algebra. The book contains 30 different variants and each variant consists of 8 exercises (16 tasks).

This practice book helps students to develop practical skills in solving basic exercises: to calculate determinants of different order using such methods as the Rule of Sarrus, expanding determinant into elements of some row or some column and the upper-triangulation; matrix operations, including finding the inverse matrix and solving matrix equations; to check the consistency of systems of linear algebraic equations and find their general solutions.

GENERAL RECOMMENDATIONS

The practice book is designed to control and improve the knowledge of university students in the study of Elements of Linear Algebra in the course of Higher Mathematics. The main goal is to develop and consolidate the skills of independent work of students in the study of educational material.

In order to successfully complete the exercises, students need to thoroughly study the lecture material and analyze the examples solved in practical classes. Only after that students can start solving their individual tasks.

Students have to adhere to the following requirements:

1. The number of the variant of the individual exercises corresponds to the ordinal number of the student in the list of the study group;
2. Individual work is written in a separate notebook, which should contain:
   - the title page;
   - the results table;
   - solved exercises (the solution of each exercise starts from a new page).
3. Before solving each exercise, the condition and all specific data for the corresponding variant are completely rewritten.
4. The solution of each task must contain detailed explanations and necessary formulas.
5. Completed work must be handed over to the teacher for verification within the prescribed time limit.

Students who do not submit their completed individual work on time will not be allowed to take the exam.

Elements of Linear Algebra

Variant №1

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 2 & -5 \\ 3 & -1 \end{vmatrix}$; **b)** $\begin{vmatrix} 2 & 0 & -2 \\ -2 & 1 & 3 \\ 3 & -2 & 1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 1 & -1 & 0 \\ -3 & -1 & 5 & -2 \\ 0 & -2 & -2 & 4 \\ 0 & 1 & -2 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 0 & 1 \\ -1 & 4 \\ -3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 & 2 \\ 3 & -1 & 0 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -3 \\ 4 & 1 \end{pmatrix}$, $C = \begin{pmatrix} -4 & -1 \\ 3 & -2 \end{pmatrix}$, $D = \begin{pmatrix} -11 & 0 \\ 0 & -11 \end{pmatrix}$:

a) calculate the expression $G = 3B^2 - AC - 2D^T + A - C$;

b) determine the value of the polynomial $f(x) = x^2 - 6x + 27$ evaluated at the matrix A.

Exercise 4. For the matrices $A,\ B,\ C,\ D$ given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $A \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 1 & 5 \\ 2 & 1 & 2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - 2x_2 + x_3 = -2, \\ x_1 - x_2 + 2x_3 = -2, \\ 2x_1 + x_2 + x_3 = 2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 + 3x_2 - 2x_3 = 0, \\ 7x_1 + 5x_2 + 2x_3 = 0, \\ 2x_1 + 6x_2 - 4x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} -x_1 + x_2 - x_3 + x_4 = 2, \\ x_1 - 3x_2 + x_3 + x_4 = 2, \\ 2x_1 - 2x_2 + x_3 - x_4 = -3, \\ 3x_1 + 2x_2 - x_3 - x_4 = 0; \end{cases}$ **b)** $\begin{cases} x_1 - 2x_2 + 3x_3 + 4x_4 = 1, \\ 4x_1 - 7x_2 + 2x_3 + x_4 = 3, \\ 3x_1 - 5x_2 - x_3 - 3x_4 = 2. \end{cases}$

Elements of Linear Algebra

Variant №2

Exercise 1. Calculate determinants.
Calculate the second determinant (b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant (c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 5 & 7 \\ -2 & 3 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 2 & 4 \\ 3 & -5 & 1 \\ 2 & 7 & 8 \end{vmatrix}$; **c)** $\begin{vmatrix} -1 & 0 & 0 & 1 \\ 2 & -2 & 1 & 0 \\ 3 & 2 & -2 & -3 \\ 1 & 2 & 2 & -3 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 \\ 0 & 2 \\ 3 & -4 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix}$, $B = \begin{pmatrix} -5 & -2 \\ 5 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 5 & 2 \end{pmatrix}$, $D = \begin{pmatrix} 15 & 8 \\ -20 & -9 \end{pmatrix}$:

a) calculate the expression $G = 2AC - B^2 - D + C^T - 2E$;

b) determine the value of the polynomial $f(x) = x^2 + 6x + 22$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $A \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 4 & 1 & 5 \\ -1 & 1 & -6 \\ 3 & 0 & 4 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + 4x_2 + x_3 = 5, \\ x_1 - x_2 - x_3 = -2, \\ 2x_1 + x_2 - 4x_3 = -3. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 3x_1 - 6x_2 + x_3 = 0, \\ x_1 - 2x_2 + 5x_3 = 0, \\ -2x_1 + 4x_2 - 10x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + 2x_2 + x_3 + 3x_4 = 5, \\ 2x_1 - x_2 + x_3 - x_4 = -6, \\ 3x_1 + x_2 + x_3 + x_4 = -1, \\ 4x_1 + x_2 + x_3 + 2x_4 = -1; \end{cases}$ **b)** $\begin{cases} x_1 + x_2 - 3x_3 - 4x_4 = 1, \\ 4x_1 + 5x_2 - 2x_3 - x_4 = 3, \\ 3x_1 + 4x_2 + x_3 + 3x_4 = 2. \end{cases}$

Elements of Linear Algebra

Variant №3

Exercise 1. Calculate determinants.
Calculate the second determinant (b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant (c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 2 & -8 \\ 4 & 1 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 1 \\ 2 & 3 & 2 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 2 & 2 \\ 0 & -2 & 2 & 1 \\ -3 & 1 & -2 & -3 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -5 & 2 \\ 3 & -4 \\ 1 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 4 & 0 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -3 \\ -3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} -3 & 1 \\ -7 & 4 \end{pmatrix}$, $D = \begin{pmatrix} 25 & -15 \\ -15 & 10 \end{pmatrix}$:

a) calculate the expression $G = B^2 - AC + D - 5B^T + 13E$;

b) determine the value of the polynomial $f(x) = x^2 - 4x + 13$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $A \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 1 & 5 \\ -1 & 1 & -1 \\ 3 & 1 & 4 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - x_2 - x_3 = -3, \\ x_1 + 4x_2 + 5x_3 = 2, \\ 2x_1 + 4x_2 + x_3 = 0. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 6x_1 - 2x_2 + 10x_3 = 0, \\ 2x_1 + x_2 - 10x_3 = 0, \\ 3x_1 - x_2 + 5x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 - x_2 + x_3 - x_4 = 3, \\ x_1 + 3x_2 + x_3 - x_4 = -1, \\ 2x_1 + 3x_2 - x_3 + 3x_4 = 2, \\ 3x_1 + 2x_2 - 2x_3 + x_4 = -3; \end{cases}$ **b)** $\begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 6, \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 = 4, \\ 9x_1 + 4x_2 + x_3 + 7x_4 = 2. \end{cases}$

Elements of Linear Algebra

Variant №4

Exercise 1. Calculate determinants.

Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.

Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 4 & 7 \\ 1 & -3 \end{vmatrix}$; **b)** $\begin{vmatrix} 2 & 0 & 1 \\ 1 & -4 & -1 \\ -1 & 8 & 3 \end{vmatrix}$; **c)** $\begin{vmatrix} -1 & -1 & 0 & 0 \\ 2 & 2 & 1 & 1 \\ 3 & 2 & -1 & -3 \\ 1 & 0 & 2 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 0 & -2 & -5 \\ 1 & 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ 1 & 5 \\ 0 & -3 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -2 & 1 \\ -2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 2 \\ -6 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 3 & 0 \\ 3 & -2 \end{pmatrix}$, $D = \begin{pmatrix} -3 & -2 \\ 6 & -8 \end{pmatrix}$:

a) calculate the expression $G = 2AC - B^2 - D + C^T + B$;

b) determine the value of the polynomial $f(x) = x^2 - 2x + 13$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} 5x_1 - 3x_2 + 2x_3 = 15, \\ x_1 + 2x_2 - x_3 = -1, \\ 2x_1 + x_2 + x_3 = 4. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 + 2x_2 - x_3 = 0, \\ 3x_1 + 4x_2 - x_3 = 0, \\ 3x_1 + 6x_2 - 3x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + x_2 - x_3 - x_4 = -1, \\ 2x_1 - x_2 + 3x_3 - x_4 = -8, \\ 2x_1 + x_2 + 2x_3 + x_4 = 2, \\ 3x_1 + x_2 + 2x_3 + x_4 = 3; \end{cases}$ **b)** $\begin{cases} 9x_1 - 3x_2 + 5x_3 + 6x_4 = 4, \\ 6x_1 - 2x_2 + 3x_3 + 4x_4 = 5, \\ 3x_1 - x_2 + 3x_3 + 14x_4 = -8. \end{cases}$

Elements of Linear Algebra

Variant №5

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 3 & 7 \\ -5 & -8 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 5 & 1 \\ 0 & 3 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 2 & 1 & 1 & 0 \\ -2 & -1 & 2 & 2 \\ 0 & -1 & 2 & -3 \\ 0 & 1 & -2 & 4 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & 0 \\ 1 & -2 \\ 2 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 & 2 \\ 4 & 0 & 1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -1 & 1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 \\ -4 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 4 & -1 \\ -3 & -1 \end{pmatrix}$, $D = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}$:

a) calculate the expression $G = 2AC - D - B^2 + A + C + 17E$;

b) determine the value of the polynomial $f(x) = x^2 - 3x + 13$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & -3 & 3 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + x_2 - 2x_3 = 6, \\ 2x_1 + 3x_2 - 7x_3 = 16, \\ 5x_1 + 2x_2 + x_3 = 16. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 21x_1 + 6x_2 + 3x_3 = 0, \\ 7x_1 + 2x_2 + x_3 = 0, \\ 5x_1 - x_2 + 8x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + x_2 + x_3 + x_4 = 6, \\ -3x_2 + x_3 - x_4 = -1, \\ 2x_1 - x_2 - x_3 - x_4 = -3, \\ -3x_1 - 2x_2 + x_3 + x_4 = 5; \end{cases}$ **b)** $\begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 = 3, \\ 6x_1 + 8x_2 + 2x_3 + 5x_4 = 7, \\ 9x_1 + 12x_2 + 3x_3 + 10x_4 = 13. \end{cases}$

Elements of Linear Algebra

Variant №6

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -5 & -8 \\ 2 & 3 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & -3 & 3 \\ 1 & 2 & -1 \\ 2 & -1 & -2 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & -1 & 0 \\ 2 & 1 & -2 & 2 \\ -1 & -1 & 1 & -3 \\ -2 & 4 & 0 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 1 & 3 & -5 \\ -3 & 0 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 8 \\ 2 & -1 \\ 0 & 1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 4 & 3 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -4 \\ -2 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 3 & 5 \\ -1 & -4 \end{pmatrix}$, $D = \begin{pmatrix} 9 & 8 \\ 4 & 9 \end{pmatrix}$:

a) calculate the expression $G = 3B^2 - AC - 2D + B^T + E$;

b) determine the value of the polynomial $f(x) = x^2 - 3x + 14$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 5 & 0 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + 2x_2 + x_3 = 4, \\ 3x_1 - 5x_2 + 3x_3 = 1, \\ 2x_1 + 7x_2 - x_3 = 8. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 3x_1 + 3x_2 - 21x_3 = 0, \\ x_1 + x_2 - 7x_3 = 0, \\ 2x_1 + 3x_2 + 4x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 - x_2 + x_3 + x_4 = 3, \\ x_1 + 2x_2 - x_3 + x_4 = 2, \\ -2x_1 - 2x_2 + x_3 + x_4 = 1, \\ -3x_1 - x_2 - 2x_3 - x_4 = -2; \end{cases}$ **b)** $\begin{cases} 3x_1 - 2x_2 + 5x_3 + 4x_4 = 2, \\ 6x_1 - 4x_2 + 4x_3 + 3x_4 = 3, \\ 9x_1 - 6x_2 + 3x_3 + 2x_4 = 4. \end{cases}$

Elements of Linear Algebra

Variant №7

Exercise 1. Calculate determinants.

Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.

Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -3 & 5 \\ 7 & 1 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 2 & 2 \\ 10 & 2 & 12 \\ 4 & -2 & 4 \end{vmatrix}$; **c)** $\begin{vmatrix} 0 & 1 & -1 & 3 \\ 1 & -2 & 3 & 0 \\ 2 & -3 & 2 & 4 \\ 0 & 2 & -5 & 3 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -5 & 2 \\ 7 & -1 \\ 5 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 2 & -1 \\ 3 & 3 & 0 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 \\ -1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 1 \\ 0 & -6 \end{pmatrix}$, $D = \begin{pmatrix} -4 & 10 \\ -2 & -4 \end{pmatrix}$:

a) calculate the expression $G = 2AC - B^2 - D + 3B^T - 3E$;

b) determine the value of the polynomial $f(x) = x^2 - 5x + 28$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + x_2 - x_3 = 0, \\ x_1 - 4x_2 + x_3 = 1, \\ 2x_1 + 3x_2 - x_3 = 4. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 2x_1 + 3x_2 - x_3 = 0, \\ 10x_1 + 15x_2 - 5x_3 = 0, \\ 6x_1 + 2x_2 - 3x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + x_2 + 2x_3 - x_4 = 3, \\ 2x_1 + x_2 + x_3 - x_4 = 1, \\ 2x_1 - x_2 + 2x_4 = -2, \\ -2x_1 - x_2 + x_3 + x_4 = 1; \end{cases}$ **b)** $\begin{cases} 6x_1 - 3x_2 + x_3 - 4x_4 = 7, \\ 2x_1 - x_2 + 3x_3 - 7x_4 = 5, \\ 4x_1 - 2x_2 + 14x_3 - 31x_4 = 18. \end{cases}$

Elements of Linear Algebra

Variant №8

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 3 & 7 \\ 2 & -8 \end{vmatrix}$; **b)** $\begin{vmatrix} -2 & 1 & 3 \\ 1 & 3 & -2 \\ 0 & -2 & 2 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & -2 & 0 & 0 \\ 3 & -7 & 1 & 2 \\ 2 & 0 & -1 & -3 \\ -1 & 3 & -1 & -3 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 3 & -4 & 6 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 5 \\ 0 & -3 \\ 1 & 2 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -1 \\ -5 & 1 \end{pmatrix}$, $C = \begin{pmatrix} -4 & 2 \\ 1 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$:

a) calculate the expression $G = 3B^2 - AC - 2D^T + 23E$;

b) determine the value of the polynomial $f(x) = x^2 - 3x + 17$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 3 & 1 \\ 2 & -1 & 0 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - x_2 - x_3 = 0, \\ x_1 - 2x_2 + 3x_3 = 2, \\ x_1 - x_2 + 2x_3 = 3. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 4x_1 + 5x_2 + 3x_3 = 0, \\ x_1 - 3x_2 + 5x_3 = 0, \\ 3x_1 - 9x_2 + 15x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + x_2 + x_3 + x_4 = 4, \\ x_1 - 3x_2 - x_3 + x_4 = 4, \\ -x_1 + x_2 + 2x_3 - x_4 = 0, \\ 2x_1 + 2x_2 + x_3 + x_4 = 2; \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 + x_3 + x_4 = 1, \\ x_1 + 2x_2 - x_3 + 4x_4 = 2, \\ x_1 + 7x_2 - 4x_3 + 11x_4 = 5. \end{cases}$

Elements of Linear Algebra

Variant №9

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 2 & 3 \\ -6 & 7 \end{vmatrix}$; **b)** $\begin{vmatrix} 5 & -3 & 3 \\ 3 & -2 & 2 \\ 3 & -1 & 7 \end{vmatrix}$; **c)** $\begin{vmatrix} -1 & 1 & -1 & 2 \\ 1 & -2 & 1 & 0 \\ 0 & -3 & 2 & 4 \\ 0 & 2 & -1 & -2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 8 & -1 \\ 2 & 1 \\ 7 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -2 & 3 \\ 0 & 1 & -1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -3 \\ 2 & -5 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 5 \\ -3 & 2 \end{pmatrix}$, $D = \begin{pmatrix} -5 & 18 \\ -12 & 19 \end{pmatrix}$:

a) calculate the expression $G = 2D - AC - B^2 + C^T + 2E$;

b) determine the value of the polynomial $f(x) = x^2 - 6x + 35$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 1 & 2 \\ -1 & 1 & 1 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + x_2 - x_3 = 0, \\ 2x_1 - x_2 + x_3 = 6, \\ 3x_1 - 3x_2 + x_3 = 4. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 6x_1 - 9x_2 + 6x_3 = 0, \\ 7x_1 + 2x_2 - 3x_3 = 0, \\ 2x_1 - 3x_2 + 2x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 - x_2 - x_3 - x_4 = -3, \\ -x_1 + 2x_2 + x_3 + x_4 = 4, \\ 2x_1 - x_3 - 2x_4 = -1, \\ -2x_1 + x_2 + x_3 + x_4 = 4; \end{cases}$ **b)** $\begin{cases} 2x_1 - 3x_2 + 5x_3 + 7x_4 = 1, \\ 4x_1 - 6x_2 + 2x_3 + 3x_4 = 2, \\ 2x_1 - 3x_2 - 11x_3 - 15x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №10

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 3 & 7 \\ 7 & 8 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 0 & 1 \\ 1 & 5 & 2 \\ -1 & 3 & 0 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & 1 & 0 \\ 3 & 1 & 1 & 2 \\ 2 & 0 & -1 & 3 \\ -1 & 3 & -1 & -3 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 6 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -5 & 2 \\ 2 & -1 \\ 3 & 0 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -2 \\ -3 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 4 & 3 \\ -3 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 7 & 4 \\ 6 & 7 \end{pmatrix}$:

a) calculate the expression $G = AC - B^2 + D^T - 7E$;

b) determine the value of the polynomial $f(x) = x^2 - 3x + 30$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 3 & 5 & -2 \\ 1 & -3 & 2 \\ 6 & 7 & -3 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - x_2 + x_3 = 0, \\ 7x_1 + 3x_2 + 7x_3 = 10, \\ 2x_1 + x_2 + x_3 = 2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 - x_2 + 3x_3 = 0, \\ 3x_1 + 5x_2 + x_3 = 0, \\ 9x_1 + 15x_2 + 3x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 - 2x_2 + 3x_3 + x_4 = 0, \\ x_1 + x_2 + x_3 + x_4 = 4, \\ -x_1 - 3x_2 + x_3 - 3x_4 = -4, \\ 2x_1 - x_2 + x_3 = 3; \end{cases}$ **b)** $\begin{cases} x_1 + 3x_2 + x_3 - x_4 = 5, \\ 2x_1 + 5x_2 - x_3 - 4x_4 = 9, \\ x_1 + 2x_2 - 2x_3 - 3x_4 = 4. \end{cases}$

Elements of Linear Algebra

Variant №11

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -5 & -3 \\ -2 & 4 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 2 & -1 \\ 1 & -3 & 3 \\ 2 & -1 & -2 \end{vmatrix}$; **c)** $\begin{vmatrix} 0 & 1 & -1 & 2 \\ 1 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ -3 & 2 & -1 & -2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -6 & 2 \\ -1 & 0 \\ 6 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 & 0 \\ 3 & 1 & -1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -1 \\ -1 & -3 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 1 & 4 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 3 \\ 3 & 10 \end{pmatrix}$:

a) calculate the expression $G = 3AC - B^2 - 2D + B^T + C$;

b) determine the value of the polynomial $f(x) = x^2 - 4x + 27$ evaluated at the matrix A.

Exercise 4. For the matrices $A,\ B,\ C,\ D$ given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $À \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - x_2 + x_3 = 0, \\ 2x_1 + x_2 - 2x_3 = 3, \\ 4x_1 - x_2 - x_3 = 0. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 - x_2 + 3x_3 = 0, \\ 3x_1 - 3x_2 + 9x_3 = 0, \\ x_1 + x_2 - 15x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + x_2 + x_3 + x_4 = 2, \\ x_1 - 3x_2 - 3x_3 + x_4 = 2, \\ -x_1 + 2x_2 - 2x_3 + x_4 = 0, \\ x_1 + x_3 + 2x_4 = 0; \end{cases}$ **b)** $\begin{cases} 3x_1 + 7x_2 - 2x_3 - x_4 = 4, \\ 2x_1 + 5x_2 + x_3 + 3x_4 = 3, \\ x_1 + 2x_2 - 3x_3 - 4x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №12

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 2 & 13 \\ 1 & -7 \end{vmatrix}$; **b)** $\begin{vmatrix} -3 & 5 & 3 \\ -2 & 3 & 2 \\ -1 & 3 & 7 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & 1 & 0 \\ -1 & 3 & 2 & -3 \\ 1 & 1 & -1 & 2 \\ -2 & 2 & 0 & -1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & 1 & -1 \\ -7 & 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 9 & 2 \\ 2 & 1 \\ -4 & 1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -3 & -1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 2 \\ 3 & -1 \end{pmatrix}$, $C = \begin{pmatrix} -6 & 3 \\ 3 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 15 & -8 \\ -12 & 7 \end{pmatrix}$:

a) calculate the expression $G = 2B^2 - AC - D + C^T - B$;

b) determine the value of the polynomial $f(x) = x^2 + x + 24$ evaluated at the matrix A.

Exercise 4. For the matrices $A,\ B,\ C,\ D$ given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -3 & 2 \\ 3 & -4 & 2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} 3x_1 + x_2 - x_3 = 2, \\ x_1 + x_2 - x_3 = 0, \\ 2x_1 - 2x_2 + x_3 = 3. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 6x_1 + x_2 + 19x_3 = 0, \\ 5x_1 - x_2 + 3x_3 = 0, \\ 15x_1 - 3x_2 + 9x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + x_2 + x_3 + x_4 = 3, \\ x_1 + x_2 + x_3 + 2x_4 = -2, \\ -x_1 - x_2 + x_3 - 2x_4 = 2, \\ x_1 - x_2 + x_3 = 2; \end{cases}$ **b)** $\begin{cases} x_1 + 4x_2 - 3x_3 - x_4 = 2, \\ x_1 + 3x_2 - x_3 - 2x_4 = 1, \\ 2x_1 + 7x_2 - 4x_3 - 3x_4 = 3. \end{cases}$

Elements of Linear Algebra

Variant №13

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -11 & 5 \\ -4 & 4 \end{vmatrix}$; **b)** $\begin{vmatrix} 3 & -2 & 1 \\ -2 & 1 & 3 \\ 2 & 0 & -2 \end{vmatrix}$; **c)** $\begin{vmatrix} 0 & 1 & -1 & 3 \\ 1 & 2 & 1 & 1 \\ -3 & -3 & -1 & 0 \\ 2 & 2 & -1 & 0 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 & -2 \\ -2 & 0 & 1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 3 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -2 \\ -4 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 \\ -3 & 3 \end{pmatrix}$, $D = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}$:

a) calculate the expression $G = B^2 - AC + D^T + 19E$;

b) determine the value of the polynomial $f(x) = x^2 - 4x + 37$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 3 & 1 & -2 \\ 4 & 2 & 5 \\ 3 & -1 & 1 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - x_2 + x_3 = 0, \\ 2x_1 + x_2 - 2x_3 = 3, \\ 4x_1 - x_2 - x_3 = 0. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 - 2x_2 + 4x_3 = 0, \\ 6x_1 + 2x_2 - 4x_3 = 0, \\ 3x_1 + x_2 - 2x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 - x_2 - 4x_3 + 9x_4 = 22, \\ x_1 + 2x_2 - 4x_4 = -3, \\ 2x_1 - 3x_2 + x_3 + 5x_4 = -3, \\ 3x_1 - 2x_2 - 5x_3 + x_4 = 3; \end{cases}$ **b)** $\begin{cases} 3x_1 - 5x_2 + 3x_3 + 7x_4 = 1, \\ 2x_1 - 3x_2 + x_3 + 4x_4 = 1, \\ x_1 - 2x_2 + 2x_3 + 3x_4 = 0. \end{cases}$

Elements of Linear Algebra

Variant №14

Exercise 1. Calculate determinants.

Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.

Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 2 & -3 \\ 5 & 7 \end{vmatrix}$; **b)** $\begin{vmatrix} 4 & -2 & 4 \\ 12 & 2 & 10 \\ 2 & 2 & 1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 3 & 0 & 0 \\ -1 & 3 & 2 & -3 \\ 1 & -3 & -1 & 2 \\ -2 & 0 & 1 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 4 & -1 & 0 \\ -1 & 2 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 5 & 0 \\ 3 & 2 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -2 & 1 \\ 4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & -7 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 6 \\ 4 & 7 \end{pmatrix}$, $D = \begin{pmatrix} 0 & -5 \\ 20 & 45 \end{pmatrix}$:

a) calculate the expression $G = 3AC - B^2 - 2D + B + C$;

b) determine the value of the polynomial $f(x) = x^2 - x + 19$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + x_2 + x_3 = 2, \\ x_1 - 2x_2 + 4x_3 = 2, \\ 5x_1 + 7x_2 - 5x_3 = 2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 3x_1 - 12x_2 + 15x_3 = 0, \\ 2x_1 - 3x_2 + 15x_3 = 0, \\ x_1 - 4x_2 + 5x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0, \\ 7x_1 + 14x_2 + 20x_3 + 27x_4 = 0, \\ 5x_1 + 10x_2 + 16x_3 + 19x_4 = -2, \\ 3x_1 + 5x_2 + 6x_3 + 13x_4 = 5; \end{cases}$ **b)** $\begin{cases} 3x_1 - 5x_2 + x_3 + 4x_4 = 1, \\ 2x_1 - 3x_2 - x_3 + x_4 = 1, \\ x_1 - 2x_2 + 2x_3 + 3x_4 = 0. \end{cases}$

Elements of Linear Algebra

Variant №15

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -7 & 6 \\ -6 & 8 \end{vmatrix}$; **b)** $\begin{vmatrix} 0 & 1 & 1 \\ 5 & 1 & 2 \\ 3 & -1 & 0 \end{vmatrix}$; **c)** $\begin{vmatrix} 2 & 1 & -1 & 1 \\ -1 & -1 & 2 & 0 \\ 0 & -3 & -1 & -4 \\ 0 & 2 & -1 & 2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & -1 \\ 3 & 2 & 4 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -4 & 3 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -3 \\ -3 & -2 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 3 \\ 2 & 5 \end{pmatrix}$, $D = \begin{pmatrix} 10 & 3 \\ 3 & 13 \end{pmatrix}$:

a) calculate the expression $G = 3B^2 - AC - 2D + B^T + C$;

b) determine the value of the polynomial $f(x) = x^2 + 2x + 19$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $À \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \\ 3 & 4 & 1 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - 3x_2 + x_3 = 2, \\ x_1 + 2x_2 + x_3 = 2, \\ 2x_1 - 3x_2 - x_3 = 1. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 5x_1 - 2x_2 - 11x_3 = 0, \\ 2x_1 + 4x_2 - 14x_3 = 0, \\ x_1 + 2x_2 - 7x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 - 2x_2 + 3x_3 + x_4 = 5, \\ x_1 - 2x_2 - 5x_3 + x_4 = 5, \\ 2x_1 + 3x_2 - x_3 + x_4 = 2, \\ 3x_1 - x_2 - 5x_3 + x_4 = 8; \end{cases}$ **b)** $\begin{cases} x_1 - 3x_2 + x_3 + 2x_4 = 4, \\ 2x_1 - 5x_2 + 4x_3 + 3x_4 = 7, \\ x_1 - 2x_2 + 3x_3 + x_4 = 3. \end{cases}$

Elements of Linear Algebra

Variant №16

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 5 & 1 \\ -2 & 3 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 0 & 2 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & -1 & 1 \\ -2 & 0 & 2 & 1 \\ 1 & -2 & -2 & 2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ -4 & 5 \\ 2 & 0 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 \\ 1 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 6 \\ 1 & 5 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 4 \\ 4 & 26 \end{pmatrix}$:

a) calculate the expression $G = 2B^2 - AC - D + B^T - C$;

b) determine the value of the polynomial $f(x) = x^2 - 3x - 5$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $À \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 1 \\ 1 & -3 & -4 \end{pmatrix}$ find the inverse matrix and check that

A^{-1} $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 1, \\ 2x_1 + 3x_2 - 4x_3 = -8, \\ 3x_1 - 2x_2 - 5x_3 = -11. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 - 2x_2 + 3x_3 = 0, \\ -2x_1 + 4x_2 - 6x_3 = 0, \\ 2x_1 + x_2 - 4x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 - 2x_2 + 3x_3 + x_4 = 1, \\ 2x_1 - 3x_2 - x_3 + x_4 = -8, \\ 3x_1 - 5x_2 - 5x_3 + x_4 = -11, \\ x_1 - 2x_2 - 5x_3 + x_4 = 1; \end{cases}$ **b)** $\begin{cases} x_1 - 2x_2 + 3x_3 + x_4 = 1, \\ 3x_1 - x_2 - 5x_3 + x_4 = -11, \\ x_1 - 2x_2 - 5x_3 + x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №17

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 11 & 3 \\ 1 & -2 \end{vmatrix}$; **b)** $\begin{vmatrix} 5 & 3 & -1 \\ 7 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 3 & 1 & -1 \\ 0 & 3 & 2 & -3 \\ 0 & -3 & -1 & 2 \\ -2 & 0 & 1 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -6 & 2 \\ -1 & 0 \\ 6 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 & 0 \\ 1 & 3 & 3 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & -1 \\ 6 & 1 \end{pmatrix}$, $D = \begin{pmatrix} -4 & -3 \\ 15 & -1 \end{pmatrix}$:

a) calculate the expression $G = 2AC - B^2 - D + C^T - E$;
b) determine the value of the polynomial $f(x) = x^2 - 4x + 9$ evaluated at the matrix A.

Exercise 4. For the matrices $A,\ B,\ C,\ D$ given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 4 & 1 \\ -2 & 1 & 1 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - 4x_2 + x_3 = 1, \\ 2x_1 + 3x_2 - x_3 = 4, \\ x_1 + x_2 - x_3 = 0. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 3x_1 - x_2 + 5x_3 = 0, \\ 2x_1 + x_2 - 10x_3 = 0, \\ 9x_1 - 3x_2 + 15x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} 2x_1 - 2x_2 + x_3 - x_4 = -3, \\ 3x_1 + 2x_2 - x_3 - x_4 = 0, \\ -x_1 + x_2 - x_3 + x_4 = 2, \\ x_1 - 3x_2 + x_3 + x_4 = 2; \end{cases}$ **b)** $\begin{cases} x_1 + 2x_2 - 2x_3 - 3x_4 = 4, \\ 2x_1 + 5x_2 - x_3 - 4x_4 = 9, \\ x_1 + 3x_2 + x_3 - x_4 = 5. \end{cases}$

Elements of Linear Algebra

Variant №18

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 7 & 1 \\ -2 & -5 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 2 & 1 \\ 3 & -5 & 3 \\ 2 & 7 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & 2 & 0 \\ 2 & 2 & -1 & 1 \\ 4 & 2 & 2 & 1 \\ 1 & -4 & -2 & 2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & 2 & -1 \\ 0 & 5 & -8 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 \\ 3 & 2 \\ 0 & 1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -2 & 1 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ -2 & 5 \end{pmatrix}$, $C = \begin{pmatrix} -4 & 6 \\ -2 & 5 \end{pmatrix}$, $D = \begin{pmatrix} 6 & -7 \\ -14 & 27 \end{pmatrix}$:

a) calculate the expression $G = 3AC - B^2 - 2D + C - B$;

b) determine the value of the polynomial $f(x) = x^2 - x - 5$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & -2 \\ -1 & 1 & 2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + x_2 - x_3 = 0, \\ 3x_1 + x_2 - x_3 = 2, \\ 2x_1 - 2x_2 + x_3 = 3. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 + 2x_2 - x_3 = 0, \\ 3x_1 + 4x_2 - x_3 = 0, \\ 2x_1 + 4x_2 - 2x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} 3x_1 + x_2 + x_3 + x_4 = -1, \\ 2x_1 - x_2 + x_3 - x_4 = -6, \\ x_1 + 2x_2 + x_3 + 3x_4 = 5, \\ 4x_1 + x_2 + x_3 + 2x_4 = -1; \end{cases}$ **b)** $\begin{cases} x_1 + 2x_2 - 3x_3 - 4x_4 = 1, \\ 3x_1 + 7x_2 - 2x_3 - x_4 = 4, \\ 2x_1 + 5x_2 + x_3 + 3x_4 = 3. \end{cases}$

Elements of Linear Algebra

Variant №19

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -8 & 6 \\ -5 & 4 \end{vmatrix}$; **b)** $\begin{vmatrix} -1 & 8 & 3 \\ 2 & 0 & 1 \\ 1 & -4 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 1 & -1 & -1 \\ -4 & -5 & 2 & 0 \\ 0 & -3 & -5 & -1 \\ 0 & 1 & 1 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 & 1 \\ 2 & -1 & 0 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 1 & 4 \\ -2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 \\ 3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} -3 & -4 \\ 6 & 5 \end{pmatrix}$, $D = \begin{pmatrix} 21 & 16 \\ 12 & 13 \end{pmatrix}$:

a) calculate the expression $G = 2D - B^2 - AC + A^T - E$;

b) determine the value of the polynomial $f(x) = x^2 - 2x + 13$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & -1 \\ -2 & 1 & -2 \end{pmatrix}$ find the inverse matrix and check that A^{-1} $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - x_2 + 2x_3 = 3, \\ x_1 - x_2 - x_3 = 0, \\ x_1 - 2x_2 + 3x_3 = 2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 7x_1 + 2x_2 + x_3 = 0, \\ 5x_1 - x_2 + 8x_3 = 0, \\ 14x_1 + 4x_2 + 2x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} 2x_1 + 3x_2 - x_3 + 3x_4 = 2, \\ x_1 + 3x_2 + x_3 - x_4 = -1, \\ 3x_1 + 2x_2 - 2x_3 + x_4 = -3, \\ x_1 - x_2 + x_3 - x_4 = 3; \end{cases}$ **b)** $\begin{cases} x_1 + 3x_2 - x_3 - 2x_4 = 1, \\ 2x_1 + 7x_2 - 4x_3 - 3x_4 = 3, \\ x_1 + 4x_2 - 3x_3 - x_4 = 2. \end{cases}$

Elements of Linear Algebra

Variant №20

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 11 & -6 \\ 5 & -3 \end{vmatrix}$; **b)** $\begin{vmatrix} 0 & -1 & 1 \\ 5 & 1 & 1 \\ 3 & 2 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & 0 & 2 \\ 2 & -1 & 1 & -1 \\ 4 & 2 & 2 & 1 \\ 1 & -1 & 1 & -2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ -3 & 0 \\ 1 & -1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$, $D = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$:

a) calculate the expression $G = 3B^2 - AC - 2D + B^T - 2E$;

b) determine the value of the polynomial $f(x) = x^2 - 3x + 10$ evaluated at the matrix A.

Exercise 4. For the matrices $A,\ B,\ C,\ D$ given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -4 \\ -2 & 1 & -2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} 3x_1 - 2x_2 + x_3 = 1, \\ x_1 - x_2 + 3x_3 = 2, \\ x_1 - x_2 + x_3 = 0. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 5x_1 + 3x_2 + 11x_3 = 0, \\ -x_1 + 4x_2 + 7x_3 = 0, \\ 2x_1 - 8x_2 - 14x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} 3x_1 + x_2 + 2x_3 + x_4 = 3, \\ 2x_1 - x_2 + 3x_3 - x_4 = -8, \\ 2x_1 + x_2 + 2x_3 + x_4 = 2, \\ x_1 + x_2 - x_3 - x_4 = -1; \end{cases}$ **b)** $\begin{cases} x_1 - 2x_2 + 2x_3 + 3x_4 = 0, \\ 2x_1 - 3x_2 + x_3 + 4x_4 = 1, \\ 3x_1 - 5x_2 + 3x_3 + 7x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №21

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 2 & -1 \\ 11 & 3 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & -1 & 1 \\ 7 & 1 & 2 \\ 5 & 3 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 0 & 1 & -1 & -1 \\ -1 & -5 & 2 & 0 \\ 0 & -2 & -1 & -1 \\ -2 & -11 & 6 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -1 & -2 \\ 2 & 1 \\ -3 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 \\ -6 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 4 \\ -16 & 0 \end{pmatrix}$:

a) calculate the expression $G = 3AC - 2D - B^2 + B^T - 2E$;

b) determine the value of the polynomial $f(x) = x^2 - 5x + 14$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 4 & 2 & 1 \\ -1 & -1 & 1 \\ -2 & 1 & -2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} 3x_1 - 3x_2 + x_3 = 4, \\ x_1 + x_2 - x_3 = 0, \\ 2x_1 - x_2 + x_3 = 6. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 + x_2 - 7x_3 = 0, \\ 2x_1 + 3x_2 + 4x_3 = 0, \\ 2x_1 + 2x_2 - 14x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} -3x_1 - 2x_2 + x_3 + x_4 = 5, \\ 2x_1 - x_2 - x_3 - x_4 = -3, \\ -3x_2 + x_3 - x_4 = -1, \\ x_1 + x_2 + x_3 + x_4 = 6; \end{cases}$ **b)** $\begin{cases} x_1 - 2x_2 + 2x_3 + 3x_4 = 0, \\ 3x_1 - 5x_2 + x_3 + 4x_4 = 1, \\ 2x_1 - 3x_2 - x_3 + x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №22

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix}$; **b)** $\begin{vmatrix} 4 & 2 & 1 \\ 1 & -5 & 3 \\ 8 & 7 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 3 & 0 & 0 \\ 2 & 4 & -1 & -1 \\ 4 & 4 & -3 & -5 \\ 1 & -1 & 1 & -2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ -1 & 4 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} -2 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -2 & -3 \\ -1 & -4 \end{pmatrix}$, $C = \begin{pmatrix} -3 & -7 \\ 1 & 4 \end{pmatrix}$, $D = \begin{pmatrix} 7 & 18 \\ 6 & 19 \end{pmatrix}$:

a) calculate the expression $G = 2B^2 - AC - D + B + C$;

b) determine the value of the polynomial $f(x) = x^2 - x + 2$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 5 & 2 & 1 \\ -1 & -1 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - 2x_2 + x_3 = 1, \\ x_1 + x_2 - x_3 = 0, \\ 2x_1 - x_2 + x_3 = 6. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify:

$$\begin{cases} 6x_1 + 2x_2 - 3x_3 = 0, \\ 2x_1 + 3x_2 - x_3 = 0, \\ 4x_1 + 6x_2 - 2x_3 = 0. \end{cases}$$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $$\begin{cases} -3x_1 - x_2 - 2x_3 - x_4 = -2, \\ -2x_1 - 2x_2 + x_3 + x_4 = 1, \\ x_1 + 2x_2 - x_3 + x_4 = 2 \\ x_1 - x_2 + x_3 + x_4 = 3; \end{cases}$$

b) $$\begin{cases} x_1 - 3x_2 + x_3 + 2x_4 = 4, \\ 2x_1 - 5x_2 + 4x_3 + 3x_4 = 7, \\ x_1 - 2x_2 + 3x_3 + x_4 = 3. \end{cases}$$

Elements of Linear Algebra

Variant №23

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 1 & 7 \\ -1 & -3 \end{vmatrix}$; **b)** $\begin{vmatrix} 2 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 2 & 2 \end{vmatrix}$; **c)** $\begin{vmatrix} 0 & 1 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 2 & -2 & -1 & -1 \\ -2 & 3 & 6 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 & -2 \\ 4 & 1 & 0 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 6 & -2 \\ 0 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 6 & -2 \\ 6 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 36 & -10 \\ 0 & 1 \end{pmatrix}$:

a) calculate the expression $G = 2AC - D - B^2 + C - B$;
b) determine the value of the polynomial $f(x) = x^2 - 5x + 14$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 4 \\ 1 & -3 & 4 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - 3x_2 + 7x_3 = -2, \\ 5x_1 + x_2 + x_3 = 24, \\ 2x_1 + x_2 - x_3 = 11. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 - 3x_2 + 5x_3 = 0, \\ 4x_1 + 5x_2 + 3x_3 = 0, \\ 2x_1 - 6x_2 + 10x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} -2x_1 - x_2 + x_3 + x_4 = 1, \\ 2x_1 - x_2 + 2x_4 = -2, \\ x_1 + x_2 + 2x_3 - x_4 = 3, \\ 2x_1 + x_2 + x_3 - x_4 = 1; \end{cases}$ **b)** $\begin{cases} x_1 + 4x_2 - 2x_3 - 3x_4 = 2, \\ 2x_1 + 9x_2 - x_3 - 4x_4 = 5, \\ x_1 + 5x_2 + x_3 - x_4 = 3. \end{cases}$

Elements of Linear Algebra

Variant №24

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 4 & 2 \\ 1 & -5 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & -4 & -1 \\ -1 & 8 & 3 \\ 2 & 0 & 1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & -1 \\ -2 & 5 & 0 & -5 \\ 1 & -1 & 1 & -2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 0 & -1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}$, $C = \begin{pmatrix} -1 & -3 \\ 2 & 1 \end{pmatrix}$, $D = \begin{pmatrix} -3 & -4 \\ 4 & -3 \end{pmatrix}$:

a) calculate the expression $G = 3B^2 - AC - 2D + B + C$;

b) determine the value of the polynomial $f(x) = x^2 - 4x + 8$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 3 & -1 & 1 \end{pmatrix}$ find the inverse matrix and check that A^{-1} $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + x_2 - x_3 = 0, \\ -x_1 + 2x_2 - x_3 = -4, \\ x_1 + 2x_2 - x_3 = 2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify:
$$\begin{cases} 7x_1 + 2x_2 - 3x_3 = 0, \\ 2x_1 - 3x_2 + 2x_3 = 0, \\ -4x_1 + 6x_2 - 4x_3 = 0. \end{cases}$$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} 2x_1 + 2x_2 + x_3 + x_4 = 2, \\ -x_1 + x_2 + 2x_3 - x_4 = 0, \\ x_1 - 3x_2 - x_3 + x_4 = 4, \\ x_1 + x_2 + x_3 + x_4 = 4; \end{cases}$ **b)** $\begin{cases} x_1 - x_2 + 3x_3 + 4x_4 = 0, \\ 2x_1 - x_2 + 2x_3 + x_4 = 1, \\ 4x_1 - 3x_2 + 8x_3 + 9x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №25

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 6 & -4 \\ 2 & -3 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & -1 & 0 \\ 2 & 1 & 5 \\ 0 & 2 & 3 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 1 & 2 & -1 \\ -2 & -1 & 2 & 2 \\ 0 & -2 & -1 & 0 \\ 0 & 1 & 6 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} -6 & 2 \\ -1 & 0 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$:

a) calculate the expression $G = 2B^2 - D - AC + B^T + C^T$;

b) determine the value of the polynomial $f(x) = x^2 - 4x + 11$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $À \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 6 & 2 & 1 \\ -1 & -1 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ find the inverse matrix and check that A^{-1} $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} 4x_1 - x_2 - x_3 = 0, \\ x_1 - x_2 + x_3 = 0, \\ 2x_1 + x_2 - 2x_3 = 3. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 3x_1 + 5x_2 + x_3 = 0, \\ x_1 - x_2 + 3x_3 = 0, \\ 6x_1 + 10x_2 + 2x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} -2x_1 + x_2 + x_3 + x_4 = 4, \\ x_1 - x_2 - x_3 - x_4 = -3, \\ -x_1 + 2x_2 + x_3 + x_4 = 4, \\ 2x_1 - x_3 - 2x_4 = -1; \end{cases}$ **b)** $\begin{cases} x_1 - 4x_2 + 2x_3 + 3x_4 = 5, \\ 2x_1 - 7x_2 + 4x_3 + x_4 = 9, \\ x_1 - 3x_2 + 2x_3 - 2x_4 = 4. \end{cases}$

Elements of Linear Algebra

Variant №26

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix}$; **b)** $\begin{vmatrix} 7 & 1 & 2 \\ 1 & -1 & 1 \\ 5 & 3 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & -1 \\ -2 & 4 & -2 & 3 \\ 1 & -1 & 0 & -2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & 2 & -1 \\ 0 & 5 & -8 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 \\ 0 & 1 \\ 2 & 3 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 8 & 6 \\ -6 & 8 \end{pmatrix}$:

a) calculate the expression $G = 3B^2 - AC - 2D + B^T - 3E$;

b) determine the value of the polynomial $f(x) = x^2 + 3x + 1$ evaluated at the matrix A.

Exercise 4. For the matrices $A,\ B,\ C,\ D$ given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 1 & -1 \\ -4 & 2 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 - 2x_2 + 2x_3 = 7, \\ 2x_1 - 9x_2 - x_3 = -1, \\ x_1 + x_2 - x_3 = -2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 + x_2 - 15x_3 = 0, \\ x_1 - x_2 + 3x_3 = 0, \\ 2x_1 - 2x_2 + 6x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} 2x_1 - x_2 + x_3 = 3, \\ -x_1 - 3x_2 + x_3 - 3x_4 = -4, \\ x_1 - 2x_2 + 3x_3 + x_4 = 0, \\ x_1 + x_2 + x_3 + x_4 = 4; \end{cases}$ **b)** $\begin{cases} x_1 - 5x_2 + 3x_3 + 4x_4 = 4, \\ 2x_1 - 9x_2 + 2x_3 + x_4 = 7, \\ x_1 - 4x_2 - x_3 - 3x_4 = 3. \end{cases}$

Elements of Linear Algebra

Variant №27

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 7 & 8 \\ 3 & 7 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & 4 & 1 \\ 3 & 1 & 3 \\ 2 & 8 & -1 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 1 & 2 & 0 \\ 0 & -1 & 2 & 2 \\ 0 & -2 & -1 & 2 \\ -3 & 1 & 6 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 1 & -3 \\ 2 & -4 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 & 0 \\ 2 & 4 & -5 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -4 & -1 \\ 2 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 3 \\ -2 & 2 \end{pmatrix}$, $D = \begin{pmatrix} 14 & 1 \\ -2 & 7 \end{pmatrix}$:

a) calculate the expression $G = 2B^2 - AC - D + C^T - 2E$;
b) determine the value of the polynomial $f(x) = x^2 - 5x + 22$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 5 & 1 & -1 \\ -1 & 2 & -2 \\ 2 & 1 & -3 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} 5x_1 + 7x_2 - 5x_3 = 2, \\ x_1 + x_2 + x_3 = 2, \\ x_1 - 2x_2 + 4x_3 = 2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 5x_1 - x_2 + 3x_3 = 0, \\ 6x_1 + x_2 + 19x_3 = 0, \\ 10x_1 - 2x_2 + 6x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + x_3 + 2x_4 = 0, \\ -x_1 + 2x_2 - 2x_3 + x_4 = 0, \\ x_1 - 3x_2 - 3x_3 + x_4 = 2, \\ x_1 + x_2 + x_3 + x_4 = 2; \end{cases}$ **b)** $\begin{cases} x_1 + x_2 + 4x_3 + 2x_4 = 0, \\ 3x_1 + 4x_2 + x_3 + 3x_4 = 1, \\ 2x_1 + 3x_2 - 3x_3 + x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №28

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -5 & 8 \\ -4 & 7 \end{vmatrix}$; **b)** $\begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & 2 \\ 2 & 1 & 3 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & -1 \\ -2 & 1 & -6 & 3 \\ 1 & 0 & 1 & -2 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & -2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 1 \\ 4 & 0 \\ 3 & 2 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -4 & 2 \\ 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 6 & -1 \\ -4 & 2 \end{pmatrix}$, $D = \begin{pmatrix} 20 & -6 \\ -6 & 5 \end{pmatrix}$:

a) calculate the expression $G = AC - B^2 + D + 3B^T + 5E$;

b) determine the value of the polynomial $f(x) = x^2 - 5x + 21$ evaluated at the matrix A.

Exercise 4. For the matrices $A,\ B,\ C,\ D$ given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & -2 & -2 \\ 2 & 1 & -4 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} x_1 + x_2 + 2x_3 = 1, \\ x_1 - 2x_2 + x_3 = -1, \\ x_1 - x_2 + 2x_3 = -1. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} 9x_1 + 3x_2 - 6x_3 = 0, \\ x_1 - 2x_2 + 4x_3 = 0, \\ 3x_1 + x_2 - 2x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} -x_1 - x_2 + x_3 - 2x_4 = 2, \\ x_1 + x_2 + x_3 + 2x_4 = -2, \\ x_1 - x_2 + x_3 = 2, \\ x_1 + x_2 + x_3 + x_4 = 3; \end{cases}$ **b)** $\begin{cases} x_1 - x_2 + 4x_3 + 3x_4 = 0, \\ 3x_1 - 2x_2 + x_3 + 2x_4 = 1, \\ 2x_1 - x_2 - 3x_3 - x_4 = 1. \end{cases}$

Elements of Linear Algebra

Variant №29

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} -7 & 5 \\ -2 & 3 \end{vmatrix}$; **b)** $\begin{vmatrix} 4 & -2 & 4 \\ 5 & 1 & 6 \\ 1 & 2 & 2 \end{vmatrix}$; **c)** $\begin{vmatrix} 0 & 1 & -1 & 2 \\ 0 & -1 & 2 & -2 \\ 2 & -2 & -2 & 0 \\ -1 & 1 & -2 & 1 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 1 & 3 \\ -2 & 2 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 5 & 6 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -4 \\ 2 & -2 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 2 \\ 1 & -6 \end{pmatrix}$, $D = \begin{pmatrix} -7 & 12 \\ -6 & -4 \end{pmatrix}$:

a) calculate the expression $G = 2B^2 - AC - D + A^T + B$;

b) determine the value of the polynomial $f(x) = x^2 - 5x + 24$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -5 & 3 \\ 2 & -1 & -1 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:

a) Cramer's Rule; **b)** using matrices; **b)** Gaussian elimination.

$$\begin{cases} x_1 + 2x_2 + x_3 = 2, \\ 2x_1 - 3x_2 - x_3 = 1, \\ -x_1 + 3x_2 - x_3 = -2. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 - 4x_2 + 5x_3 = 0, \\ 2x_1 - 3x_2 + 15x_3 = 0, \\ 2x_1 - 8x_2 + 10x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} 3x_1 - 2x_2 - 5x_3 + x_4 = 3, \\ 2x_1 - 3x_2 + x_3 + 5x_4 = -3, \\ x_1 + 2x_2 - 4x_4 = -3, \\ x_1 - x_2 - 4x_3 + 9x_4 = 22; \end{cases}$ **b)** $\begin{cases} x_1 - 3x_2 + 4x_3 + 3x_4 = 2, \\ 3x_1 - 8x_2 + x_3 + 2x_4 = 5, \\ 2x_1 - 5x_2 - 3x_3 - x_4 = 3. \end{cases}$

Elements of Linear Algebra

Variant №30

Exercise 1. Calculate determinants.
Calculate the second determinant b) in two ways: using the Rule of Sarrus and expanding determinant into elements of some row or some column.
Calculate the third determinant c) in two ways: expanding it into elements of some row or some column and using the upper-triangulation.

a) $\begin{vmatrix} 11 & 2 \\ -7 & 1 \end{vmatrix}$; **b)** $\begin{vmatrix} 2 & -1 & -2 \\ 1 & 2 & -1 \\ 1 & -3 & 3 \end{vmatrix}$; **c)** $\begin{vmatrix} 1 & 0 & 0 & 1 \\ -2 & -1 & 1 & -1 \\ -2 & 1 & -1 & 3 \\ 1 & 3 & -5 & 0 \end{vmatrix}$.

Exercise 2. Find all possible products of the matrices $A = \begin{pmatrix} 2 & 3 & 0 \\ -2 & 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ -5 & 2 \\ -1 & 3 \end{pmatrix}$.

Exercise 3. For the matrices $A = \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 7 & -3 \\ 1 & -2 \end{pmatrix}$, $C = \begin{pmatrix} 6 & -1 \\ 7 & -3 \end{pmatrix}$, $D = \begin{pmatrix} 46 & -15 \\ 5 & 1 \end{pmatrix}$:

a) calculate the expression $G = AC - B^2 + D - 5B + 4E$;

b) determine the value of the polynomial $f(x) = x^2 - 2x + 5$ evaluated at the matrix A.

Exercise 4. For the matrices A, B, C, D given in the Exercise 3 solve the matrix equations:

a) $A \cdot X = B$; **b)** $X \cdot B = C$; **c)** $\grave{A} \cdot X \cdot B = D$.

Exercise 5. For the given matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ -5 & 1 & 3 \\ 2 & 1 & 2 \end{pmatrix}$ find the inverse matrix A^{-1} and check that $A^{-1} \cdot A = E$.

Exercise 6. Solve the system of linear algebraic equations (SLAE) in three ways:
a) Cramer's Rule; **b)** using matrices; **c)** Gaussian elimination.

$$\begin{cases} 2x_1 - x_2 + 3x_3 = 3, \\ -x_1 + 2x_2 + 3x_3 = -3, \\ x_1 + x_2 + x_3 = 0. \end{cases}$$

Exercise 7. Solve the homogeneous SLAE and verify: $\begin{cases} x_1 + 2x_2 - 7x_3 = 0, \\ 5x_1 - 2x_2 - 11x_3 = 0, \\ 3x_1 + 6x_2 - 21x_3 = 0. \end{cases}$

Exercise 8. Check the consistency of SLAE, find their general solutions and verify:

a) $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0, \\ 7x_1 + 14x_2 + 20x_3 + 27x_4 = 0, \\ 5x_1 + 10x_2 + 16x_3 + 19x_4 = -2, \\ 3x_1 + 5x_2 + 6x_3 + 13x_4 = 5; \end{cases}$ **b)** $\begin{cases} x_1 - x_2 + 3x_3 + 4x_4 = 0, \\ 4x_1 - 3x_2 + x_3 + 2x_4 = 1, \\ 3x_1 - 2x_2 - 2x_3 - 2x_4 = 1. \end{cases}$

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Електронне мережне навчальне видання

ВИЩА МАТЕМАТИКА

ЕЛЕМЕНТИ ЛІНІЙНОЇ АЛГЕБРИ

Практикум

(Англійською мовою)

Укладачі: Массалітіна Є.В., Новикова Г.К., Пилипенко В.А.

Практикум до розділу «Елементи лінійної алгебри» з курсу «Вища математика» для студентів технічних спеціальностей містить 30 варіантів, кожен варіант складається з 8 завдань (16 задач). Самостійне виконання цих завдань забезпечує свідоме оволодіння навчальним матеріалом, який передбачено робочою програмою з вищої математики.

Практикум може бути рекомендований в якості розрахункової роботи за темою «Елементи лінійної алгебри» для студентів першого курсу технічних спеціальностей.

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і розповсюджувачів видавничої продукції ДК № 5354 від 25.05.2017 р.