

**ФУНКЦІОНАЛЬНА ЕЛЕКТРОНІКА. МІКРО- ТА  
НАНОЕЛЕКТРОННА ТЕХНІКА**

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**USE DIRECT VOLUME MAGNETOSTATIC WAVES  
FOR CONVOLVER CONSTRUCTION**

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**ВИКОРИСТАННЯ ПРЯМИХ ОБ'ЄМНИХ МАГНІТОСТАТИЧНИХ ХВИЛЬ  
ДЛЯ ПОБУДОВИ КОНВОЛЬВЕРА**

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**Introduction**

Magnetostatic waves (MSW), or another name, nonexchange spin waves [1], are those for which the most appropriate medium for excitation and propagation are magnetized to saturation monocrystalline epitaxial films Yttrium Iron Garnet (YIG). They can be used to create microwave devices as linear well as nonlinear: delay lines, resonators, filters, power limiters, generators and others [2]. The use of nonlinear properties of medium in which MSW propagate, can create based on them such device as a nonlinear signal processor convolver. The output of this device corresponds to the function of the convolution of two signals at the inputs of device. Thus, the nonlinear functional convolver performs the conversion of signals:

$$V_3(t) = \int_{-\infty}^{\infty} V_1(t) \cdot V_2(t - \tau) d\tau \quad (1)$$

where  $V_3(t)$  - the output signal or signal convolution, and  $V_1(t)$  and  $V_2(t)$  - input signals of convolver.

Convolver inputs are located on opposite ends of the film elements, and the formation of the convolution signal occurs when MSW propagate towards each other in the nonlinear medium. Let us note that in the case of the interaction of waves in a flat layer - epitaxial ferrite film - from three major classes of magnetostatic waves [1], the class of the surface MSW is not suitable for studying the interaction, because the opposite direction surface waves propagate along the oppo-

site surfaces of the layer, and almost do not interact. There is a good interaction only between forward and backward volume waves

### Problem statement

Possibility of creating convolver based on backward volume magnetostatic waves (BVMSW) has been investigated [3,4], but there are reasons to expect that convolver with forward volume magnetostatic waves (FVMSW) should be structurally simpler with the advantage in the signal/noise ratio convolution signal. The goal of this work was to consider the principle of convolver construction based on FVMSW and to calculate the signal of convolution for this convolver.

### Nonlinear interaction FVMSW propagating toward each other

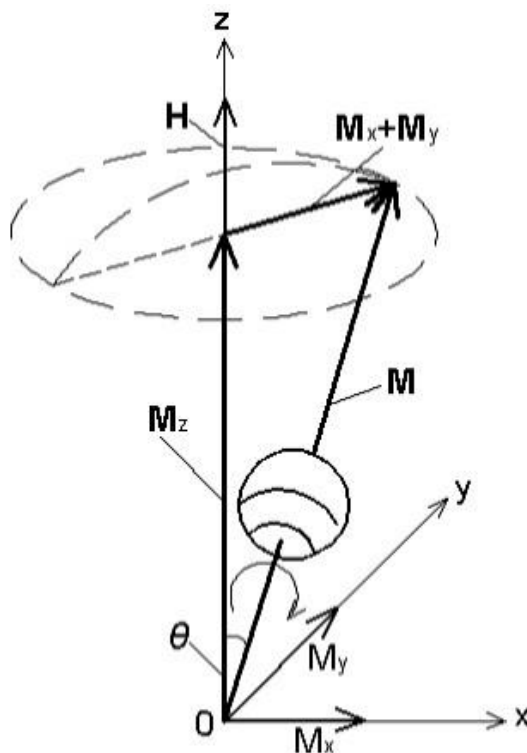


Fig.1 Precession of magnetic moment  $\mathbf{M}$  in a static magnetic field  $\mathbf{H}$

The magnetic properties interesting for us of magnetized to saturation static field  $\mathbf{H}$  ferromagnetic medium are defined by features of the precession of the magnetic moment of the electron, or, equivalently, a small but macroscopic volume of the medium around the balance direction of the magnetic moment, which coincides with the direction of the static magnetic field [5]. The magnetic moment  $\mathbf{M}$  precesses under the influence of the magnetic component of the electromagnetic wave, while the modulus of the magnetic moment (equal to the value of the saturation magnetization) does not change, but change the position

vector in space - vector uniformly rotates under the precession angle  $\theta$ , in this case the end of the vector is moving on trajectory, located on a spherical surface (Fig. 1). Suppose that the equilibrium direction of the magnetic moment coincides with the  $z$  axis of the Cartesian coordinate system. Then, in view of the above, we have the relation

$$M^2 = M_x^2 + M_y^2 + M_z^2 \quad (2)$$

where  $M$  - the modulus of the magnetic moment,  $M_x, M_y, M_z$ , vector components along the coordinate axes.

Because  $M_x$  and  $M_y$  the significantly small compared to  $M$ , then, laying found in (2) the value  $M_z$  in the Taylor series and keeping only the value of the quadratic term (in this case due to the very small precession angle error due to the neglected terms of higher order does not exceed 1%), we have:

$$M_z \approx M - \frac{M_x^2 + M_y^2}{2M} \quad (3)$$

Thus, the component  $M_z$  associated with  $M_x$  and  $M_y$  nonlinear relationship, which can be considered as approximately quadratic. The process of interaction of waves propagating toward each other in a medium with quadratic non-linearity can be described as follows.

Suppose that  $u_i$  the amplitude function of a Cartesian component of the magnetic field  $\mathbf{h}$  magnetostatic wave in the layer where  $i = 1, 2$  correspond the waves that propagate along the positive and negative directions of the axis  $y$ , respectively. Then

$$u_i(y, t) = U_i \left( t \mp \frac{y}{v_{ig}} \right) \exp j\omega_i t \quad (4)$$

Here  $U_i$  - the envelope of the amplitude function,  $v_{ig}$  - the group velocity. Upper signs in brackets refer to the wave traveling along positive direction of axis  $y$ .

As is known, the ratio of high-frequency component of the magnetic moment  $\mathbf{M}_t = \mathbf{M}_x + \mathbf{M}_y$  and the field  $\mathbf{h}$  of the wave in the medium:  $\mathbf{M}_t = \vec{\chi} \cdot \mathbf{h}$  where  $\vec{\chi}$  - magnetic susceptibility tensor. In turn, for the high-frequency magnetic field  $\mathbf{h}$  magnetostatic wave in the absence of external currents is true equation of magnetostatic  $\nabla \times \mathbf{h} = 0$ , so it can be expressed in terms of the magnetostatic potential  $\psi$ , that is  $\mathbf{h} = \nabla \cdot \psi$ , which in the case of a gyromagnetic medium magnetized to saturation at constant field along the  $z$  axis be found from the equation of Walker [5]

$$\mu \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + \frac{\partial^2 \psi}{\partial z^2} = 0, \quad (5)$$

where  $\mu$  - the diagonal component of the permeability tensor:  $\vec{\mu} = \mu_0(1 + \vec{\chi})$ ,  $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$ .

The solution for the magnetostatic potential corresponding FVMSW with unit amplitude function in normal magnetized layer, excluding losses on the distribution can be written as

$$\psi_i = (A \cos k_{iz} z + B \sin k_{iz} z) \exp(\mp j k_{iy} y), \quad (6)$$

that, as is easily seen, satisfies equation of Walker under condition

$$k_{iz}^2 = -\mu(\omega) k_{iy}^2, \quad (7)$$

Then (6) can be written as

$$\psi_i = (A \cos k_{iz} z + B \sin k_{iz} z) \exp(\mp \frac{j k_{iz} y}{\sqrt{-\mu(\omega)}})$$

Outside layer, taking into account that  $\mu = 1$  the equation (5) becomes the Laplace equation  $\nabla^2 \cdot \psi = 0$ , and the solution for the magnetostatic potential should be

$$\begin{aligned} \psi_{i0} &= C \exp(k_{iz0} z \mp j k_{iy} y) && \text{for } z < 0, \text{ end} \\ \psi_{i0} &= D \exp(-k_{iz0} z \mp j k_{iy} y) && \text{for } z > d. \end{aligned}$$

where  $d$  - thickness of the ferrite layer (film). When applying the appropriate boundary conditions

$$\psi_i = \psi_{i0} \text{ и } \partial \psi_i / \partial z = \partial \psi_{i0} / \partial z \text{ at } z = 0 \text{ and } z = d$$

will get the system of equations to determine the constants A, B, C, D, and the condition of its compatibility is the dispersion equation for the transverse wave number  $k_{iz}$ , and, as shown in [5]:

$$k_{iz0} = k_{iy} = k_{iz} / \sqrt{-\mu(\omega)}$$

Introducing  $M_z$  from (3) how  $M_0 + m_z$ , where  $M_0$ , and  $m_z$  there is constant and variable components of the  $z$ -component of the magnetic moment, and  $M \approx M_0$ , for the modulus  $m_z$  from (3) can be written

$$m_z \cong -\frac{1}{2M_0} [(\text{Re } M_{1x} + \text{Re } M_{2x})^2 + (\text{Re } M_{1y} + \text{Re } M_{2y})^2],$$

where  $\text{Re } M_{ix,y}$  - the real part of the component of  $x$  and  $y$  components of the magnetic moment parts, that match waves along the positive (index 1) and negative (subscript 2) direction of the axis  $y$ .

The relationship between the high frequency component of the magnetic moment  $\mathbf{M}_t = \mathbf{x}_0 M_x + \mathbf{y}_0 M_y$  imagine how  $\mathbf{M}_t = \tilde{\chi} \nabla \cdot \psi$ , where  $\mathbf{x}_0, \mathbf{y}_0$  - the unit vectors of the respective axes of the coordinate system. Since

$\mathbf{h}_x = \mathbf{x}_0(\partial\psi/\partial x) = 0$ , that  $\nabla \cdot \psi = \nabla_y \cdot \psi = \partial\psi/\partial y$ , then, by (6), and going from the tensor notation to writing via the components of the tensor, we obtain

$$m_z = -\frac{1}{2M_0} \{ [\eta_1 \operatorname{Re}(j\chi_{a1} \frac{\partial}{\partial y} e^{\omega_1 t - jk_{1y} y}) + \eta_2 \operatorname{Re}(j\chi_{a2} \frac{\partial}{\partial y} e^{\omega_2 t - jk_{2y} y})]^2 +$$

$$+ [\eta_1 \operatorname{Re}(\chi_1 \frac{\partial}{\partial y} e^{\omega_1 t - jk_{1y} y}) + \eta_2 \operatorname{Re}(\chi_2 \frac{\partial}{\partial y} e^{\omega_2 t - jk_{2y} y})]^2 \} \quad (8)$$

where  $\chi$  and  $\chi_a$  - diagonal and antisymmetric components of tensor, which, without excluding loss in media:

$$\chi = \frac{\omega_M \omega_0}{\omega_0^2 - \omega^2}, \quad \chi_a = \frac{\omega_M \omega}{\omega_0^2 - \omega^2}, \quad (9)$$

$\omega_0 = \mu_0 \gamma (H - M_0)$ ,  $\omega_M = \mu_0 \gamma M_0$ ,  $\gamma = 1.76 \cdot 10^{11}$  C / kg – gyro-magnetic ratio,  $M_0$  - the saturation magnetization,  $H$  - external constant magnetic field, and where, according to (6)

$$\eta_{1,2} = A \cos k_{1,2z} z + B \sin k_{1,2z} z \quad (10)$$

and the indices 1 and 2 for the diagonal  $\chi$  and asymmetric  $\chi_a$  tensor components correspond to the wavelengths (frequencies) of waves in the positive and negative directions of the axis of  $y$

Separating the real part, for  $m_z$  we have:

$$m_z = -\frac{1}{2M_0} \{ [\eta_1 k_{1y} \chi_{a1} \cos(\omega_1 t - k_{1y} y) - \eta_2 k_{2y} \chi_{a2} \cos(\omega_2 t + k_{2y} y)]^2 +$$

$$+ [\eta_1 k_{1y} \chi_1 \sin(\omega_1 t - k_{1y} y) - \eta_2 k_{2y} \chi_2 \sin(\omega_2 t + k_{2y} y)]^2 \}$$

It is a variable part of the  $z$ -component of the magnetic moment for the wave of unit amplitude. And with MSW, which carry signals and propagating toward each other, from (4) for the amplitude functions of these waves  $U_1 = U_1(t - y/v_{1g})$ ,  $U_2 = U_2(t - y/v_{2g})$ , the variable part of the  $z$ -component

of the magnetic moment  $m_z^A$

$$m_z^A = -\frac{1}{2M_0} \{ [U_1 \eta_1 k_{1y} \chi_{a1} \cos(\omega_1 t - k_{1y} y) - U_2 \eta_2 k_{2y} \chi_{a2} \cos(\omega_2 t + k_{2y} y)]^2 +$$

$$+ [U_1 \eta_1 k_{1y} \chi_1 \sin(\omega_1 t - k_{1y} y) - U_2 \eta_2 k_{2y} \chi_2 \sin(\omega_2 t + k_{2y} y)]^2 \} \quad (11)$$

It is clear that in addition to the quadratic component  $m_z^A$  includes a component proportional to the product of summands - combinational component, that is, from (11), we can write

$$m_z^A \sim \frac{U_1 U_2}{M_0} \eta_1 \eta_2 k_{1y} k_{2y} [\chi_{a1} \chi_{a2} \cos(\omega_1 t - k_{1y} y) \cos(\omega_2 t + k_{2y} y) - \chi_1 \chi_2 \sin(\omega_1 t - k_{1y} y) \sin(\omega_2 t + k_{2y} y)]$$

In the degenerate case  $\omega_1 = \omega_2 = \omega$ ,  $k_{1y} = k_{2y} = k_y$ ,  $\eta_1 = \eta_2 = \eta$ ,  $\chi_1 = \chi_2 = \chi$ ,  $\chi_{a1} = \chi_{a2} = \chi_a$ ,  $v_{1g} = v_{2g} = v_g$ , have

$$m_z^A \sim -\frac{U_1 U_2}{2M_0} (\eta k_y)^2 (\chi^2 - \chi_a^2) (\cos 2\omega t + \cos 2k_y y),$$

or, equivalently

$$m_z^A \sim -\frac{U_1 U_2}{M_0} (\eta k_y)^2 (\chi^2 - \chi_a^2) \cos(\omega t - k_y y) \cos(\omega t + k_y y)$$

Substituting from (9) the values of the tensor components, final expression for  $m_z^A$  will present in the form

$$m_z^A \sim -\frac{U_1(t - \frac{y}{v_g}) U_2(t + \frac{y}{v_g})}{2M_0} (\eta k_y)^2 \frac{\omega_M^2}{\omega_0^2 - \omega^2} (\cos 2\omega t + \cos 2k_y y) \quad (12)$$

### Convolver construction principle and the signal of convolution

Convolver construction principle based on FVMSW shown in Figure 2, where the parallel coordinate plane  $xy$ , that is parallel to the plane of the ferrite layer or film magnetized along the axis  $z$ , along the contour of the area propagation of two opposing FVMSW, who are carriers of signals, is located the loop as antenna.

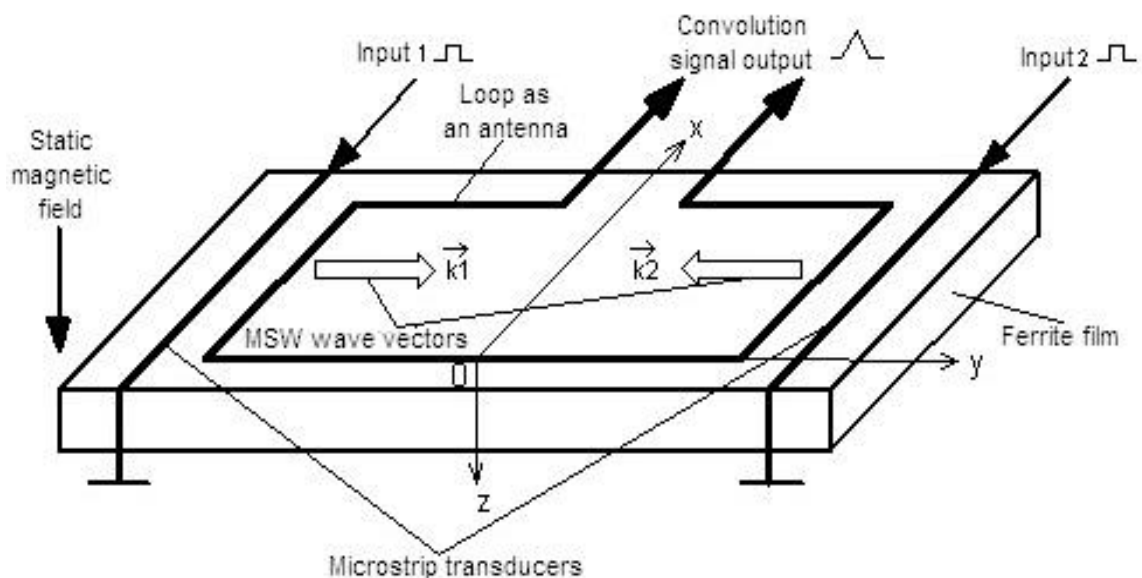


Fig.2 Convolver based on ferrite film, in which FVMSW excited

The induced voltage in the loop is equal to the time derivative of the normal component of the magnetic flux  $\phi_z$  through the plane of the loop, that is  $u_3(t) = -(d\phi_z / dt)$ .

Normal variable component flow through the plane of the loop  $\phi_z = \int_S \mathbf{z}_0 \mathbf{B} \cdot d\mathbf{S}$ , where  $\mathbf{B}$  - the variable component of the magnetic induction.

Then the flow through the element of plane of the loop  $dS$  :

$$d\phi_z = \mu_0 [\mathbf{z}_0 \nabla(\psi_{10} + \psi_{20}) + m_{Z0}^A] dS,$$

where  $\psi_{10}$  та  $\psi_{20}$  - is magnetostatic potentials, and  $m_{Z0}^A$  - the magnetic moment in the plane of the loop (at  $z = 0$ ) opposing FVMSW. With this in mind

$$u_3(t) = -\mu_0 \frac{\partial}{\partial t} \int_S [\mathbf{z}_0 \nabla(\psi_{10} + \psi_{20}) + m_{Z0}^A] \cdot d\mathbf{S} \quad (13)$$

Write the expression (13) for  $u_3(t)$  as

$$u_3(t) = -\mu_0 \frac{\partial}{\partial t} \left\{ \int_0^w dx \int_{-\frac{L}{2}}^{\frac{L}{2}} [\mathbf{z}_0 \nabla(\psi_{10} + \psi_{20}) + m_{Z0}^A] \cdot dy \right\}, \quad (14)$$

where  $W$  and  $L$  – width and length of the loop, respectively.

Layer or film, in which the FVMSW is propagating, can be considered as a homogeneous waveguide structure and when integrate along the direction of FVMSW because of rapid changes in the phase along the axis  $y$  quadratic components, and component  $\mathbf{z}_0 \nabla(\psi_{10} + \psi_{20})$  will give a very small contribution to the voltage induced in the loop, as opposed to combinational components with a uniform spatial distribution. Therefore, the signal at the output of the antenna in the form of a loop, neglecting small components, which include the time derivatives of the envelope amplitude functions  $U_{1,2}$ , combining (12) and (14) can be written as

$$u_3(t) = \sin 2\omega t \cdot K \int_{-\frac{L}{2}}^{\frac{L}{2}} U_1\left(t - \frac{y}{v_g}\right) U_2\left(t + \frac{y}{v_g}\right) dy \quad (15)$$

where, by (7), (9), (10), (12),

$$K(\omega) = \mu_0 \omega \frac{\omega_M^2}{\omega_M^2 - \omega^2} \frac{k_z^2 w}{\mu(\omega)} \frac{A^2}{M_0}$$

If make replace in (15)

$$(t - y/v_g) = \tau, \quad u_3(t) = U_3(t) \sin 2\omega t$$

we can write

$$U_3(t) = K \int_{-\frac{L}{2}}^{\frac{L}{2}} U_1(\tau) U_2(2t - \tau) d\tau$$

If we assume that the interaction area of wave packets along the  $y$  is small compared to  $L$ , the limits of integration can be replaced by infinity and write

$$U_3(t) = K \int_{-\infty}^{\infty} U_1(\tau) U_2(2t - \tau) d\tau$$

from which, in comparison with (1), we see that  $U_3(t)$  - it is a signal that corresponds to the convolution of signals  $U_1(t)$  and  $U_2(t)$ , the double compressed in time due to the counter motion of the wave packets.

### **Conclusions**

The possibility of constructing convolver using direct volume MSW in a ferrite film is shown. In [3] considered the possibility of convolver construction with using of backward volume MSW with the magnetization of the film along the axis  $y$ . But because of the possible occurrence of unstable nonlinear phenomena associated with parametric excitation of exchange spin waves [5] is significantly limited the level of signals that can be fed to the inputs of convolver and, accordingly, the signal / noise ratio at the output will be low. For direct volume MSW limit signals, beyond which there is an unstable nonlinear phenomenon is much higher, so you can give signals to a higher level and get a much better signal / noise ratio. Besides, creating a uniform magnetic field along the axis  $y$  is structurally more complex than the magnetization normal to the film. And although because of demagnetization factor along the normal to the film must be the higher value of external magnetic field this is not a difficulty, by reason of the small gap of the magnetic system,.

The peculiarity a magnetized ferrite film as a waveguide structure is visible, that at certain ratios of the carrier frequency of signal and value of the magnetizing field is considerable frequency dispersion, which leads to a distortion of the signals that are pass in the structure [6], and this is reflected in the formation of the signal convolution the input signals of convolver. It is therefore interesting to study the effect of the influence dispersion characteristics of the waveguide structure on the formation of convolution signal

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*Кудінов Є.В. Використання прямих об'ємних магнітостатичних хвиль для побудови конвольвера. Показано, що використання прямих об'ємних магнітостатичних хвиль (ПОМСХ) в епітаксіальній феритовій плівці може бути основою побудови конвольвера. В результаті теоретичного аналізу підтверджено, що вихідний сигнал відповідає стислій в часі в два рази (через зустрічний рух хвиль, що несуть сигнали) функції згортки сигналів, що надходять на входи. Визначено вираз для обчислення сигналу згортки, що враховує частотні характеристики намагніченої плівки як хвилеведучої структури, розміри плівки та антени у вигляді прямокутної петлі, індукована напруга в якій відповідає сигналу згортки. Так як для ПОМСХ граничний рівень сигналів, при перевищенні якого виникають нестабільні нелінійні явища, пов'язані з параметричним збудженням обмінних спінових хвиль, високий, можна подавати сигнали більш високого рівня і отримати гарне співвідношення сигнал/шум для сигналу згортки.*

**Ключові слова:** епітаксіальна феритова плівка, нелінійні властивості, магнітостатичні хвилі, функція згортки сигналів, конвольвер.

*Кудинов Е.В. Использование прямых объемных магнитостатических волн для построения конвольвера. Показано, что использование прямых объемных магнитостатических волн (ПОМСВ) в эпитаксиальной ферритовой пленке может быть основой построения конвольвера. В результате теоретического анализа подтверждено, что выходной сигнал соответствует сжатой во времени в два раза (из-за встречного движения волн, несущих сигналы) функции свертки, поступающих на входы сигналов. Определено выражение для вычисления сигнала свертки, учитывающее частотные характеристики намагниченной пленки как волноведущей структуры, размеры пленки и антенны в виде прямоугольной петли, индуцированное напряжение в которой соответствует сигналу свертки. Так как для ПОМСВ предельный уровень сигналов, при превышении которого возникают нестабильные нелинейные явления, связанные с параметрическим возбуждением обменных спиновых волн, высокий, можно подавать сигналы более высокого уровня и получить хорошее соотношение сигнал / шум для сигнала свертки.*

**Ключевые слова:** эпитаксиальная ферритовая пленка, нелинейные свойства, магнитостатические волны, функция свертки сигналов, конвольвер.

*E.Kudinov. Use direct volume magnetostatic waves for convolver construction It is shown that the use of direct volume magnetostatic waves (FVMSW) in epitaxial ferrite film can be the basis for convolver construction. A theoretical analysis confirmed that the output signal corresponds to the double compressed in time (because of counter motion waves that carry signals) convolution function at the input signals. Defined expression for calculating signal of convolution, which takes into account the frequency characteristics of a magnetized film as waveguide structure, the sizes of the film and the antenna in form a rectangular loop, the induced voltage which corresponds to the signal convolution. For FVMSW limit signals, beyond which there is an unstable nonlinear phenomena associated with parametric excitation of exchange spin waves, tall, can give signals of a higher level and get a good signal/noise ratio for the signal convolution.*

**Keywords:** epitaxial ferrite film, nonlinear properties, magnetostatic waves, convolution signals, convolver.