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«THE ROTATIONAL MOTION OF A SOLID BODY»

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METHODOLOGICAL INSTRUCTIONS FOR LABORATORY WORK NO.5(1)

«THE ROTATIONAL MOTION OF A SOLID BODY»

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The tutorial outlines a methodology for carrying out laboratory work devoted to the study of physical laws that describe the rotational motion of a solid. The proposed method is based on an experimental study of the motion of a system of solids fixed on the Oberbeck pendulum.

The publication gives a brief description of the theoretical data, which are needed to describe the phenomenon that is being studied in laboratory work. The theoretical calculations that are necessary for the interpretation of the corresponding experimental data and confirmation of the known theoretical regularities are given. Also the step-by-step instruction for conducting experimental measurements, and corresponding calculations of physical quantities is presented.

For the students of higher educational institutions with technical profile.

The rotational motion of a solid body

The aim of the work:

experimental verification of the fundamental equation of the rotation dynamics motion of a solid body around a fixed axis.

Brief theoretical information and method of measurement.

For a rotational motion, in addition to the mass of the body and force acting on the body, physical quantities are introduced, that depend on the point of application of force and on the distribution of the mass of the body. These values are the torque and moment of inertia (rotational inertia) [1, 2].

The vector product of the radius vector $\overrightarrow{r}$ drawn from the point O to the point A of application of force $\overrightarrow{F}$, on this force is called the torque relative to a stationary point O:

$$\overrightarrow{M} = \left[\overrightarrow{r}\,\overrightarrow{F}\right].$$

The vector $\overrightarrow{M}$ is directed perpendicular to the plane of the vectors $\overrightarrow{r}$ and $\overrightarrow{F}$ (Fig. 1). Its absolute value is equal [1, 2]

$$M = Fr\sin\alpha = Fl, \tag{5.1}$$

where α is the angle between $\overrightarrow{r}$ and $\overrightarrow{F}$ and $l = r\sin\alpha$ is called the lever arm (moment arm) of the force $\overrightarrow{F}$, which is equal to the length of the perpendicular, traced from the point O on the line of action of the force. The torque M_z relative to the axis OZ is the projection of the torque on this axis relative to any point selected on this axis.

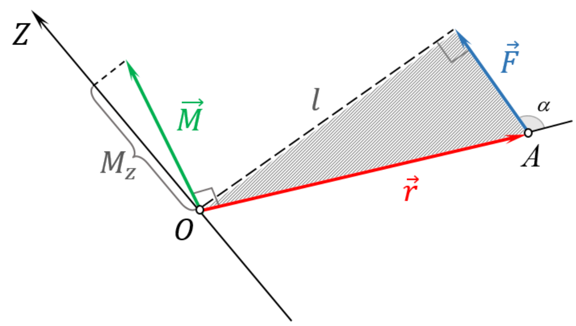

Figure 1: Torque of a force

The moment of inertia characterizes inertial properties of rotating bodies, that is, the ability of bodies to resist attempts to change the velocity of their rotational motion, including attempts to give them a rotational motion. The moment of inertia of the system relative to the axis OZ is the quantity I_z, that is equal to the sum of products of masses m_i of all material points forming the system and the squares of their distances r to the axis OZ: $I_z = \sum\limits_{i=1}^{n} m_i r_i^2$. In the case of a solid body: $I_z = \int r^2 dm$ [1, 2].

According to the fundamental equation of rotational motion of a solid body, the product of the moment of inertia of the system I_z and the angular acceleration β_z (both are the projections on the axis OZ) is equal to the total torque M_z of the external forces relative to this axis [1, 2]:

$$I_z \beta_z = M_z. \tag{5.2}$$

For an experimental verification of this relationship, we use the Oberbeck's pendulum (Fig. 2). It consists of four rods S and two sheave wheels with radii R_1 and R_2 fixed on a common horizontal axis. There are four identical masses m_0 on the rods (one for each rod). These masses can be moved or be fixed in the required position on the rods. The pendulum can be launched with the help of the mass m, fixed at the end of a rope which is spooled on one of the sheave wheels.

Neglecting the forces of friction, we can write the torque (5.1) for this case [1, 2]:

$$M_z = RT,$$

where T — is the tension force of the rope with attached mass m, R — is the radius of the corresponding sheave wheel.

Then considering the weightless and inextensible rope, we can obtain the equation of the rotational motion of the pendulum in a scalar form [1, 2]:

$$I_z \beta_z = RT. \tag{5.3}$$

Equation of the translational motion of the mass m on the rope is:

$$ma = mg - T, \tag{5.4}$$

where a — is the linear acceleration of the mass, g — is the free fall acceleration.

Equation of kinematic connection is:

$$a = a_\tau = \beta_z R, \tag{5.5}$$

where a_τ — tangential acceleration of the sheave wheel.

It follows from the system of equations (5.3 – 5.5) that the mass m moves with constant acceleration:

$$a = \frac{mR^2 g}{I_z + mR^2}. \tag{5.6}$$

The fundamental equation of the rotational motion (5.3) was written without taking into account the torque of friction forces in the axis of the pendulum and the torque of the viscous friction forces of the air. To prove the validity of this approach in the process of performing of our work it is necessary to make sure that the total torque of the friction forces M_{fr} is much smaller than the torque of the tension force of the rope M_z, which, taking into account (5.4) and (5.6), is equal to [1, 2]:

$$M_z = RT = Rm\,(g - a) = mgR\frac{I_z}{I_z + mR^2}. \tag{5.7}$$

Taking into account the inequality $mR^2 \ll I_z$, we can write that $M_z \approx mgR$.

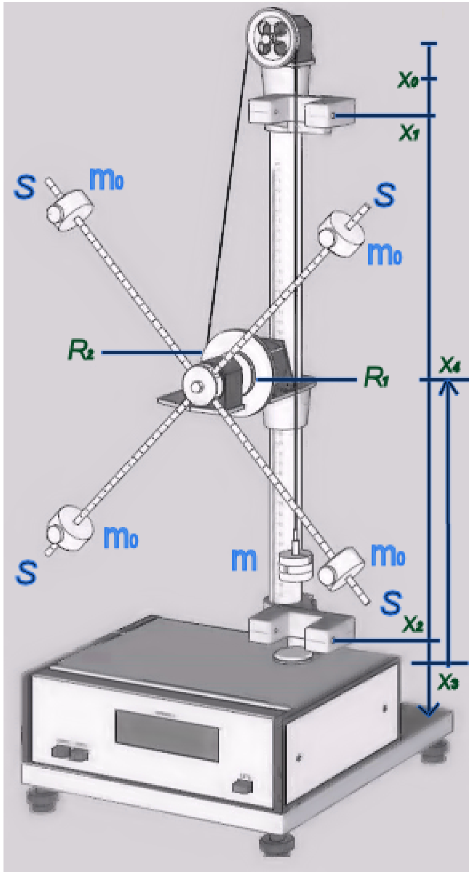

Figure 2: Oberbeck's pendulum

Let's compare the magnitude of the torque of friction force in the assumption that it remains unchangeable during motion. When the mass m falls from the position x_0 to x_3 (the entire length of the rope), and then lifts to the position x_4, the change in its potential energy is equal to the work of friction force:

$$\Delta U = A_{fr},$$

$$mg\,(x_4 - x_0) = M_{fr} \cdot \varphi$$

where φ — is the full angle of rotation of the Oberbeck's pendulum, and:

$$R\varphi = (x_3 - x_0) + (x_3 - x_4)\,.$$

Thus, the condition of the insignificance of the torque of friction forces will be [3, 4]:

$$M_{fr} = mgR\frac{x_4 - x_0}{2x_3 - (x_0 + x_4)}. \tag{5.8}$$

Taking into account that $M_z \approx mgR$, we can obtain

$$M_{fr} \ll M_z. \tag{5.8a}$$

The method of measurement.

Task 1. Checking the law of motion.

It follows from equations (5.3 - 5.5) that the rotation of the Oberbeck's pendulum occurs with constant angular acceleration β_z, while the mass m is falling with constant linear acceleration a. The coordinate x varies according to the law (axis is directed downwards):

$$x = x_0 + \frac{at^2}{2}. \tag{5.9}$$

Table 1:

№	x_{1i},m	x_{4i},m	$\langle x_4\rangle$,m	Δt_i,s	$\langle \Delta t\rangle$,s	$\sqrt{x_{1i}-x_0}$, $\sqrt{\text{m}}$	$\frac{\langle x_4\rangle - x_0}{2x_3-(x_0+\langle x_4\rangle)}$	a_i,m/s^2	$\langle a\rangle$,m/s^2	β_i,s^{-2}	$\langle\beta\rangle$,s^{-2}
1											
2											
3											
1											
2											
3											
1											
2											
3											
1											
2											
3											
1											
2											
3											

Using (5.9), we can define the time Δt of the mass motion between the marks x_1 and x_2:

$$\Delta t = \sqrt{\frac{2}{a}}\left(\sqrt{x_2 - x_0} - \sqrt{x_1 - x_0}\right). \tag{5.10}$$

It follows from (5.10) that in the case of a uniformly accelerated motion (a = const) and fixed positions x_0 and x_2, the dependence Δt on $\sqrt{x_1 - x_0}$ is linear and is presented on the graph by a straight line [3, 4].

The measurement procedure.

1. Install masses m_0 in the middle position, placing them at equal distances from the axis so that the pendulum is in an equilibrium position. Being in this state, the pendulum, brought to rotational motion, stops every time in a new position, and its stop is not accompanied by fluctuations around the equilibrium position. The motion of the mass m begins from the fixed position x_0, which must be recorded in the protocol. The rope should be spooled on the sheave wheel of a larger diameter.

2. Release the mass m and measure the time Δt of the motion between the positions x_1 and x_2. Results of measurements should be recorded in tab. 5.1. Measure time Δt for different 5 positions x_1 of the upper sensor (it is recommended to change x_1 with the step of 3 cm). Measure Δt at least $n = 3$ times for each position x_1 of the sensor.

3. For the first 5-7 experiments, measure the value of x_4 – the position which is reached by the mass when it is raised due to the rotation of the pendulum. Enter the results in tab. 5.1.

4. Measure the value of x_3 – the maximum position, to which the mass falls during its motion.

Bracket position: $x_0 =$ ·   m, $x_2 =$ ·   m.
Maximum position which is reached by the mass m during its falling: $x_3 =$ ·   m.

Processing of results.

1. According to the experimental data for each position of sensor x_1, calculate the average value Δt using the formula

$$\langle \Delta t \rangle = \frac{1}{n} \sum_{i=1}^{n} \Delta t_i,$$

where n — is the number of measurements at each fixed position x_1. Enter the results of the calculations to the table. 5.1.

2. Plot the dependence $\langle \Delta t \rangle$ on $\sqrt{x_1 - x_0}$, and make sure that the motion of mass is uniformly accelerated.

3. Find the average value $\langle x_4 \rangle$ and estimate the value $\frac{M_{fr}}{mgR}$ using the equation (5.8). Make sure that the value of the torque of friction forces is small in comparison with the initial torque of gravity force mgR.

4. Determine the linear acceleration of the mass a and the angular acceleration of the sheave wheel β:

$$a_i = \frac{2}{\Delta t_i^2} \left(\sqrt{x_2 - x_0} - \sqrt{x_1 - x_0} \right)^2 ;$$

$$\beta_i = \frac{a}{R} = \frac{2}{R \Delta t_i^2} \left(\sqrt{x_2 - x_0} - \sqrt{x_1 - x_0} \right)^2 .$$

5. Calculate the average value of linear acceleration $\langle a \rangle$ and the average value of the angle acceleration $\langle \beta \rangle$. Enter the results of the calculations to the table. 5.1.

Task 2. Experimental verification of the independence of the inertial properties of the pendulum (moment of inertia) from the torque of external forces.

From equation (5.3) we have

$$\frac{M_{z1}}{\beta_1} = \frac{M_{z2}}{\beta_2} = I_z.$$

From equations (5.6), (5.10) it follows that

$$I_z = mR^2 \left(\frac{gt^2}{2 \left(\sqrt{x_2 - x_0} - \sqrt{x_1 - x_0} \right)^2} - 1 \right). \tag{5.11}$$

All quantities included in the equation (5.11) are determined experimentally (except m and g) [3, 4].

The measurement procedure.

1. Measure the radii R_1 and R_2 of sheave wheels by dial caliper and record results in the protocol.

2. Set the maximum distance between the brackets. Record the coordinates of the positions of the brackets x_1, x_2 and x_0 in protocol.

3. Install masses m_0 in the middle position, placing them at equal distances from the axis so that the pendulum is in an equilibrium position.

4. Attach the mass m_1 to the end of the rope spooled on the sheave wheel of radius R_1, and measure the time Δt of mass motion between the brackets x_1 and x_2 . Simultaneously measure the coordinate of the position x_4 to which the mass will rise. Measurements should be repeated 3 times and the results should be recorded in table 5.2.

5. Spool the rope on the other sheave wheel (of radius R_2) and measure the time Δt and the coordinate x_4 (3 times). Enter results in table 5.2.

Table 2:

Combinations	№	Δt_i,s	x_{4i},m	$\langle x_4 \rangle$,m	I_{zi},kg$\cdot$ m^2	$\langle I_z \rangle$,kg$\cdot$ m^2	$\frac{\langle x_4 \rangle - x_0}{2x_3 - (x_0 + \langle x_4 \rangle)}$
R_1, m_1	1						
	2						
	3						
R_2, m_2	1						
	2						
	3						
R_1, m_2	1						
	2						
	3						
R_2, m_2	1						
	2						
	3						

6. Perform measurements, similar to steps 4 and 5, attaching the mass m_2 to the end of the rope. Enter the results of measurements in table 5.2.

$$R_1 = \_\_\cdot\_\_\text{m};\ R_2 = \_\_\cdot\_\_\text{m};$$

$$m_1 = \_\_\cdot\_\_\text{kg};\ m_2 = \_\_\cdot\_\_\text{kg};$$

$$x_1 = \_\_\cdot\_\_\text{m};\ x_2 = \_\_\cdot\_\_\text{m};\ x_0 = \_\_\cdot\_\_\text{m};\ x_3 = \_\_\cdot\_\_\text{m}.$$

Processing of results.

1. Calculate the values of the moment of inertia of the pendulum I_{zi} by the equation (5.11), as well as the average value of the moment of inertia $\langle I_z \rangle$ for each experiment.
2. Determine the value $\langle x_4 \rangle$ for each experiment, and evaluate the ratio $\frac{M_{fr}}{mgr}$ by equation (5.8). Make the conclusion.
3. Enter the results of the calculations in table 5.2.

Task 3. Examination of the fundamental equation of rotational motion and Huygens-Steiner theorem.

The moment of inertia of the body depends on the material, shape and size of the body, as well as on the location relative to the axis of rotation. The calculation of the moment of inertia of a body relative to an axis that does not pass through the center of masses is carried out with the help of Steiner's theorem: the moment of inertia of a body relative to an arbitrary axis OZ is equal to the sum of the moment of inertia I_0 of the body relative to its parallel axis passing through the center of the mass of the body, and the product of the mass of the body m on the square of the distance between these axes. Assume that I_0' is the total moment of inertia of four masses m_0 with respect to the axis passing through their center of mass. If the centers of masses are moving at a distance $l = l_1$, from the axis of rotation, the moment of inertia of these masses can be found according to the Huygens-Steiner theorem as [3, 4]:

$$I_{1z}' = I_0' + 4m_0 l_1^2. \tag{5.12}$$

If I_{OZ} - is the moment of inertia of the pendulum without masses, then the total moment of inertia of the pendulum will be equal to:

$$I_{1z} = I_{OZ} + I_0' + 4m_0 l_1^2. \tag{5.13}$$

If the centers of masses are moving at a distance $l = l_2$, from the axis of rotation we can obtain:

$$I_{2z} = I_{OZ} + I_0' + 4m_0 l_2^2. \tag{5.14}$$

Taking into account equations (5.6) and (5.10), the dependence of the square of the time of motion between the marks x_1 and x_2 on the distance l has the form [3, 4]:

$$\Delta t^2 = \frac{2\left(\sqrt{x_2 - x_0} - \sqrt{x_1 - x_0}\right)^2}{g}\left(1 + \frac{I_{OZ} - I'_{OZ}}{mR^2} + \frac{4m'^2}{mR^2}\right). \tag{5.15}$$

In case if $l_1 > l_2$, then

$$I_{1z} - I_{2z} = 4m_0\left(l_1^2 - l_2^2\right). \tag{5.16}$$

The equations (5.15) and (5.16) give

$$\Delta t_1^2 - \Delta t_2^2 = 4\frac{m_0}{m}\frac{\left(l_1^2 - l_2^2\right)}{R^2 g}\left(\sqrt{x_2 - x_0} - \sqrt{x_1 - x_0}\right)^2, \tag{5.17}$$

where Δt_1 and Δt_2 – are times of motion between the sensors for the cases when $l = l_1$ and $l = l_2$ respectively. Equation (5.17) contains values that are determined experimentally.

The measurement procedure.

1. Attach the greatest mass m to the end of the rope spooled on the sheave wheel of radius R_2.
2. Set the minimum value of the moment of inertia of the pendulum. To do this, four masses m_0 must be set as close as possible to the axis. Measure the distance l from the centers of the masses to the axis and record the value to table 5.3.
3. Measure the time Δt of motion of the mass m between the brackets x_1 and x_2. Simultaneously measure the position x_4 to which the mass will rise. Measurements should be repeated 3 times and the results should be recorded in table 5.3.
4. Perform measurements, similar to step 3, for different values of distance l. Enter the results of measurements in table 5.3.

Processing of results.

1. Find the average value $\langle \Delta t \rangle$ and $\left(\langle \Delta t \rangle\right)^2$ for each position of masses m_0, according to the experimental data.
2. Plot the dependence of $\left(\langle \Delta t \rangle\right)^2$ on the value of l^2.
3. Check the relationship (5.17) for several pairs of values $\left(\langle \Delta t \rangle\right)^2$ and l^2.
4. For each moment of inertia (i.e., for each position of masses m_0), determine $\langle x_4 \rangle$ and estimate the ratio $\frac{M_{fr}}{mgR}$ in accordance with (5.8). Make shure that the approximation $\frac{M_{fr}}{mgR} \ll 1$ is fulfilled.

Table 3:

№	Δt_i,s	x_{4i},m	l,m	$\langle \Delta t \rangle$,s	$\left(\langle \Delta t \rangle\right)^2$,s^2	l^2,m^2	$\langle x_4 \rangle$,m	$\frac{\langle x_4 \rangle - x_0}{2x_3 - (x_0 + \langle x_4 \rangle)}$
1								
2								
3								
1								
2								
3								
1								
2								
3								
1								
2								
3								
1								
2								
3								

Control questions.

1. Give the determinations of angular velocity and angular acceleration.
2. How are the directions of the vectors of angular velocity and angular acceleration determined?
3. Give the determinations of the torque of a force relative to some point. How is it directed?
4. Give the determinations of the torque of a force relative to the fixed axis.
5. Give the determinations of the moment of inertia of the body relative to the fixed axis. On what quantities does the moment of inertia depend? How does the moment of inertia influence on the rotational motion?
6. Formulate and prove Huygens-Steiner theorem.
7. Derive the equation of moments and the fundamental equation of rotational motion of a solid body relative to the fixed axis.

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