MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
«IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE»

# HIGHER MATHEMATICS

## DIFFERENTIAL EQUATIONS

**Practice exercises collection**

Recommended by the Methodological Council
of the Igor Sikorsky Kyiv Polytechnic Institute
as a study aid for bachelor's degree applicants
on the technical specialties

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The practice book offers additional individual exercises for university students studying Differential Equations in the course of Higher Mathematics of Igor Sikorsky KPI. The book contains 30 different variants and each variant consists of 9 exercises (21 tasks). Students master the material being studied and consolidate the acquired knowledge by solving such individual tasks.

The practice book can be recommended as an individual work on Differential Equations for first-year students of technical specialties.

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# CONTENT

# INTRODUCTION

The Differential Equations section is included in the course of Higher Mathematics for engineering students of Igor Sikorsky KPI. An important factor in the successful assimilation of the educational material by the students is solving practical tasks on their own.

The practice book offers a systematized set of exercises that students of technical specialties should be able to solve when studying Differential Equations. The book contains 30 different variants and each variant consists of 9 exercises (21 tasks).

This practice book helps students to develop practical skills in solving basic types of first-order differential equations, such as separable differential equations, homogeneous, linear and Bernoulli equations, as well as higher-order differential equations, linear differential equations with constant coefficients and systems of ordinary differential equations.

## GENERAL RECOMMENDATIONS

The practice book is designed to control and improve the knowledge of university students in the study of Differential Equations in the course of Higher Mathematics. The main goal is to develop and consolidate the skills of independent work of students in the study of educational material.

In order to successfully complete the exercises, students need to thoroughly study the lecture material and analyze the examples solved in practical classes. Only after that students can start solving their individual tasks.

Students have to adhere to the following requirements:

1. The number of the variant of the individual exercises corresponds to the ordinal number of the student in the list of the study group;
2. Individual work is written in a separate notebook, which should contain:
    - the title page;
    - the results table;
    - solved exercises (the solution of each exercise starts from a new page).
3. Before solving each exercise, the condition and all specific data for the corresponding variant are completely rewritten.
4. The solution of each task must contain detailed explanations and necessary formulas.
5. Completed work must be handed over to the teacher for verification within the prescribed time limit.

Students who do not submit their completed individual work on time will not be allowed to take the exam.

# Differential Equations

## Variant № 1

**Exercise 1.** Solve the first-order differential equations:

a) $2\,\mathrm{e}^x\,\mathrm{tg}\,y\,dx + (1+\mathrm{e}^x)\sec^2 y\,dy = 0$;  b) $y' = \cos(2x+2y-1)$;

c) $4x^2y' = y^2 + 10xy + 5x^2$;  d) $y' = \dfrac{x+y+2}{x+1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' - y\,\mathrm{ctg}\,x = 3x^2\sin x,\quad y\left(\dfrac{\pi}{2}\right) = \dfrac{\pi^3}{8}$;  b) $y' + \dfrac{y}{x} = y^2\dfrac{\ln x}{x},\quad y(1)=1$;

c) $dx = (x\,\mathrm{tg}\,y + 2y - y^2\,\mathrm{tg}\,y)dy,\quad y(1)=0$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \dfrac{1+\cos^2 x}{1+\cos 2x}$;  b) $xy'' = y'\ln\dfrac{y'}{x}$.

**Exercise 4.** Find the solution to the Cauchy problem: $2(y')^2 = y''(y-1),\ y(1)=2,\ y'(1)=-1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $2y'' - 7y' + 3y = 0$;  b) $4y'' + 20y' + 25y = 0$;  c) $y'' + y' + y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + y' = 0,\quad y(0)=2,\quad y'(0)=0,\quad y''(0)=-1.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + y = \frac{1}{\sin x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(4)} - 4y''' + 4y'' = 48x^2 - 8$;  b) $y'' - 3y' + 2y = (6x+1)\,\mathrm{e}^{-x}$;

c) $y'' + 2y' + 2y = 8\,\mathrm{e}^x\cos x$;  d) $y'' + 9y = 18x + 12\cos 3x$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases}\dot{x} = 24y - x,\\ \dot{y} = x - 3y;\end{cases}$  b) $\begin{cases}\dot{x} = 2x - y,\\ \dot{y} = y - 2x + 18t.\end{cases}$

# Differential Equations

## Variant № 2

**Exercise 1.** Solve the first-order differential equations:

**a)** $x^3\sin 2y dy - x^2\sin^2 y dx = x^2 dx - 7\sin 2y dy$; **b)** $y' = \sqrt[4]{2x+y-1}$;

**c)** $y' = \dfrac{x^2+xy-5y^2}{x^2-6xy}$; **d)** $y' = \dfrac{2y-2}{x+y-2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' - \dfrac{2}{x+1}y = \mathrm{e}^{x+1}(x+1)^2, \quad y(0) = \mathrm{e}$; **b)** $y' + 2y\,\mathrm{cth}\,x = y^2\,\mathrm{ch}\,x, \quad y(1) = \dfrac{1}{\mathrm{sh}\,1}$;

**c)** $2y^2dx = -(\sqrt{y}+2xy)dy, \quad y(-1) = 1$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = \dfrac{1+2x^2}{x^2(1+x^2)}$; **b)** $2xy'y'' = (y')^2 + 1$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' = \mathrm{e}^{2y}, \quad y(0) = 0, \quad y'(0) = 1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $4y'' + 4y' - 3y = 0$; **b)** $36y'' - 12y' + y = 0$; **c)** $y'' - 4y' + 7y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y^V - y' = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1, \quad y^{IV}(0) = 2.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 3y' + 2y = \frac{1}{3+\mathrm{e}^{-x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(4)} + 2y''' + 2y'' = 12x$; **b)** $y'' + 4y' + 3y = (12-16x)\mathrm{e}^x$;

**c)** $y'' - 2y' + 17y = \mathrm{e}^{4x}(24\cos x - 6\sin x)$; **d)** $y'' - 8y' + 15y = 96\,\mathrm{ch}\,3x$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = y - 2x, \\ \dot{y} = 10x + y; \end{cases}$ **b)** $\begin{cases} \dot{x} = 2x - y, \\ \dot{y} = x + 2\mathrm{e}^t. \end{cases}$

## Differential Equations

### Variant № 3

**Exercise 1.** Solve the first-order differential equations:

**a)** $(1+y)(e^x\,dx - e^{2y}\,dy) - (1+y^2)dy = 0$;

**b)** $y' = (9x+4y-1)^2$;

**c)** $xy' = \dfrac{3y^3+14yx^2}{2y^2+7x^2}$;

**d)** $y' = \dfrac{x+2y-3}{x-1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' - \dfrac{2xy}{x^2+1} = 3\operatorname{arctg}^3 x, \quad y(0)=0$;

**b)** $y' - y\operatorname{tg} x = -\dfrac{2}{3}y^4\sin x, \quad y(0)=1$;

**c)** $ydx = 2(x+\ln^2 y - \ln y)dy, \quad y(1)=1$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = \log_2 x$;

**b)** $y''(1+x^2) = 2xy'$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' = -2\sin y\cos^3 y, \; y(0)=0, \; y'(0)=1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $3y''-4y'+y=0$;

**b)** $4y''-4y'+y=0$;

**c)** $y''+5y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y'''-y'=0, \quad y(0)=3, \quad y'(0)=-1, \quad y''(0)=1.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+y=4\operatorname{ctg} x.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(5)}-4y^{(4)}+3y'''=72x-42$;

**b)** $y''-2y'+5y=(10x-6)e^{2x}$;

**c)** $y''-5y'+6y=2e^{3x}\sin x$;

**d)** $y''+4y'=8(e^{-4x}+1)$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = 3x + y, \\ \dot{y} = 5y - 5x; \end{cases}$

**b)** $\begin{cases} \dot{x} = x + 2y + \sin t, \\ \dot{y} = 4x - y. \end{cases}$

# Differential Equations

## Variant № 4

**Exercise 1.** Solve the first-order differential equations:

**a)** $3x^2ydy - 5xy^2dx = 20xdx - 3ydy$;

**b)** $y' = (\cos(8x + 8y - 1))^{-1}$;

**c)** $xy' - y = 4\sqrt{x^2 + y^2}$;

**d)** $y' = \dfrac{5y + 5}{4x + 3y - 1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' - \dfrac{y}{x} = -\dfrac{\ln x}{x}, \quad y(1) = 1$;

**b)** $y' + xy = \dfrac{y^2}{2}(x - 1)\,\mathrm{e}^x, \quad y(0) = 2$;

**c)** $(x + \mathrm{e}^{\frac{1}{y}})y' = -2y^2, \quad y(\mathrm{e}) = 1$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = \dfrac{4}{\sqrt{1 - x^2}}$;

**b)** $y'' = \dfrac{y'}{x} + \dfrac{x^2}{y'}$.

**Exercise 4.** Find the solution to the Cauchy problem: $4y^3y'' + 1 = y^4, \ y(0) = \sqrt{2}, \ y'(0) = 8^{-\frac{1}{2}}$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $3y'' - y = 0$;

**b)** $25y'' + 30y' + 9y = 0$;

**c)** $y'' - 2y' + 4y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + 2y'' + 5y' = 0, \quad y(0) = -1, \quad y'(0) = 2, \quad y''(0) = 0.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 6y' + 8y = \frac{4}{2 + \mathrm{e}^{-2x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(5)} + y''' = 6x^2 + 24x$;

**b)** $y'' + 3y' = (12x + 2)\,\mathrm{e}^{-3x}$;

**c)** $y'' + 2y' + y = 4\,\mathrm{e}^{-x}\cos 2x$;

**d)** $y'' + 4y' + 13y = 40\cos 3x + 25\,\mathrm{e}^{2x}$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = 6x - 2y, \\ \dot{y} = x + 3y; \end{cases}$

**b)** $\begin{cases} \dot{x} = 3x - 2y + t, \\ \dot{y} = 3x - 4y. \end{cases}$

# Differential Equations

## Variant № 5

**Exercise 1.** Solve the first-order differential equations:

**a)** $(xy^2+4y^2)dx+(2x^2-x^2y)dy=0;$ **b)** $y'=\sqrt[3]{12x+3y+2};$

**c)** $2x^2y'=y^2+8xy+8x^2;$ **d)** $y'=\dfrac{x+y-4}{x-2}.$

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y'-\dfrac{y}{x}=-\dfrac{4}{x^2},\quad y(1)=2;$ **b)** $y'-y=2xy^2,\quad y(0)=\dfrac{1}{2};$

**c)** $ydx=(2\sin^2 y+y\sin 2y-2x)dy,\quad y(1)=\dfrac{\pi}{2}.$

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y''=x\cos x;$ **b)** $xy''+x(y')^2=y'.$

**Exercise 4.** Find the solution to the Cauchy problem: $y^3y''+16=y^4,\ y(0)=2\sqrt{2},\ y'(0)=\sqrt{2}.$

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $2y''+3y'+y=0;$ **b)** $25y''+40y'+16y=0;$ **c)** $y''+2y'+3y=0.$

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y'''-y''-y'+y=0,\quad y(0)=-1,\quad y'(0)=0,\quad y''(0)=13.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+9y=\frac{9}{\cos 3x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(5)}-4y^{(4)}+y'''+6y''=36x-6;$ **b)** $y''-10y'+25y=(16x+24)e^{x};$

**c)** $y''-2y'+2y=8e^{-x}(\cos x-\sin x);$ **d)** $y''+36y=36+24\sin 6x.$

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases}\dot{x}=8y-3x,\\ \dot{y}=x-y;\end{cases}$ **b)** $\begin{cases}\dot{x}=2x-4y,\\ \dot{y}=x-3y+3e^{t}.\end{cases}$

# Differential Equations

## Variant № 6

**Exercise 1.** Solve the first-order differential equations:

a) $xydy = (1+y^2)dx + x\sqrt{1+y^2}\,dy$;  b) $y' = \sqrt[3]{(25x+3y-1)^2}$;

c) $y' = \dfrac{x^2+3xy-y^2}{3x^2-2xy}$;  d) $y' = \dfrac{y+2}{2x+y-4}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' + \dfrac{y}{x} = \dfrac{x+2}{x}\mathrm{e}^x, \quad y(1) = 2\,\mathrm{e}$;  b) $y' + 2xy = 2y^3x^3, \quad y(0) = \sqrt{2}$;

c) $dx = (3x + \sin y + 3\cos y)dy, \quad y(-1) = 0$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y''' = \dfrac{1}{\sqrt[3]{\mathrm{e}^{2x}}}$;  b) $y'' - 2y'\operatorname{ctg} x = \sin^3 x$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' - 18y^3 = 0, \quad y(1) = 1, \quad y'(1) = 3$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $5y'' - 6y' = 0$;  b) $25y'' - 20y' + 4y = 0$;  c) $4y'' + 4y' + 5y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + y' = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 1.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - y' = \frac{\mathrm{e}^{-x}}{2+\mathrm{e}^{-x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(4)} - 4y''' + 5y'' = 30x + 6$;  b) $y'' + 2y' + 10y = (-25x - 8)\,\mathrm{e}^{3x}$;

c) $y'' - 5y' + 4y = \mathrm{e}^{-4x}(13\cos x + 91\sin x)$;  d) $y'' - y' - 2y = 24\operatorname{sh} 2x$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = 4x - y, \\ \dot{y} = x + 6y; \end{cases}$  b) $\begin{cases} \dot{x} = 2x - 3y, \\ \dot{y} = x - 2y + 2\sin t. \end{cases}$

# Differential Equations

## Variant № 7

**Exercise 1.** Solve the first-order differential equations:

**a)** $\sqrt{3-x^2}\,y' = 16x - 4xy^2$;  **b)** $y' = (\sin(2y-2x-7))^{-2}$;

**c)** $xy' = \dfrac{3y^3 + 10yx^2}{2y^2 + 5x^2}$;  **d)** $y' = \dfrac{2x+y-3}{2x-2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' - \dfrac{2y}{x+1} = 3(x+1)^4, \quad y(0) = 1$;  **b)** $y' + \dfrac{y}{2}\cos x = \dfrac{(1+\sin x)\cos x}{2y}, \quad y(0) = 1$;

**c)** $ydx = (x + 2\ln y - \ln^2 y)dy, \quad y(1) = 1$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = \mathrm{tg}^2\, x$;  **b)** $xy''' + y'' = \sqrt{x}$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' = 8\cos y \sin^3 y, \; y(1) = \dfrac{\pi}{2}, \; y'(1) = 2$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $2y'' + 5y' - 3y = 0$;  **b)** $4y'' + 4y' + y = 0$;  **c)** $3y'' + 4y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y^{IV} + 10y'' + 9y = 0, \quad y(0) = 1, \quad y'(0) = 3, \quad y''(0) = -9, \quad y'''(0) = -27.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + 3y' + 2y = \frac{e^{-x}}{2 + e^{x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(4)} - 2y''' + y'' = 12x^2 - 12x - 20$;  **b)** $y'' + 2y' = (4x+2)e^{-2x}$;

**c)** $y'' + 4y' + 13y = 24e^{-2x}\sin x$;  **d)** $y'' - 2y' + 17y = 16e^{x} + 65\sin 4x$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = 3x + y, \\ \dot{y} = 6y; \end{cases}$  **b)** $\begin{cases} \dot{x} = x + 2y + t, \\ \dot{y} = 2x + y + t. \end{cases}$

## Differential Equations

### Variant № 8

**Exercise 1.** Solve the first-order differential equations:

a) $2x^2ydy - 3xy^2dx = 6xdx - 6ydy$; b) $y' = \sqrt{9x + 2y + 4}$;

c) $xy' - y = 3\sqrt{2x^2 + y^2}$; d) $y' = \dfrac{y}{2x + 2y - 2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' - 4xy = 8x^3, \quad y(0) = -1$; b) $y' - \dfrac{3}{2x}y = -\dfrac{(5x^2+3)}{2x}y^3, \quad y(1) = \dfrac{1}{\sqrt{2}}$;

c) $\sin y \cos y dx = (\cos 2y \cos^2 y - x)dy, \quad y\left(\dfrac{1}{2}\right) = \dfrac{\pi}{4}$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = x\,\mathrm{e}^x + x$; b) $y'''(1 + \sin x) = y'' \cos x$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''y^3 + 9 = 0, \quad y(1) = 1, \quad y'(1) = 3$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $3y'' - 2y' - 8y = 0$; b) $9y'' - 30y' + 25y = 0$; c) $4y'' - 8y' + 5y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' - y'' - 2y' = 0, \quad y(0) = 4, \quad y'(0) = -4, \quad y''(0) = -2.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + 4y = 8\,\mathrm{ctg}\,2x.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)} + 3y^{(4)} - 4y''' = 96x$; b) $y'' + 5y' - 6y = (-20x - 4)\mathrm{e}^{-x}$;

c) $y'' - 6y' + 10y = \mathrm{e}^x(4\cos 3x + 28\sin 3x)$; d) $y'' - 7y' = 14 - 7\mathrm{e}^{7x}$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = -2x + y, \\ \dot{y} = -9x - 2y; \end{cases}$ b) $\begin{cases} \dot{x} = -x + 3y + 2\mathrm{e}^{-t}, \\ \dot{y} = x + y + 1. \end{cases}$

# Differential Equations

## Variant № 9

**Exercise 1.** Solve the first-order differential equations:

a) $\sin x(\ln^2 y+3)dy-ydx=0$; b) $y'=(5x+2y-1)^{-1}$;

c) $x^2y'=y^2+8xy+12x^2$; d) $y'=\dfrac{x+y-2}{2x-2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y'+\dfrac{2xy}{1+x^2}=\dfrac{3x^2}{1+x^2}, \quad y(0)=1$; b) $y'+4x^3y=4y^2(x^3+1)\mathrm{e}^{-4x}, \quad y(0)=1$;

c) $y^2dx=(\sqrt{y}+xy)dy, \quad y\left(-\dfrac{2}{3}\right)=1$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y''=\dfrac{\cos 2x}{\cos^2 x\sin^2 x}$; b) $xy'''+y''=\dfrac{1}{\sqrt{x}}$.

**Exercise 4.** Find the solution to the Cauchy problem: $2(y')^2=y''(y-2), \; y(1)=3, \; y'(1)=-1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $2y''-5y'+3y=0$; b) $36y''+12y'+y=0$; c) $y''-6y'+11y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y'''-6y''+11y'-6y=0, \quad y(0)=2, \quad y'(0)=6, \quad y''(0)=20.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+3y'=\frac{9\mathrm{e}^{3x}}{1+\mathrm{e}^{3x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(4)}+6y'''+9y''=108x^2+90x+6$; b) $y''-2y'+10y=(25x-33)\mathrm{e}^{-3x}$;

c) $y''-y'-6y=\mathrm{e}^x(2\cos 2x-10\sin 2x)$; d) $y''+y=x+2\cos x$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases}\dot{x}=-2x-10y,\\ \dot{y}=x-9y;\end{cases}$ b) $\begin{cases}\dot{x}=y-5\cos t,\\ \dot{y}=2x+y.\end{cases}$

# Differential Equations

## Variant № 10

**Exercise 1.** Solve the first-order differential equations:

**a)** $x^2ydy = (x+5)\sqrt{y^2-1}dx;$ **b)** $y' = (\operatorname{tg}(4x+4y-1))^{-2};$

**c)** $y' = \dfrac{x^2+xy-3y^2}{x^2-4xy};$ **d)** $y' = \dfrac{2x+y-3}{4x-4}.$

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' + 3x^2y = 3x^2(x^3+1), \quad y(0) = 1;$ **b)** $y' - \dfrac{y}{x} = -y^2(\ln x + 2)\dfrac{\ln x}{x}, \quad y(1) = 1;$

**c)** $y\cos^2 y dx = (x\cos^2 y - y^2)dy, \quad y(\pi) = \pi.$

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = \dfrac{2x}{x^2+1};$ **b)** $y''\operatorname{cth} x + y' = \operatorname{ch} x.$

**Exercise 4.** Find the solution to the Cauchy problem: $y'' - e^{4y} = 0, \quad y(0) = 0, \quad y'(0) = \dfrac{1}{\sqrt{2}}.$

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $4y'' - 5y' + y = 0;$ **b)** $25y'' - 70y' + 49y = 0;$ **c)** $y'' + 2y' + 6y = 0.$

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + 9y' = 0, \quad y(0) = 0, \quad y'(0) = 9, \quad y''(0) = -18.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 3y' + 2y = \frac{1}{2+e^{-x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(5)} - y^{(4)} - 2y''' = 48x - 24;$ **b)** $y'' - 8y' + 16y = (-64x - 48)e^{-4x};$

**c)** $y'' + 4y' + 5y = -40e^{2x}\sin x;$ **d)** $y'' - 2y' - 3y = 8\operatorname{ch} x.$

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = 5x + y, \\ \dot{y} = 6x + 4y; \end{cases}$ **b)** $\begin{cases} \dot{x} = 2x - 4y + 4e^{-2t}, \\ \dot{y} = 2x - 2y. \end{cases}$

# Differential Equations

## Variant № 11

**Exercise 1.** Solve the first-order differential equations:

**a)** $\sqrt{6+y^2}\,dx+4(x^2y+9y)dy=0$; **b)** $y'=\sqrt[4]{2x+y-1}$;

**c)** $xy'=\dfrac{3y^3+12yx^2}{2y^2+6x^2}$; **d)** $y'=\dfrac{y-2x+3}{x-1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y'-\dfrac{y}{x}=3\ln^2 x,\quad y(1)=1$; **b)** $y'+\dfrac{3}{2}y\cos x=y^{-1}\,e^{2x}\left(1+\dfrac{3}{2}\cos x\right),\quad y(0)=1$;

**c)** $y^2dx=-(x+e^{\frac{y}{2}})dy,\quad y(e^2)=1$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y''=\dfrac{1}{\sqrt{1-x^2}}$; **b)** $x^4y''+x^3y'=4$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''=-8\sin y\cos^3 y,\ y(0)=0,\ y'(0)=2$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $3y''-2y'-y=0$; **b)** $25y''-40y'+16y=0$; **c)** $3y''+16y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y^{IV}-y=0,\quad y(0)=0,\quad y'(0)=0,\quad y''(0)=0,\quad y'''(0)=-4.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+16y=\frac{16}{\sin 4x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(5)}+4y'''=96x+24$; **b)** $y''-y'=(4x+1)e^x$;

**c)** $y''+6y'+9y=5e^{-3x}(\cos x+\sin x)$; **d)** $y''-4y'+13y=40\sin 3x-9e^{2x}$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases}\dot{x}=15y,\\ \dot{y}=2y+x;\end{cases}$ **b)** $\begin{cases}\dot{x}=3x+5y+2t,\\ \dot{y}=3x+y+1.\end{cases}$

## Differential Equations

### Variant № 12

**Exercise 1.** Solve the first-order differential equations:

a) $e^{2x}\,dx = y(e^{x}+4)dy$; b) $y' = (x+16y+3)^{-2}$;

c) $xy' - y = 2\sqrt{3x^2+y^2}$; d) $y' = \dfrac{x+y-2}{3x-y-2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' - \dfrac{y}{x} = -\dfrac{18}{x^3}, \quad y(1)=6$; b) $y' + xy = y^2\,e^{x}(x-1), \quad y(0)=1$;

c) $\sin 2y\,dx = (2x - 2\sin^2 y + \sin^2 2y)dy, \quad y\left(\dfrac{1}{2}\right) = \dfrac{\pi}{4}$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \dfrac{2}{\operatorname{ctg}^3 x \sin^2 x}$; b) $y''' + \dfrac{y''}{x} = 1 + \dfrac{1}{x}$.

**Exercise 4.** Find the solution to the Cauchy problem: $4y^3y''+1 = 16y^4,\ y(0)=\dfrac{1}{\sqrt{2}},\ y'(0)=\dfrac{1}{\sqrt{2}}$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $2y'' + y' - y = 0$; b) $4y'' - 20y' + 25y = 0$; c) $5y'' - 10y' + 6y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' - 6y'' + 13y' = 0, \quad y(0)=6, \quad y'(0)=-1, \quad y''(0)=-19.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 6y' + 8y = \frac{4}{1+e^{-2x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(4)} + 6y''' + 10y'' = 120x^2 - 24x - 8$; b) $y'' + 2y' + 5y = (13x+6)e^{2x}$;

c) $y'' - 7y' + 12y = 2e^{4x}\cos x$; d) $y'' - 4y = 4 + 12x - 4e^{-2x}$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = 3x + 9y, \\ \dot{y} = x + 3y; \end{cases}$ b) $\begin{cases} \dot{x} = -5x - y + e^{t}, \\ \dot{y} = -2y + e^{2t}. \end{cases}$

# Differential Equations

## Variant № 13

**Exercise 1.** Solve the first-order differential equations:

a) $y'y\sqrt{16-x^2}+\sqrt{7+y^2}=0$;

b) $y'=(\operatorname{ctg}(2x+2y-1))^{-2}$;

c) $2x^2y'=y^2+6xy+3x^2$;

d) $y'=\dfrac{x+3y+4}{3x-6}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y'+y\operatorname{tg}x=x\cos^2 x,\quad y(0)=1$;

b) $y'+\dfrac{y}{x}=\dfrac{y^2}{3x}\ln x,\quad y(1)=3$;

c) $dx=2(xy+y^3-y)dy,\quad y(-2)=0$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y''=\dfrac{1+\sin^2 x}{1+\cos 2x}$;

b) $xy''-y'=x^2\,\mathrm{e}^x$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''-2y^3=0,\quad y(-1)=1,\quad y'(-1)=1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $4y''+5y'-6y=0$;

b) $9y''+12y'+4y=0$;

c) $3y''-6y'+7y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y'''+3y''+3y'+y=0,\quad y(0)=-1,\quad y'(0)=2,\quad y''(0)=3.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+4y=\frac{4}{\cos 2x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)}-3y^{(4)}-6y'''+8y''=48x-4$;

b) $y''-4y'=(24x-6)\,\mathrm{e}^{4x}$;

c) $y''+2y'+37y=35\,\mathrm{e}^{-x}(\sin x-\cos x)$;

d) $y''-6y'+10y=30\cos 2x+\mathrm{e}^{3x}$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases}\dot{x}=10x+y,\\ \dot{y}=4y-9x;\end{cases}$

b) $\begin{cases}\dot{x}=x+2y,\\ \dot{y}=x-5\sin t.\end{cases}$

## Differential Equations

### Variant № 14

**Exercise 1.** Solve the first-order differential equations:

**a)** $x^3dx - 4\sin 2y dy = x^4 \sin 2y dy - x^3 \sin^2 y dx$; **b)** $y' = (\cos(2y - 2x - 7))^{-2}$;

**c)** $y' = \dfrac{x^2 + 2xy - 5y^2}{2x^2 - 6xy}$; **d)** $y' = \dfrac{x - 2y + 3}{-2x - 2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' - \dfrac{y}{x} = -\dfrac{\ln x}{x}$, $\quad y(1) = 1$; **b)** $y' + \dfrac{2}{3}xy = \dfrac{2}{3}y^{-2}x\,\mathrm{e}^{-2x^2}$, $\quad y(0) = -1$;

**c)** $4ydx = (13y^3 - x)dy$, $\quad y(1) = 1$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = \dfrac{2\cdot 3^x - 9^x x}{3^x}$; **b)** $xy'' - y' = x\sin\dfrac{y'}{x}$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' = 2\cos y \sin^3 y$, $\quad y(1) = \dfrac{\pi}{2}$, $\quad y'(1) = 1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $2y'' + 9y' - 5y = 0$; **b)** $4y'' - 12y' + 9y = 0$; **c)** $y'' + 2y' + 7y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + y' = 0, \quad y(0) = 2, \quad y'(0) = 0, \quad y''(0) = -1.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + 6y' + 8y = \frac{4\,\mathrm{e}^{-2x}}{2 + \mathrm{e}^{2x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(5)} - y^{(4)} - 6y''' = 144x - 48$; **b)** $y'' + 6y' + 9y = (36x - 60)\,\mathrm{e}^{3x}$;

**c)** $y'' - 2y' + 5y = \mathrm{e}^{-2x}(6\cos x + 18\sin x)$; **d)** $y'' + 81y = 18\sin 9x + 81$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = 2x + 16y, \\ \dot{y} = x - 4y; \end{cases}$ **b)** $\begin{cases} \dot{x} = y + 2\,\mathrm{e}^t, \\ \dot{y} = x + t^2. \end{cases}$

# Differential Equations

## Variant № 15

**Exercise 1.** Solve the first-order differential equations:

a) $2\,\mathrm{e}^{y}\operatorname{ctg}x dy + (1+\mathrm{e}^{y})\sec^2 x dx = 0$; b) $y' = \sqrt[3]{(4x-y+1)^2}$;

c) $xy' = \dfrac{3y^3+4yx^2}{2y^2+2x^2}$; d) $y' = \dfrac{x+2y-3}{2x-2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' + \dfrac{y}{x} = -\sin x, \quad y\left(\dfrac{\pi}{2}\right) = -\dfrac{2}{\pi}$; b) $y' + xy = y^2(1+x)\,\mathrm{e}^{-x}, \quad y(0) = 1$;

c) $y^2dx + 2xydy = \dfrac{8}{y^2+4}dy, \quad y(1) = 2$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \operatorname{ctg}^2 x$; b) $x^3y''' + x^2y'' = \sqrt{x}$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''y^3 + 4 = 0, \quad y(0) = -1, \quad y'(0) = -2$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $3y'' - y' - 2y = 0$; b) $16y'' + 40y' + 25y = 0$; c) $y'' - y' + y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' - 6y'' + 12y' - 8y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 4.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + y = 2\operatorname{ctg}x.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)} + 4y^{(4)} + 4y''' = 96x + 48$; b) $y'' - 2y' + 2y = (30x+2)\,\mathrm{e}^{-2x}$;

c) $y'' + 2y' - 15y = \mathrm{e}^{-5x}(\cos x - 8\sin x)$; d) $y'' - 2y' - 8y = 96\operatorname{sh}4x$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = 3x + y, \\ \dot{y} = 3y - 16x; \end{cases}$ b) $\begin{cases} \dot{x} = 2x + 3y + \cos t, \\ \dot{y} = 4x - 2y. \end{cases}$

# Differential Equations

## Variant № 16

**Exercise 1.** Solve the first-order differential equations:

**a)** $(1+x^2)dy + y\sqrt{1+x^2}\,dx = xy\,dx;$ **b)** $y' = \cos(4x-4y+3);$

**c)** $xy' - y = 2\sqrt{x^2+y^2};$ **d)** $y' = \dfrac{x+3y-4}{5x-y-4}.$

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' + y\cos x = \sin x\cos x, \quad y(0) = -1;$ **b)** $y' + \dfrac{y}{x} = 2y^2\dfrac{\ln x}{x}, \quad y(1) = \dfrac{1}{2};$

**c)** $2y\,dx = \left(6x + \dfrac{7}{\sqrt{y}}\right)dy, \quad y(-1) = 1.$

**Exercise 3.** Find general solutions of the differential equations:

**a)** $x^2y'' = -1;$ **b)** $y'''x\ln x = y''.$

**Exercise 4.** Find the solution to the Cauchy problem: $y''(y-3) = 2(y')^2, \; y(1)=4, \; y'(1)=-1.$

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $2y'' - 9y' - 5y = 0;$ **b)** $25y'' - 60y' + 36y = 0;$ **c)** $y'' + 4y' + 7y = 0.$

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + 3y'' + 2y' = 0, \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 2.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 2y' = \frac{4e^{-2x}}{1+e^{-2x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(4)} - 2y''' + 2y'' = 24x^2 + 4;$ **b)** $y'' + y' = (2x+1)e^{-x};$

**c)** $y'' + 8y' + 16y = e^{-4x}(3\sin x - 4\cos x);$ **d)** $y'' + 4y' + 5y = 16\sin x + 17e^{2x}.$

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = 9y - 5x, \\ \dot{y} = x - 5y; \end{cases}$ **b)** $\begin{cases} \dot{x} = 2y + t^2, \\ \dot{y} = 2x + 3y + 1. \end{cases}$

# Differential Equations

## Variant № 17

**Exercise 1.** Solve the first-order differential equations:

a) $(1+x^2)dx=(1+x)(\mathrm{e}^{2y}\,dy-\mathrm{e}^{4x}\,dx)$;  b) $y'=\sqrt[4]{3x+y-3}$;

c) $x^2y'=y^2+4xy+2x^2$;  d) $y'=\dfrac{3x+2y-1}{x+1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y'-\dfrac{y}{x+2}=x(x+2)\,\mathrm{e}^x,\quad y(0)=-2$;  b) $y'-y\,\mathrm{tg}\,x+y^2\cos x=0,\quad y(0)=1$;

c) $y^2dx=(4-xy)dy,\quad y(1)=1$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y''=\dfrac{1}{\cos 2x+\sin^2 x}$;  b) $y'''+\dfrac{y''}{x}+1=0$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''=\mathrm{e}^{9y},\quad y(0)=0,\quad y'(0)=\dfrac{\sqrt{2}}{3}$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $4y''-y'-3y=0$;  b) $9y''-6y'+y=0$;  c) $5y''+4y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y'''-y''=0,\quad y(0)=0,\quad y'(0)=0,\quad y''(0)=-1.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''-3y'+2y=\frac{1}{1+\mathrm{e}^{-x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)}-y'''=60x^2-24x$;  b) $y''+3y'-4y=(48x+22)\,\mathrm{e}^{4x}$;

c) $y''-2y'+10y=\mathrm{e}^{-x}(4\cos 3x-28\sin 3x)$;  d) $y''-9y'=9\,\mathrm{e}^{9x}-9$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases}\dot{x}=-7x+y,\\ \dot{y}=-x-7y;\end{cases}$  b) $\begin{cases}\dot{x}=4x+y-\mathrm{e}^{2t},\\ \dot{y}=y-2x.\end{cases}$

# Differential Equations

## Variant № 18

**Exercise 1.** Solve the first-order differential equations:

**a)** $(yx^2+5x^2)dy=(y^2x-y^2)dx$;

**b)** $y'=(9x+25y-1)^2$;

**c)** $y'=\dfrac{x^2+xy-y^2}{x^2-2xy}$;

**d)** $y'=\dfrac{3y+3}{2x+y-1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y'-\dfrac{y}{x+1}=\mathrm{e}^x\,x(x+1),\quad y(0)=-1$;

**b)** $y'-y\,\mathrm{tg}\,x=-\dfrac{2}{3}y^4\sin x,\quad y(0)=1$;

**c)** $\mathrm{ch}\,y\,dx=(x\,\mathrm{sh}\,y+4)dy,\quad y(4)=0$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y''=\dfrac{(1+x)^2}{x(1+x^2)}$;

**b)** $y'''\,\mathrm{tg}\,5x=5y''$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''=-18\sin y\cos^3 y,\ \ y(0)=0,\ \ y'(0)=3$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $5y''-4y=0$;

**b)** $9y''+30y'+25y=0$;

**c)** $y''+2y'+4y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y'''+y''-4y'-4y=0,\quad y(0)=0,\quad y'(0)=0,\quad y''(0)=12.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+y=\frac{1}{\cos x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(4)}+3y'''+2y''=24x^2-12$;

**b)** $y''-4y'+8y=(16x+12)\,\mathrm{e}^{2x}$;

**c)** $y''-2y'-3y=20\,\mathrm{e}^{-x}\cos 2x$;

**d)** $y''+16y=16(x+1)+24\cos 4x$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases}\dot{x}=-6x,\\ \dot{y}=-6y+x;\end{cases}$

**b)** $\begin{cases}\dot{x}=2x+4y-8,\\ \dot{y}=3x+6y.\end{cases}$

# Differential Equations

## Variant № 19

**Exercise 1.** Solve the first-order differential equations:

a) $(\mathrm{e}^x - \ln 5)\operatorname{ctg} y\,dy + \mathrm{e}^x \ln\sin y\,dx = 0$;

b) $y' = (\cos(4x - 4y + 3))^{-1}$;

c) $xy' = \dfrac{3y^3 + 2yx^2}{2y^2 + x^2}$;

d) $y' = \dfrac{2x + y - 3}{x - 1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' - \dfrac{y}{x} = x^2 \sin x, \quad y\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}$;

b) $y' + y = x\sqrt{y}, \quad y(0) = \dfrac{1}{4}$;

c) $dx = (8y + 8y^2 - 2x)dy, \quad y(4) = 0$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = x\left(\cos\dfrac{x}{2} + \sin\dfrac{x}{2}\right)^2$;

b) $(x+1)y''' + y'' = x + 1$.

**Exercise 4.** Find the solution to the Cauchy problem: $y^3 y'' + 4 = 4y^4, \; y(0) = \sqrt{2}, \; y'(0) = \sqrt{2}$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $3y'' - 4y' = 0$;

b) $16y'' + 8y' + y = 0$;

c) $4y'' - 4y' + 5y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' - y' = 0, \quad y(0) = 3, \quad y'(0) = -1, \quad y''(0) = 1.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - y' = \frac{\mathrm{e}^x}{1 + \mathrm{e}^x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)} - 2y^{(4)} - 5y''' + 6y'' = 36x + 18$;

b) $y'' - 3y' = (10 - 6x)\mathrm{e}^{3x}$;

c) $y'' - 2y' + 50y = 48\,\mathrm{e}^x \sin x$

d) $y'' + 2y' + 5y = 8\,\mathrm{e}^{-x} + 17\cos 2x$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = 15y - x, \\ \dot{y} = x - 3y; \end{cases}$

b) $\begin{cases} \dot{x} = -3x - y + t^2, \\ \dot{y} = x - y + 1. \end{cases}$

# Differential Equations

## Variant № 20

**Exercise 1.** Solve the first-order differential equations:

**a)** $yx^2dy - 3xy^2dx = 6xdx - 2ydy$; **b)** $y' = \sqrt[3]{9x+3y-5}$;

**c)** $xy' - y = \sqrt{x^2+y^2}$; **d)** $y' = \dfrac{5y+5}{4x+3y-1}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' - y\cos x = \sin 2x, \quad y(0) = -2$; **b)** $y' + \dfrac{y}{x} = y^2, \quad y(1) = 1$;

**c)** $4y^2dx = -(x + e^{\frac{1}{2y}})dy, \quad y(\sqrt{e}) = 1$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = \dfrac{1}{\sin^2 x\cos^2 x}$; **b)** $y''\operatorname{cth} x + \dfrac{1}{\operatorname{ch} x} = y'$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' - 32y^3 = 0, \quad y(4) = 1, \quad y'(4) = 4$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $3y'' + 2y' - y = 0$; **b)** $25y'' + 60y' + 36y = 0$; **c)** $y'' - 2y' + 3y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + y'' - 5y' + 3y = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -2.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 6y' + 8y = \frac{4e^{2x}}{1+e^{-2x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(4)} + 2y''' + 5y'' = 60x + 4$; **b)** $y'' + 8y' + 16y = (-64x - 16)e^{4x}$;

**c)** $y'' + 4y' + 20y = e^{-x}(4\cos 4x + 33\sin 4x)$; **d)** $y'' - 5y' + 6y = 40\operatorname{ch} 2x$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = 6y - 2x, \\ \dot{y} = x - 3y; \end{cases}$ **b)** $\begin{cases} \dot{x} = x + y, \\ \dot{y} = 4x + y + 10\cos t. \end{cases}$

# Differential Equations

## Variant № 21

**Exercise 1.** Solve the first-order differential equations:

a) $xdy = \cos y(\ln^3 x - x^3)dx$;

b) $y' = \sqrt[3]{(16x+3y-2)^2}$;

c) $x^2y' = y^2 + 6xy + 6x^2$;

d) $y' = \dfrac{3y-x-4}{3x+3}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' - \dfrac{2x-3}{x^2}y = 3, \quad y(3) = 9$;

b) $y' + xy = \dfrac{1}{2}y^2(1+x)\mathrm{e}^{-x}, \quad y(0) = 2$;

c) $ydx = (3y\cos 2y - 2y^2 \sin 2y - 2x)dy, \quad y(0) = \dfrac{\pi}{4}$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \dfrac{x^2+6}{x^2+4}$;

b) $(1+\mathrm{e}^x)y'' + y' = 0$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' = 18\cos y \sin^3 y, \ y(1) = \dfrac{\pi}{2}, \ y'(1) = 3.$

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $4y'' + 3y' - y = 0$;

b) $25y'' + 20y' + 4y = 0$;

c) $4y'' + 8y' + 5y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + 2y'' + 10y' = 0, \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = 1.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + 4y = \frac{4}{\sin 2x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)} - 2y^{(4)} - 3y''' = 72x + 12$;

b) $y'' - 6y' + 13y = (29x+19)\mathrm{e}^{-2x}$;

c) $y'' - 3y' + 2y = 10\mathrm{e}^{x}\cos 2x$;

d) $y'' - 16y = 32 - 16x + 8\mathrm{e}^{-4x}$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = -2x + y, \\ \dot{y} = -25x - 2y; \end{cases}$

b) $\begin{cases} \dot{x} = 3x + 2y + 4\mathrm{e}^{5t}, \\ \dot{y} = x + 2y. \end{cases}$

# Differential Equations

## Variant № 22

**Exercise 1.** Solve the first-order differential equations:

a) $(y^2-1)(x+4)dx - x^2 y dy = 0$;

b) $y' = (\sin(8y-8x+1))^{-2}$;

c) $y' = \dfrac{x^2+2xy-y^2}{2x^2-2xy}$;

d) $y' = \dfrac{2x+y-1}{2x-2}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' + \dfrac{2}{x} y = \dfrac{2\ln x}{x}, \quad y(1) = -\dfrac{1}{2}$;

b) $y' + 4x^3 y = 4y^2(1-x^3)\,\mathrm{e}^{4x}, \quad y(0) = 1$;

c) $dx = (2\cos^2 y - 2x - \sin 2y)dy, \quad y(1) = 0$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \dfrac{5 - 2\operatorname{ctg} x}{\sin^2 x}$;

b) $(1+x^2)y'' + 2xy' = 12x^3$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''y^3 + 16 = 0, \quad y(1) = 2, \quad y'(1) = 2$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $5y'' - 9y' - 2y = 0$;

b) $25y'' - 10y' + y = 0$;

c) $5y'' + 9y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + 3y'' + 3y' + y = 0, \quad y(0) = -1, \quad y'(0) = 2, \quad y''(0) = 3.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + y' = \frac{\mathrm{e}^x}{2 + \mathrm{e}^x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(4)} + 2y''' + 10y'' = 120x^2 - 12x - 8$;

b) $y'' - 2y' = (8x+2)\,\mathrm{e}^{2x}$;

c) $y'' - 10y' + 25y = 6\,\mathrm{e}^{5x}(\cos x + \sin x)$;

d) $y'' + 2y' + 2y = 10\sin x + \mathrm{e}^{-x}$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = 4x + 10y, \\ \dot{y} = x + 7y; \end{cases}$

b) $\begin{cases} \dot{x} = 4x - 3y + \sin t, \\ \dot{y} = 2x - y - 2\cos t. \end{cases}$

## Differential Equations

## Variant № 23

**Exercise 1.** Solve the first-order differential equations:

a) $\sqrt{4+x^2}\,dy+2(xy^2+4x)dx=0$;

b) $y'=\sqrt{16x+2y-3}$;

c) $xy'=\dfrac{3y^3+8yx^2}{2y^2+4x^2}$;

d) $y'=\dfrac{3y-2x+1}{3x+3}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y'+\dfrac{y}{2x}=\sqrt{x}\,\mathrm{e}^x, \quad y(1)=0$;

b) $y'-y\,\mathrm{tg}\,x=-y^2\cos x, \quad y(0)=1$;

c) $ydx=(2x+4y^4\,\mathrm{e}^y)dy, \quad y(0)=1$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y''=-\dfrac{2}{\mathrm{tg}^3\,x\cos^2 x}$;

b) $y'''+\dfrac{y''}{x+1}=1$.

**Exercise 4.** Find the solution to the Cauchy problem: $2(y')^2=y''(y-4),\ \ y(1)=5,\ \ y'(1)=-1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $7y''-4y'=0$;

b) $25y''+70y'+49y=0$;

c) $4y''-8y'+7y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y'''-7y''+6y'=0, \quad y(0)=0, \quad y'(0)=0, \quad y''(0)=30.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+3y'+2y=\frac{1}{1+\mathrm{e}^x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)}+2y^{(4)}+y'''=24x-6$;

b) $y''+2y'-3y=(12x+16)\,\mathrm{e}^{-5x}$;

c) $y''-6y'+18y=\mathrm{e}^x(24\cos 3x-8\sin 3x)$;

d) $y''+25y=20\cos 5x-25x$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases}\dot{x}=6x+y,\\ \dot{y}=5x+2y;\end{cases}$

b) $\begin{cases}\dot{x}=x+3y+2\,\mathrm{e}^t,\\ \dot{y}=x-y+1.\end{cases}$

## Differential Equations

### Variant № 24

**Exercise 1.** Solve the first-order differential equations:

**a)** $y\,\mathrm{e}^{2x}\,dx + (\mathrm{e}^{2x} + 2)dy = 0;$

**b)** $y' = (6x + 3y - 2)^{-1};$

**c)** $xy' - y = 3\sqrt{x^2 + y^2};$

**d)** $y' = \dfrac{x + 4y - 5}{6x - y - 5}.$

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' + 2xy = x\,\mathrm{e}^{-x^2}\sin x, \quad y(0) = 0;$

**b)** $y' + \dfrac{y}{x} = \dfrac{1}{2}y^2, \quad y(1) = 2;$

**c)** $ydx = 2(x + y^4 \ln y)dy, \quad y\left(-\dfrac{1}{4}\right) = 1.$

**Exercise 3.** Find general solutions of the differential equations:

**a)** $y'' = x\sin x + x^2;$

**b)** $y'' = \dfrac{y - xy'}{x^2}.$

**Exercise 4.** Find the solution to the Cauchy problem: $y'' - \mathrm{e}^{16y} = 0, \quad y(0) = 0, \quad y'(0) = \dfrac{1}{\sqrt{8}}.$

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $2y'' - 5y' - 3y = 0;$ **b)** $16y'' - 40y' + 25y = 0;$ **c)** $y'' + 6y' + 11y = 0.$

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' - 6y'' + 11y' - 6y = 0, \quad y(0) = 2, \quad y'(0) = 6, \quad y''(0) = 20.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + 9y = \frac{9}{\sin 3x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(4)} + 4y''' + 5y'' = 60x^2 + 6x + 2;$

**b)** $y'' - 6y' + 25y = (-65x - 51)\,\mathrm{e}^{-4x};$

**c)** $y'' - 2y' - 8y = 37\,\mathrm{e}^{4x}\sin x;$

**d)** $y'' - y' - 2y = 12\,\mathrm{sh}\,x.$

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = y - 3x, \\ \dot{y} = 14x + 2y; \end{cases}$

**b)** $\begin{cases} \dot{x} = 2y - x + 1, \\ \dot{y} = 3y - 2x. \end{cases}$

# Differential Equations

## Variant № 25

**Exercise 1.** Solve the first-order differential equations:

a) $e^x \ln\cos y\,dx + (e^x + 2)\operatorname{tg} y\,dy = 0;$ b) $y' = (\operatorname{tg}(6x + 6y - 1))^{-2};$

c) $3x^2 y' = y^2 + 10xy + 10x^2;$ d) $y' = \dfrac{6y - 6}{5x + 4y - 9}.$

**Exercise 2.** Find solutions to the Cauchy problems:

a) $x^2 y' + (3 - 2x)y = 3x^2, \quad y(1) = 1;$ b) $y' + \dfrac{y}{x} = \dfrac{1}{2} y^2 \dfrac{\ln x}{x}, \quad y(1) = 2;$

c) $dx = (2y^2 - x + 4y)dy, \quad y(0) = 0.$

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \dfrac{3 - 2\operatorname{tg} x}{\cos^2 x};$ b) $y''' \operatorname{ctg} 2x + 2y'' = 0.$

**Exercise 4.** Find the solution to the Cauchy problem: $y'' = -32\sin y \cos^3 y, \ y(0) = 0, \ y'(0) = 4.$

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $3y'' + 2y' - 8y = 0;$ b) $25y'' - 30y' + 9y = 0;$ c) $y'' - 2y' + 6y = 0.$

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' - 6y'' + 12y' - 8y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 4.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 3y' + 2y = \frac{e^x}{1 + e^{-x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)} + y^{(4)} - 10y''' + 8y'' = 48x - 12;$ b) $y'' - 5y' = (10x - 3)e^{5x};$

c) $y'' + 2y' + 65y = 63e^{-x}(\cos x + \sin x);$ d) $y'' - 4y' + 5y = 6e^{2x} - 8\cos x.$

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = -5x + y, \\ \dot{y} = -x - 3y; \end{cases}$ b) $\begin{cases} \dot{x} = y, \\ \dot{y} = x + e^t + e^{-t}. \end{cases}$

## Differential Equations

### Variant № 26

**Exercise 1.** Solve the firs-order differential equations:

a) $\sqrt{2-x^2}\,y'+2xy^2+2x=0$; b) $y'=\sqrt[4]{3x+y-3}$;

c) $y'=\dfrac{x^2+3xy+3y^2}{x^2+2xy}$; d) $y'=\dfrac{x+y-8}{3x-y-8}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y'-\dfrac{y}{x}=\sqrt[3]{x^4}\,e^x, \quad y(1)=0$; b) $y'+y=\dfrac{1}{2}y^2x, \quad y(0)=2$;

c) $e^{y^2}\,dx-2xy\,e^{y^2}\,dy=-4y\,dy, \quad y(1)=0$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y''=\dfrac{(x+2)^2}{(x^2+4)x}$; b) $y''\operatorname{tg}x-y'=-\dfrac{1}{\sin x}$.

**Exercise 4.** Find the solution to the Cauchy problem: $4y^3y''+16=y^4,\ y(0)=2\sqrt{2},\ y'(0)=\dfrac{1}{\sqrt{2}}$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $9y''-2y=0$; b) $4y''+12y'+9y=0$; c) $3y''+6y'+7y=0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y^{IV}-y=0, \quad y(0)=0, \quad y'(0)=0, \quad y''(0)=0, \quad y'''(0)=-4.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y''+4y=4\operatorname{ctg}2x.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)}-3y^{(4)}-4y'''=96x+72$; b) $y''+4y'+4y=(32x-32)\,e^{2x}$;

c) $y''-10y'+26y=e^{-5x}(20\cos x-100\sin x)$; d) $y''+6y'=24+6\,e^{-6x}$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases}\dot{x} = x - 13y,\\ \dot{y} = x + 5y;\end{cases}$ b) $\begin{cases}\dot{x} = -2x + y + 1,\\ \dot{y} = 3x + t^2.\end{cases}$

# Differential Equations

## Variant № 27

**Exercise 1.** Solve the first-order differential equations:

**a)** $6xdx - 6ydy = 3x^2ydy - 2xy^2dx$;

**b)** $y' = (x + 9y + 5)^{-2}$;

**c)** $xy' = \dfrac{3y^3 + 6yx^2}{2y^2 + 3x^2}$;

**d)** $y' = \dfrac{x + 2y - 3}{4x - y - 3}$.

**Exercise 2.** Find solutions to the Cauchy problems:

**a)** $y' - \dfrac{2xy}{1 + x^2} = 2\,\mathrm{arctg}\,x, \quad y(0) = 0$;

**b)** $y' + \dfrac{x^3}{4}y = y^2\left(\dfrac{x^3}{4} + 2\right)\mathrm{e}^{-2x}, \quad y(0) = 1$;

**c)** $\sin 2ydx = 2(\cos^2 y \cdot \cos 2y - x)dy, \quad y\left(\dfrac{1}{2}\right) = \dfrac{\pi}{4}$.

**Exercise 3.** Find general solutions of the differential equations:

**a)** $xy''' = 1$;

**b)** $y'''\,\mathrm{tg}\,x = y'' + 1$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' - 50y^3 = 0, \quad y(3) = 1, \quad y'(3) = 5$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

**a)** $3y'' - 16y' + 5y = 0$;

**b)** $9y'' + 6y' + y = 0$;

**c)** $9y'' + 5y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + y'' - 5y' + 3y = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -2.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 3y' = \frac{9\,\mathrm{e}^{-3x}}{3 + \mathrm{e}^{-3x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

**a)** $y^{(4)} - 6y''' + 9y'' = 108x^2 - 36x + 6$;

**b)** $y'' - 4y' + 13y = (-25x - 42)\,\mathrm{e}^{-2x}$;

**c)** $y'' - 4y' + 3y = 5\,\mathrm{e}^{3x}\cos x$;

**d)** $y'' + 49y = 42\sin 7x - 49x$.

**Exercise 9.** Solve the systems of differential equations:

**a)** $\begin{cases} \dot{x} = y - 4x, \\ \dot{y} = 7y - 10x; \end{cases}$

**b)** $\begin{cases} \dot{x} = 2x + 2y + 2\,\mathrm{e}^t, \\ \dot{y} = 4y + \mathrm{e}^t. \end{cases}$

# Differential Equations

## Variant № 28

**Exercise 1.** Solve the first-order differential equations:

a) $\sqrt{2-x^2}\,y' + 8x + 2xy^2 = 0$;

b) $y' = (\operatorname{ctg}(8x+8y-7))^{-2}$;

c) $xy' - y = \sqrt{2x^2 + y^2}$;

d) $y' = \dfrac{x+2y-3}{4x-y-3}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' + 2xy = 2x^3, \quad y(0) = -1$;

b) $y' + \dfrac{5}{3x}y = y^4\left(\dfrac{4}{3} - \dfrac{5}{3x}\right), \quad y(1) = 1$;

c) $y^3dx + 3xy^2dy = \ln y\,dy, \quad y(-1) = 1$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \dfrac{4^x x - 3\cdot 2^x}{2^x}$;

b) $(1+x^2)y'' + 2xy' = x^3$.

**Exercise 4.** Find the solution to the Cauchy problem: $y'' = 32\cos y\sin^3 y, \; y(1) = \dfrac{\pi}{2}, \; y'(1) = 4$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $4y'' - 4y' - 3y = 0$;

b) $25y'' + 10y' + y = 0$;

c) $4y'' + 8y' + 7y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y''' + 2y'' + y' = 0, \quad y(0) = 0, \quad y'(0) = 2, \quad y''(0) = -3.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' - 9y' + 18y = \frac{9\,e^{3x}}{1+e^{-3x}}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(4)} - 6y''' + 10y'' = 120x^2 - 84x + 28$;

b) $y'' + 4y' = (4-32x)\,e^{-4x}$;

c) $y'' - 4y' + 4y = e^{2x}(2\sin x - 5\cos x)$;

d) $y'' + 2y' + 10y = 9\,e^{-x} - 37\sin 3x$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = 8x + y, \\ \dot{y} = 3y; \end{cases}$

b) $\begin{cases} \dot{x} = x + 3y + 3, \\ \dot{y} = x - y + 1. \end{cases}$

# Differential Equations

## Variant № 29

**Exercise 1.** Solve the first-order differential equations:

a) $3(1+e^{y})\operatorname{cosec}^2 x dx = e^{y}\operatorname{tg}^2 x dy;$ b) $y' = (\cos(4y-4x+1))^{-2};$

c) $3x^2y' = y^2 + 8yx + 4x^2;$ d) $y' = \dfrac{2x+3y-5}{5x-5}.$

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' + \dfrac{3y}{x} = \dfrac{e^{x}}{x^2}, \quad y(1)=1;$ b) $y' - \dfrac{3}{2}y\cos x = -y^{-1}\left(1+\dfrac{3}{2}\cos x\right)e^{-2x}, \quad y(0)=1;$

c) $dx = (8xy + 32y^3 - 8y)dy, \quad y(0)=0.$

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \ln x;$ b) $y''' \operatorname{th} 7x = 7y''.$

**Exercise 4.** Find the solution to the Cauchy problem: $y''y^3 + 25 = 0, \quad y(2) = -5, \quad y'(2) = -1.$

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $8y'' - 10y' + 3y = 0;$ b) $9y'' - 12y' + 4y = 0;$ c) $5y'' + 10y' + 6y = 0.$

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y^{IV} + 5y'' + 4y = 0, \quad y(0)=1, \quad y'(0)=4, \quad y''(0)=-1, \quad y'''(0)=-16.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + y = -\operatorname{ctg}^2 x.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)} - 5y^{(4)} + 6y''' = 144x - 12;$ b) $y'' + 3y' + 2y = (5-30x)e^{x};$

c) $y'' + 6y' + 25y = e^{-x}(16\cos 4x - 64\sin 4x);$ d) $y'' - 6y' + 8y = 96\operatorname{ch} 4x.$

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = 6y - x, \\ \dot{y} = x - 6y; \end{cases}$ b) $\begin{cases} \dot{x} = x - y + 8t, \\ \dot{y} = 5x - y. \end{cases}$

## Differential Equations

### Variant № 30

**Exercise 1.** Solve the first-order differential equations:

a) $x^2dx - 5\sin 2y dy = x^3 \sin 2y dy - x^2\cos^2 y dx$; b) $y' = \sqrt[3]{(9x - y + 1)^2}$;

c) $y' = \dfrac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$; d) $y' = \dfrac{x + 8y - 9}{10x - y - 9}$.

**Exercise 2.** Find solutions to the Cauchy problems:

a) $y' - y\ln 2 = 2^{\sin x}(\cos x - 1)\ln 2, \quad y(0) = 1$; b) $y' + \dfrac{y}{x} = \dfrac{1}{3}y^2, \quad y(1) = 3$;

c) $dx = (y^3 - xy)dy, \quad y(-2) = 0$.

**Exercise 3.** Find general solutions of the differential equations:

a) $y'' = \operatorname{arctg} x$; b) $y'' + \dfrac{2x}{x^2+1}y' = 2x$.

**Exercise 4.** Find the solution to the Cauchy problem: $y''(y-5) = 2(y')^2,\ y(1) = 6,\ y'(1) = -1$.

**Exercise 5.** Find general solutions of the homogeneous linear differential equations with constant coefficients:

a) $3y'' - 8y' = 0$; b) $16y'' - 8y' + y = 0$; c) $y'' - 2y' + 7y = 0$.

**Exercise 6.** Find the particular solution of the homogeneous linear differential equation with constant coefficients:

$$y^{V} - 9y''' = 0, \quad y(0) = 1, \quad y'(0) = -1, \quad y''(0) = 0, \quad y'''(0) = 0, \quad y^{IV}(0) = 0.$$

**Exercise 7.** Find the general solution of the nonhomogeneous linear differential equation with constant coefficients using the method of variation of constants

$$y'' + 16y = \frac{16}{\cos 4x}.$$

**Exercise 8.** Find the general solution of each of the nonhomogeneous linear differential equations with constant coefficients:

a) $y^{(5)} + 2y^{(4)} - 5y''' - 6y'' = 72x$; b) $y'' - 6y' + 10y = (37x + 25)\,e^{-3x}$;

c) $y'' - 3y' - 10y = e^{5x}(7\cos x - \sin x)$; d) $y'' - 9y = 27 - 9x + 6\,e^{3x}$.

**Exercise 9.** Solve the systems of differential equations:

a) $\begin{cases} \dot{x} = -x + y, \\ \dot{y} = -16x - y; \end{cases}$ b) $\begin{cases} \dot{x} = 5x - 3y + 2\,e^{3t}, \\ \dot{y} = x + y + 5\,e^{-t}. \end{cases}$

# LITERATURE


1. Фихтенгольц Г. М. Основы математического анализа: В 2 т. – М.: Наука, 1985. – Т. 1 – 440 с.; Т. 2 – 463 с.

2. Данко П. Е. Высшая математика в упражнениях и задачах [Текст]: учебное пособие для вузов в 2-х ч. Ч. II / Павел Ефимович Данко, Александр Георгиевич Попов, Татьяна Яковлевна Кожевникова. – Изд. 5-е, исп. – М.: Высш. шк., 1996. – 416 с.: ил.; 21 см. – Библиогр.: с. 416 . – 10000 экз. – ISBN 5–06–003071–7 (ч. II). – ISBN 5–06–003072.

3. Пискунов Н. С. Дифференциальное и интегральное исчисление для вузов [Текст]: учебник для вузов . Том I - II. – М. Наука, 1972, 1978.

4. Письменный Д. Т. Конспект лекций по высшей математике [Текст]: Тридцать пять лекций. 2 часть / Дмитрий Письменный; [вступ. ст. автора] – М. : Рольф, 2002. – 256 с.: ил; 21 см. – 10000 экз. – ISBN 5–7836–0312–0.

5. Каплан И. А. Практические занятия по высшей математике. [Текст]: учебник для вузов / Ч. 1-5. – Харьков, Издательство Харьковского университета, 1967–1972.

6. Справочное пособие по математическому анализу. Ч.II. Ряды. Функции нескольких переменных, кратные и криволинейные интегралы. Ляшко И. И., Боярчук А. К., Гай Я. Г., Головач Г. П. – Киев, Вища школа, 1979.

7. Кузнецов Л.А. Сборник заданий по высшей математике. Типовые расчеты: Учебное пособие. 4-е изд., стер. – СПб.: Издательство “Лань“, 2005. – 240 с.

Електронне мережне навчальне видання

# ВИЩА МАТЕМАТИКА
## ДИФЕРЕНЦІАЛЬНІ РІВНЯННЯ

Практикум

(Англійською мовою)

Укладачі: Массалітіна Є.В., Пилипенко В.А.

Практикум до розділу «Диференціальні рівняння» з курсу «Вища математика» для студентів технічних спеціальностей містить 30 варіантів, кожен варіант складається з 9 завдань (21 задачі). Самостійне виконання цих завдань забезпечує свідоме оволодіння навчальним матеріалом, який передбачено робочою програмою з вищої математики.

Практикум може бути рекомендований в якості розрахункової роботи за темою «Диференціальні рівняння» для студентів першого курсу технічних спеціальностей.

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Свідоцтво про внесення до Державного реєстру видавців, виготовлювачів
і розповсюджувачів видавничої продукції ДК № 5354 від 25.05.2017 р.