

H_∞ DESIGN METHOD APPLIED TO THE OPTIMIZATION OF THE PRIMARY VIBRATION PATTERN CORIOLIS VIBRATORY GYROSCOPE

Introduction

The Phase-Locked Loops (PLLs) are essential devices in control system of wave pattern of Coriolis vibratory gyroscope (CVG). The analysis of the dynamics of the CVG resonator was performed in the [1]. In addition, there was proposed the control system of it's vibration pattern by means of using the PLL loop. The approach of excitation of resonant vibrations in the CVG resonator on the base of self-oscillator was considered in the [2].

A PLL system is used to track the natural frequency of the resonator due to fatigue of the spring element, or through temperature variation. An analysis of a nonlinear control system based on PLL that was used to track the natural frequency of a MEMS resonator is presented in the [3].

Nowadays, due to the miniaturization process and the new advances in Microelectronics, the modern PLL system has extremely strong requirements in terms of the noise performance specifications which increase progressively. As a consequence, the classical design methods of the PLL attain their performance limits that cannot be surpassed under the classical methods. For these reasons, the PLL designers start to seek the new design methods of the PLL allowing to release the mentioned noise performance limits.

Problem formulation

The purpose of this article is to study and to understand the classical design methods of the PLL and to purpose, using the advanced Control System Theory tools, as the H_∞ method, an alternative one allowing a significant improvement of the PLL noise performance.

Classical Phase-Locked Loops

Basically, the PLL operating mechanism is based on the control of the phase of a variable frequency oscillator allowing the tracking of an external reference periodical signal [4]. The oscillator is generally controlled by a voltage, and therefore it is respectively named Voltage Controlled Oscillator (VCO).

A PLL is a feedback system in which the tracking value is the phase of a periodic signal. The Fig. 1 represents the functional scheme of a PLL.

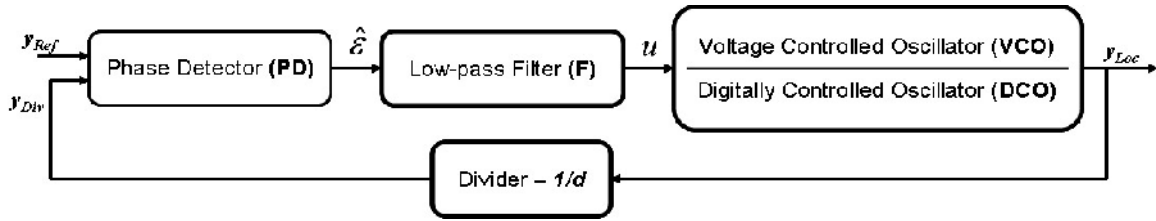


Fig. 1. Functional scheme of a PLL

The VCO output signal y_{Loc} after the *frequency division* y_{Div} is compared with the external reference signal y_{Ref} thanks to a *phase detector* (PD). The output signal of the PD $\hat{\epsilon}$, which will be called the phase error, is generally composed of 2 components: the first, at low frequencies, is proportional to the phase difference of two PD input signals $y_{Ref} - y_{Div}$, and the second, at high frequencies, is a deterministic parasite signal (also called “ripple”) whose characteristics can vary depending on the employed technology for the PD [5]. The PD output signal $\hat{\epsilon}$ is sent to the input of the *filter* F .

In case classical approach of the phase domain linearization, the time signals $y_{Loc}, y_{Div}, y_{Ref}$ are described by the harmonic function \cos (or \sin) and are replaced by their corresponding phase signals $\phi_{Loc}, \phi_{Div}, \phi_{Ref}$ according to the relation $y_x(t) = \cos(\phi_x(t))$. Similarly to the phase signal, we introduce the instantaneous angular frequency $\omega_x(t)$ defined by $\omega_x(t) = d\phi_x(t)/dt$.

The PD measures a phase difference between two periodical signals at its input. In the phase domain and for the small signals, the value of the PD output is thus proportional to this phase difference. As a consequence the linear part of the PD behavior is modeled in the phase domain by an adder followed by a simple gain K_{PD} . The filter (F) is a tuning block of the feedback loop that controls the VCO and is modeled in a quite general form by a transfer function $F(s)$. Concerning the noise of PD, it is usually modeled by a white noise b_{PD} of fixed Power Spectral Density (PSD) level L_{PD} (Fig. 2 and [6]). It's the same filter b_F and divider b_D noises.



Fig. 2. PD noise model

The VCO is a device that delivers at its output the periodical signal. The frequency of the delivered periodical signal depends on the input voltage control signal u . The dependence is usually proportional so that the output oscillator frequency ω_{Loc} is defined as: $\omega_{Loc}(t) = \omega_0 + K_{VCO}u(t)$, where the frequency $\omega_0 = 2\pi f_0$ is the central VCO angular frequency expressed in *rad/sec* and f_0 is the central VCO frequency expressed in *Hz*. It is obtained when the input voltage is zero *i.e.* $u(t) = 0$. The constant gain K_{VCO} represents the sensitivity of the oscillator.

In the phase domain, a periodical signal is represented by its phase evolution. For the stationary periodic signal it is described by a ramp with a constant slope. For a controlled oscillator, the ramp slope depends on the control applied in the oscillator input: $\varphi_{Loc}(t) = \int_0^t \omega_{Loc}(\tau) d\tau = \omega_0 t + K_{VCO} \int_0^t u(\tau) d\tau$.

In the Laplace domain, it is equivalent [6, 7] to:

$$\varphi_{Loc}(s) = \frac{K_{VCO}}{s} \left(\frac{\omega_0}{K_{VCO}} \frac{1}{s} + u(s) \right).$$

This last equality reveals that the VCO can be modeled in the phase domain by a gain K_{VCO} followed by a simple integrator. At the input of VCO integrator two signals are added: the first signal is the control signal from filter, the second signal is a constant in the Laplace domain which is proportional to the oscillator central frequency.

Although the VCOs are easy to implement in terms of integrated circuits [6, 7], they suffer from an important phase noise b_{VCO} . Indeed, the noise caused by the electronic components or the exterior signals (command, energy supply) can introduce a distortion of the frequency and the amplitude of the output signal. Often, the distortion of the amplitude is neglected, and only the distortion of the frequency is considered. It can be seen like a random variation of the period and thus the phase of the VCO output signal. Therefore, the output signal, in the sinusoidal oscillator case, can be described as follows:

$$y_{Loc}(t) = A_{Loc} \cos(\varphi_{Loc}(t) + b_{VCO}(t)),$$

where A_{Loc} is the amplitude of the generated signal, $\varphi_{Loc}(t)$ is the unperturbed ideal phase of the VCO signal and $b_{VCO}(t)$ represents the phase noise signal with a certain spectral shape.

In contrast to the PD and the filter and due to various non-linear effects, the shape of every oscillator noise PSD is much more complicated.

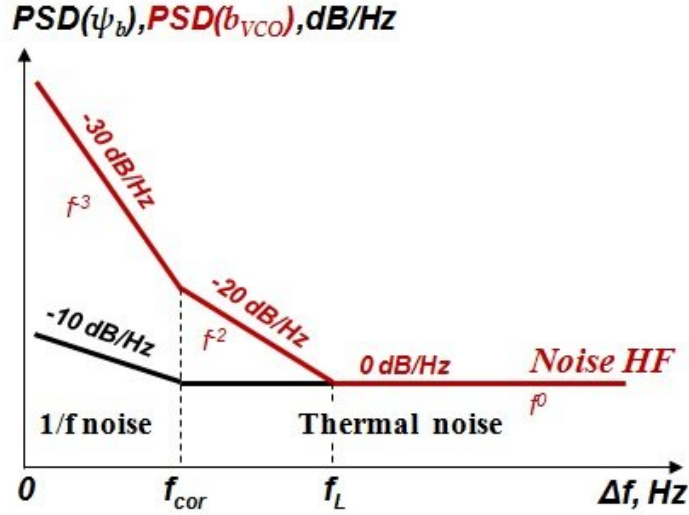


Fig. 3. PSD of the transistor noise (black line) and of the phase noise in oscillators (red line)

To understand the different sources of the phase noise, we investigate the noise of a transistor. The transistor noise ψ_b is composed of two main contributions [7]: 1) the thermal noise which implies the noise growth at the high frequencies and 2) the Flicker noise or f^{-1} noise, characterized by a decrease of its PSD in the low frequencies with a slope of -10 dB/Hz/dec up to the corner frequency f_{cor} (Fig. 3, black line). Due to various mentioned non-linear effects [9] the noise spectrum of each transistor composing the VCO is *converted* at the output of the oscillator in an overall phase noise of the oscillator b_{VCO} (Fig. 3., red line).

The overall linear model of the PLL in the phase domain based on the previous subsections is presented on the Fig. 4.

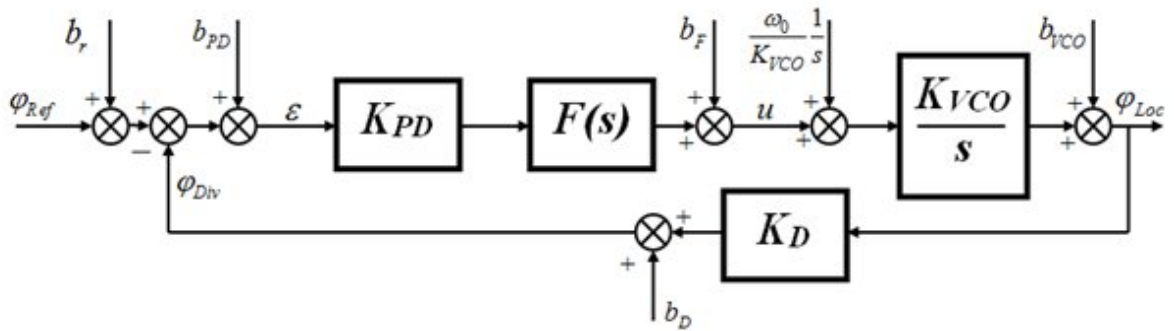


Fig. 4. Linear model of the overall PLL

From the Microelectronics point of view, the PLL has to be designed such that it satisfies the following performance specifications.

The generated PLL output signal has to be synchronized, at least in frequency, with the reference signal at the PLL input $\exists \epsilon_{stat} \in \square : \lim_{t \rightarrow +\infty} (\varphi_{Loc}(t) - \varphi_{Ref}(t)) = \epsilon_{stat}$ and the contribution of the different

noise sources has to be minimized on the PLL output signal such that its PSD respects the frequency constraint.

We thus focus on the controller design that for the given model and given noise PSD shapes ensures the described PLL performance specifications i.e. on the following problem resolution.

H_∞ -optimization approach for design PLL

The H_∞ design method is an advanced design method in the frequency domain which is essential from the application point of view. Indeed, the classical design methods of the PLL in CVG design field are mainly based on the classical frequency design methods. Formulation of the H_∞ problem synthesis is this: Find a controller K such as: a) the feedback loop $P*K$ is stable (Fig. 5.); b) The H_∞ norm of the transfer function $T_{w \rightarrow z}(s)$ between the input w and the output z is less or equal to γ : $\|T_{w \rightarrow z}(s)\|_\infty \leq \gamma$.

We take the PLL model (Fig. 4) with only two inputs and one output as presented in the Fig. 6, *a*. The chosen inputs are the VCO phase noise b_{VCO} and reference signal $r = \varphi_{Ref} + b_r$ as it is the most important noise sources in PLL.

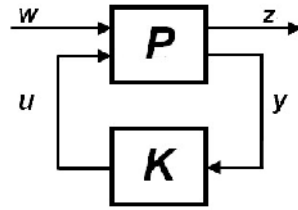


Fig. 5. Standard H_∞ control scheme

Let us define the new controller $K(s) = K_{PD}F(s)$ and the new linear operator $G(s) = \frac{K_{VCO}K_D}{s}$ as presented in the Fig. 6, *a*.

The next step is to define the plant P to be controlled as presented in the Fig. 6, *b* in order to obtain the standard H_∞ control scheme (Fig. 6) for MIMO systems.

To apply the standard H_∞ , while satisfying the specifications, we put the weighting functions of the inputs (W_r and W_b) and outputs (W_φ) of the system (Fig. 7.).

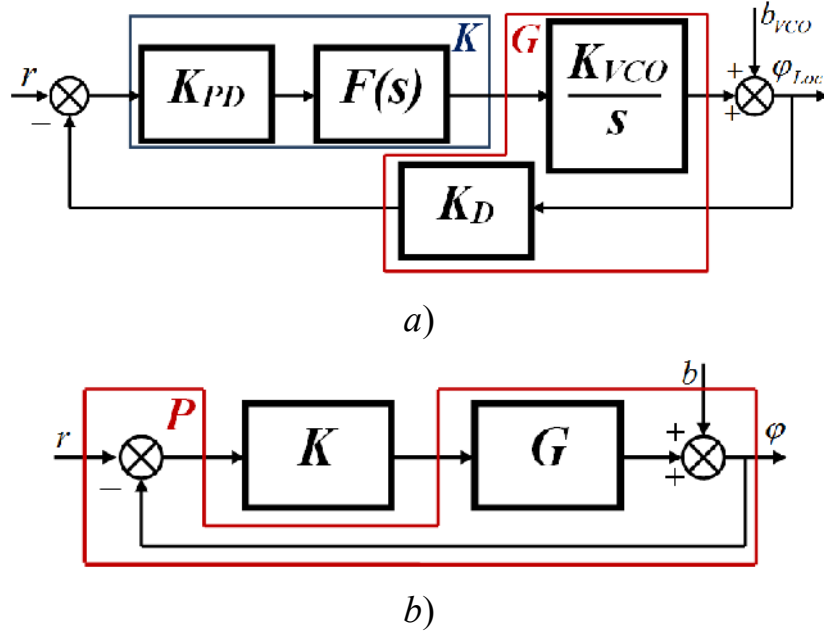


Fig. 6. Used notations

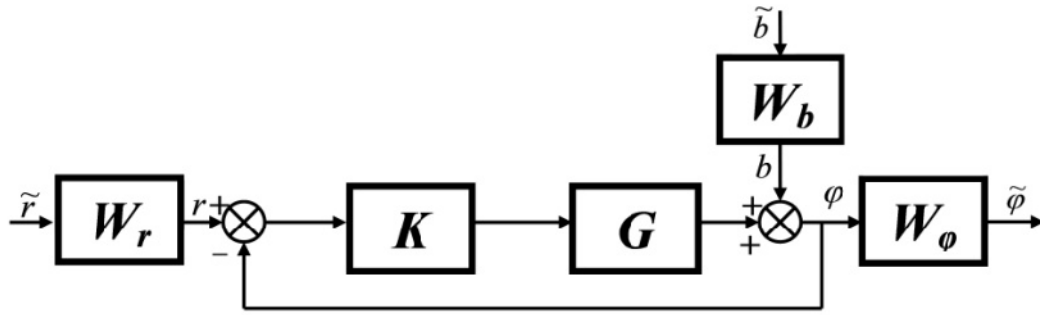


Fig. 7. PLL model with two inputs

The transfer functions are $\varphi = T_{r \rightarrow \varphi} r + T_{b \rightarrow \varphi} b$, where from Control System Theory literature $T_{b \rightarrow \varphi}(s) = (1 + G(s)K(s))^{-1}$ is often called the *sensitivity function* $S(s)$; $T_{r \rightarrow \varphi}(s) = \frac{G(s)K(s)}{1 + G(s)K(s)}$ is often called the *complementary sensitivity function* $T(s)$ since the following complementary relation is always satisfied for $\forall K$ and $\forall G$: $S(s) + T(s) = 1$.

Using the H_∞ property [10] the frequency constraints on two transfer functions S and T , *i.e.* for $\forall \omega$ and illustrated in the Fig. 8:

$$S(s) < \frac{1}{|W_\varphi(s)W_b(s)|}; \quad T(s) < \frac{1}{|W_\varphi(s)W_r(s)|}$$

Observing the shapes of the constraints imposed on the feedback transfer functions in the Fig. 7., one can conclude that they are quite complicated.

In the H_∞ design procedure, the overall order of the controller is equal to the order of the system to be controlled plus the order of the all weighting

functions [10]. Further discuss additional constraints and its simplification in the case of sensitivity S and complementary sensitivity T transfer functions.

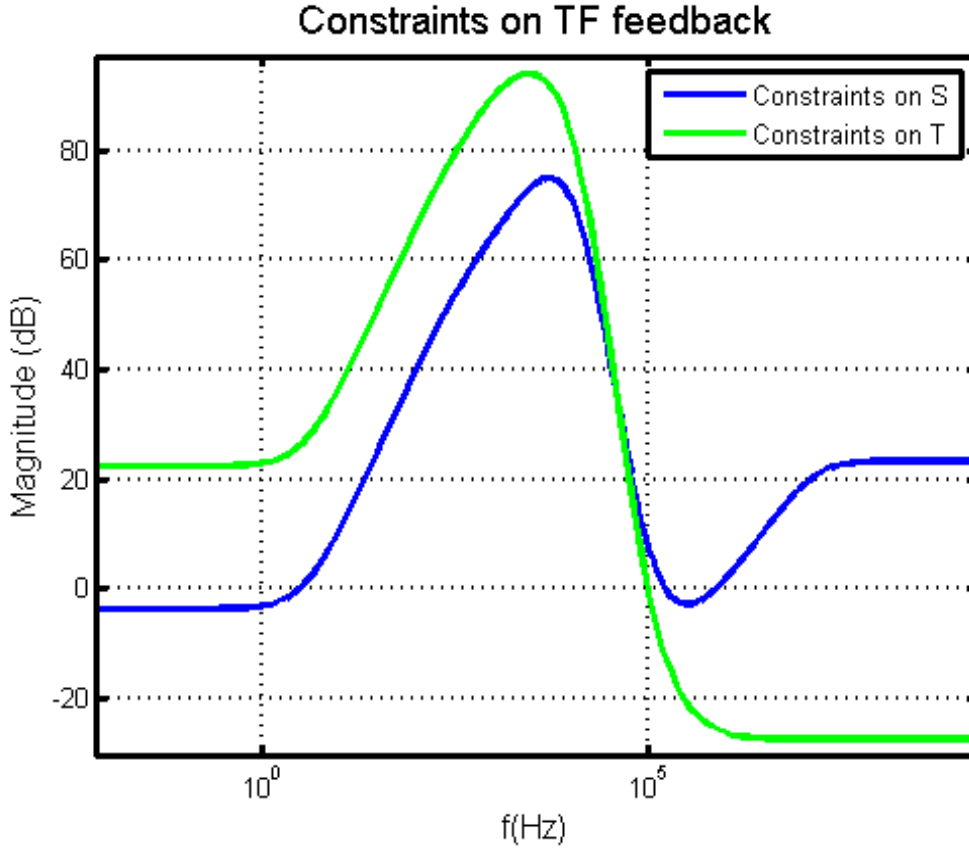


Fig. 8. Constraints on TF feedback

Sensitivity function $S(s)$ specifications besides the VCO noise, the sensitivity function S is impose the reference tracking and thus the PLL synchronization since $T_{\varphi_{Ref} \rightarrow \varepsilon}(s) = S(s)$. The PLL reference input signal is in the form of a ramp [10], and in order to be synchronized in frequency, the PLL error signal $\varepsilon(t)$ in steady state has to approach the constant value ε_{stat} . By using Final value theorem, we have $\lim_{t \rightarrow +\infty} \varepsilon(t) = \lim_{s \rightarrow 0} s\varepsilon(s) = \lim_{s \rightarrow 0} sT_{\varphi_{Ref} \rightarrow \varepsilon}(s)\varphi_{Ref}(s)$.

The Laplace transform of a ramp is $\varphi_{Ref}(s) = A/s^2$ where $A \in \mathbb{R}$ is the slope of the ramp, and thus, according to the Final value theorem we obtain $\lim_{t \rightarrow +\infty} \varepsilon(t) = A \lim_{s \rightarrow 0} T_{\varphi_{Ref} \rightarrow \varepsilon}(s)/s$.

To ensure that the tracking error trajectories $\varepsilon(t)$ tend to a constant ε_{stat} when t tends to infinity, the transfer function $T_{\varphi_{Ref} \rightarrow \varepsilon}(s)$ must have at least one zeros at zero: $T_{\varphi_{Ref} \rightarrow \varepsilon}(s) = s^k T'_{\varphi_{Ref} \rightarrow \varepsilon}(s)$ with $k \geq 1$ and all his poles must have strictly negative real parts.

The requirements of “one zero at zero” of the transfer function $T_{\varphi_{Ref} \rightarrow \varepsilon}(s)$ can be expressed as “the transfer function $T_{\varphi_{Ref} \rightarrow \varepsilon}(s)$ has a slope of at least +20 dB/dec in LF range”. Additionally, the rapidity of the reference tracking is imposed by the cut-off frequency ω_s^c . In general, the response time evolves inversely proportional to the cut-off frequency ω_s^c of the sensitivity function $S(s)$. Therefore, the more attenuation gain of the sensitivity function $S(s)$ is important in LF, the faster the system is in reference tracking [10] and thus in synchronization.

Another point that concerns the sensitivity function is the stability margins. Indeed, it can be shown [10] that the modulus margin Δ_M imposing the gain and phase margins is inversely proportional to the H_∞ -norm of the sensitivity function S . A constraint of 6-8dB on the modulus of the sensitivity function S fixes thus the acceptable stability margins of the PLL.

An important performance limitation concerning the sensitivity function has to be highlighted here. It is not possible, at high frequencies, to reduce the contribution of the VCO noise b in the PLL output φ since the gain of the sensitivity function S connecting these two signals is equal to 1. Indeed, HF gain of the open loop is generally low, i.e. negligible compared to 1. This is due to the fact that the modulus of $G(s) = K_{VCO}K_D/s$ is low at HF. As a consequence we obtain the approximate relation for HF: $|S(s)| = \left| (W_\varphi(s)W_b(s))^{-1} \right| \approx 1$.

The frequency constraint of the complementary sensitivity function T is essentially imposed according to the reference noise contribution rejection specification (Fig. 9.) since $T_{\varphi_{Ref} \rightarrow \varphi}(s) = T(s)$.

From the standpoint of meeting the requirements of a closed system, and simplify the transfer function of the controller, we choose the new weighting functions W_{r_n}, W_{b_n} and W_{φ_n} , as in (1), with minimal possible order which is sufficient to impose the noise frequency constraint presented in Fig. 8 and additional frequency constraints according to the previous discussion.

$$W_{r_n} = \frac{0.5(s + 2 \cdot 10^6)}{s + 3000}, \quad W_{b_n} = \frac{60(s + 3.749 \cdot 10^6)}{s + 3.7 \cdot 10^8}, \quad W_{\varphi_n} = 1. \quad (1)$$

The new constraints on the functions S and T are shown in Fig. 9. As we can see, if the new PSD constraints are satisfied (dashed lines in the Fig.9), it is *a fortiori* the case for the original ones which are represented by the full lines in the same Fig. 9. Additionally, the +20dB/dec slope in LF range, the fixed cut-off

frequency ω_S^c and the $8dB$ gain limitation of the sensitivity function S ensure the PLL synchronization and appropriate stability margins.

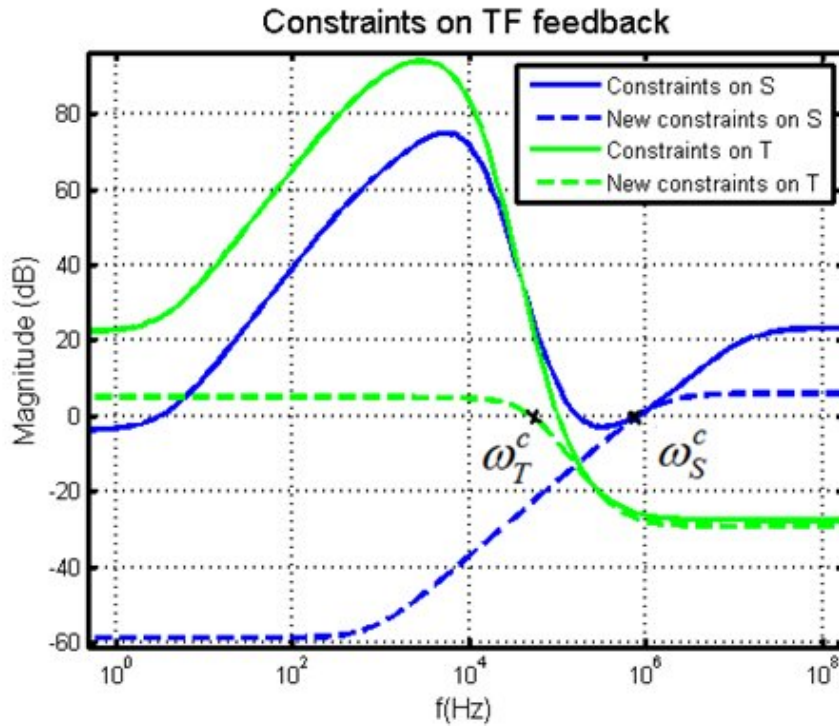


Fig. 9 New simplified constraints on feedback

The transfer function of the found controller, after the simplification, are:

$$F = \frac{20016(s + 3.7 \cdot 10^8)(s + 2.3 \cdot 10^{-6})}{(s + 2923)(s + 6.5 \cdot 10^6)}$$

In order to test the noise specifications, the PSD of each noise contribution (colored lines) as well as the overall PLL output PSD (black line) is presented in the Fig. 10. The dashed black line in the Fig. 10 shows the noise performance specifications.

The results of the Fig. 10 confirms that the controllers were chosen correctly.

Conclusion

The present document represents a new PLL design method using the Advanced Control System Theory tools.

In this work concerns an elaboration of a simple phase domain PLL model and the H_∞ control design method allowing to design the PLL controller. It turns out that this method is particularly well adapted to the PLL system design applications since it is a frequency based design approach and has an efficient

solution algorithms that can be integrated in the flow of the CAD PLL conception tools.

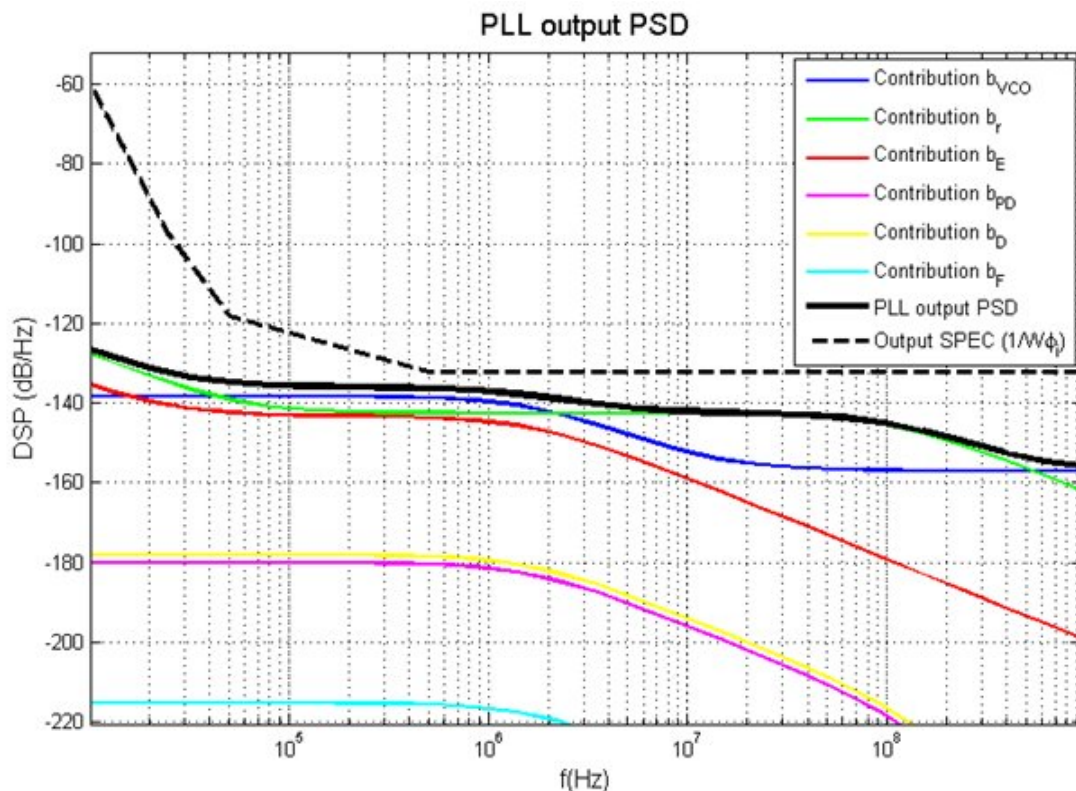


Fig. 10. PLL output PSD

The theoretical investigation of the new PLL design method shows its relevance with respect to the PLL performance limitations release and its significant improvement. Thanks to this, we managed to achieve the requirements for the control of performance of resonant vibrations CVG sensitive element to various disturbing factors.

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