

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
«IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE»



HIGHER MATHEMATICS

LIMITS AND CONTINUITY

Practice exercises collection

Recommended by the Methodological Council
of the Igor Sikorsky Kyiv Polytechnic Institute
as a study aid for bachelor's degree applicants
on the technical specialties

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The practice book offers additional individual exercises for university students studying Limits and Continuity in the course of Higher Mathematics of Igor Sikorsky KPI. The book contains 30 different variants and each variant consists of 9 exercises (26 tasks). Students master the material being studied and consolidate the acquired knowledge by solving such individual tasks.

The practice book can be recommended as an individual work on Limits and Continuity for first-year students of technical specialties.

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Limits and Continuity

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Limits and Continuity

INTRODUCTION

The Limits and Continuity section is included in the course of Higher Mathematics for engineering students of Igor Sikorsky KPI. An important factor in the successful assimilation of the educational material by the students is solving practical tasks on their own.

The practice book offers a systematized set of exercises that students of technical specialties should be able to solve when studying Limits and Continuity. The book contains 30 different variants and each variant consists of 9 exercises (26 tasks).

This practice book helps students to develop practical skills in evaluating different kinds of limits using such methods as direct substitution, factorization and cancellation, rationalization; using alternate forms of trigonometric functions, the first and the second special limits and their consequences. The practice book also contains tasks for determining the order of infinitesimal functions and examining the continuity of functions.

Limits and Continuity

GENERAL RECOMMENDATIONS

The practice book is designed to control and improve the knowledge of university students in the study of Limits and Continuity in the course of Higher Mathematics. The main goal is to develop and consolidate the skills of independent work of students in the study of educational material.

In order to successfully complete the exercises, students need to thoroughly study the lecture material and analyze the examples solved in practical classes. Only after that students can start solving their individual tasks.

Students have to adhere to the following requirements:

1. The number of the variant of the individual exercises corresponds to the ordinal number of the student in the list of the study group;
2. Individual work is written in a separate notebook, which should contain:
 - the title page;
 - the results table;
 - solved exercises (the solution of each exercise starts from a new page).
3. Before solving each exercise, the condition and all specific data for the corresponding variant are completely rewritten.
4. The solution of each task must contain detailed explanations and necessary formulas.
5. Completed work must be handed over to the teacher for verification within the prescribed time limit.

Students who do not submit their completed individual work on time will not be allowed to take the exam.

Limits and Continuity

Variant №1

Exercise 1. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \frac{10 - x^4 - 6x^5 + 6x^6}{3x^6 - 2x^2 + x}$; **b)** $\lim_{x \rightarrow \infty} \frac{x^4 + 5x^3 + x^2 - 3}{2x^5 + x^3 + 12}$;

c) $\lim_{x \rightarrow \infty} \frac{4 + 2x^3 + x^4}{x^3 + 6x^2 - 21}$; **d)** $\lim_{x \rightarrow \infty} \frac{(1-x)^4 - (1+x)^4}{(1+x)^3 - (1-x)^3}$.

Exercise 2. Evaluate limits: **a)** $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{2x^2 - x - 1}$; **b)** $\lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{\sqrt{10 - x} - 3}$.

Exercise 3. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \left(\frac{x^3}{4x^2 + 1} - \frac{2x^2}{8x - 1} \right)$;

b) $\lim_{x \rightarrow \infty} x (\sqrt[3]{x^3 + 1} - x)$; **c)** $\lim_{x \rightarrow 0} \frac{(1 - \cos x) \operatorname{tg} 4x}{\arcsin^3 x}$; **d)** $\lim_{x \rightarrow 0} \frac{x \operatorname{arctg} 4x}{\sin 3x - \sin 5x}$.

Exercise 4. Evaluate limits: **a)** $\lim_{x \rightarrow 0} (1 + 4x)^{\frac{3}{2x}}$; **b)** $\lim_{x \rightarrow \infty} \left(\frac{x + 4}{x + 3} \right)^{5x-1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x - 1}{x^2 + 5x} \right)^{6x^2 - x + 3}$; **d)** $\lim_{x \rightarrow \infty} \left(\frac{x - 3}{2x + 5} \right)^{3x-4}$; **e)** $\lim_{x \rightarrow \frac{\pi}{2}} (1 + \cos 3x)^{\operatorname{ctg} x}$.

Exercise 5. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{\ln(x^2 + 1)}{2 - \sqrt{2x^2 + 4}}$; **b)** $\lim_{x \rightarrow 2} \frac{\operatorname{arctg}(x^2 - 2x)}{\sin 3\pi x}$.

Exercise 6. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{2^{3x} - 3^{5x}}{\sin 7x - 2x}$; **b)** $\lim_{x \rightarrow 2} \frac{8^x - 64}{x - 2}$;

c) $\lim_{x \rightarrow \infty} (6x + 8)[\ln x - \ln(x - 3)]$; **d)** $\lim_{x \rightarrow 3} \frac{\log_3 x - 1}{x - 3}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \operatorname{tg}(\sqrt{4+x^3} - 2)$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} \sin x, & x < -\pi, \\ 0, & -\pi < x < 0, \\ \sqrt{x}, & x \geq 0. \end{cases}$ **b)** $f(x) = \begin{cases} -x, & x \leq 0, \\ \ln x, & 0 < x < 1, \\ 2, & x \geq 1. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 4 \frac{1}{x-3}$; **b)** $f(x) = \frac{\sqrt{7+x} - 3}{x^2 - 4}$.

Limits and Continuity

Variant №2

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{1 - 2x + 3x^6 + x^7}{8x^7 - 10x + 4}$; b) $\lim_{x \rightarrow \infty} \frac{x^2 + 3x + 14}{x^3 + 3x^2 - 3x + 1}$;

c) $\lim_{x \rightarrow \infty} \frac{4 + 3x^4 + x^5}{x^3 - 2x^2 + x - 1}$; d) $\lim_{x \rightarrow \infty} \frac{(x+1)^4 - (x-1)^4}{(x+1)^3 + (x-1)^3}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow -2} \frac{x^2 + 7x + 10}{2x^2 + 9x + 10}$; b) $\lim_{x \rightarrow -8} \frac{\sqrt[3]{x} + 2}{\sqrt{9 + x} - 1}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow 1} \left(\frac{5}{(x-1)(x-6)} - \frac{8}{(x-1)(x-9)} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{4x^2 + 9} - 2x)$; c) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x \cdot \arcsin 4x}{(1 - \cos 2x)}$; d) $\lim_{x \rightarrow 0} \frac{x \operatorname{arctg} 8x}{\cos 6x - \cos 4x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 - 6x)^{\frac{4}{x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{x+4}{x+2} \right)^x$;

c) $\lim_{x \rightarrow \infty} \left(\frac{6x^2 + x + 2}{6x^2 + x + 1} \right)^{x^2 + 4x + 1}$; d) $\lim_{x \rightarrow \infty} \left(\frac{x+2}{5x-1} \right)^{5x+3}$; e) $\lim_{x \rightarrow 2} (5 - 2x)^{\frac{3x}{x-2}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{e^{4\pi x} - 1}{\sqrt[3]{8 + 24x} - 2}$; b) $\lim_{x \rightarrow 1} \frac{1 - x^2}{\sin \pi x}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{e^{4x} - e^{2x}}{\sin 5x - \sin 7x}$; b) $\lim_{x \rightarrow 4} \frac{\ln^2(5-x)}{64 - x^3}$;

c) $\lim_{x \rightarrow \infty} (5 + 8x)[\ln(4x + 5) - \ln 4x]$; d) $\lim_{x \rightarrow 4} \frac{x-4}{\log_4 x - 1}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \ln(1 + \sqrt[5]{x^4})$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} x, & x < 1, \\ \frac{1}{x}, & 1 < x \leq 2, \\ \frac{1}{2}, & x > 2. \end{cases}$ b) $f(x) = \begin{cases} -1, & x \leq 1, \\ x, & 1 < x \leq \frac{\pi}{2}, \\ \operatorname{tg} x, & x > \frac{\pi}{2}. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 3^{-\frac{1}{x+1}}$; b) $f(x) = \operatorname{arctg} \frac{1}{2-x}$.

Limits and Continuity

Variant № 3

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{15x^3 - 4x^2 - x + 1}{3x^3 - 3x + 9}$;

b) $\lim_{x \rightarrow \infty} \frac{x^3 + 2x^2 - x + 1}{7 + 2x^3 + 11x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{x^5 + 9x^3 + 4}{2 - x - 9x^3 + x^4}$;

d) $\lim_{x \rightarrow \infty} \frac{(x + 7)^3 - (x + 2)^3}{(3x + 2)^2 + (4x + 1)^2}$.

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow -2} \frac{2x^2 + 3x - 2}{x^2 - x - 6}$;

b) $\lim_{x \rightarrow -2} \frac{\sqrt{5 - 2x} - 3}{x^2 + 7x + 10}$.

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{7x^2 - 1} - \frac{2x^2}{14x + 5} \right)$;

b) $\lim_{x \rightarrow \infty} \sqrt[3]{x^2} (\sqrt[3]{x} - \sqrt[3]{x + 2})$;

c) $\lim_{x \rightarrow 0} \frac{x \sin^2 2x}{\operatorname{tg}^3 x}$;

d) $\lim_{x \rightarrow 0} \frac{1 - \cos^3 8x}{\operatorname{arctg} 2x}$.

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 - 7 \sin \sqrt{x})^{\operatorname{cosec} \sqrt{x}}$;

b) $\lim_{x \rightarrow \infty} \left(\frac{3x - 1}{3x + 5} \right)^{2x+1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{5x^2 + 6x - 1}{5x^2 + 3x + 2} \right)^{3x+6}$;

d) $\lim_{x \rightarrow \infty} \left(\frac{4x - 1}{x + 2} \right)^{4x+1}$;

e) $\lim_{x \rightarrow -3} (4 + x)^{\frac{2}{x+3}}$.

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\ln(2x + 1)}{1 - \sqrt[3]{x + 1}}$;

b) $\lim_{x \rightarrow 1} \frac{\cos \frac{\pi x}{2}}{1 - x}$.

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{5^{3x} - 3^{2x}}{2 \sin x - \operatorname{tg} x}$;

b) $\lim_{x \rightarrow 1} \frac{e^x - e}{\sin(1 - x^2)}$;

c) $\lim_{x \rightarrow 0} \frac{1}{4x} [\ln(4x + 2) - \ln 2]$;

d) $\lim_{x \rightarrow \infty} x \ln \sqrt[4]{\frac{2x + 7}{2x - 5}}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sqrt{1 + \sqrt[5]{x}} - 1$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 2^{1-x}, & x < -1, \\ 4, & -1 < x \leq 2, \\ 2x, & x > 2. \end{cases}$

b) $f(x) = \begin{cases} \frac{1}{x-1}, & x < 1, \\ x, & 1 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 8^{\frac{1}{x-4}}$;

b) $f(x) = \frac{x^2 + x - 12}{x^3 - 27}$.

Limits and Continuity

Variant №4

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{1 - x^6 + x^8 - 7x^9}{x^9 + 3x^8 + x^5 + 5}$; b) $\lim_{x \rightarrow \infty} \frac{x^3 + 5x^2 - 11}{1 + 3x + 3x^3 + x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{x^6 + 2x^5 - 4}{x - 12x^2 + x^4}$; d) $\lim_{x \rightarrow \infty} \frac{(2x + 1)^3 + (3x + 2)^3}{(2x + 3)^3 - (x - 7)^3}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x^2 - 6x - 16}$; b) $\lim_{x \rightarrow 4} \frac{\sqrt{5 - x} - \sqrt{x - 3}}{x^2 - 3x - 4}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow 5} \left(\frac{1}{5 - x} - \frac{75}{125 - x^3} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 - 3x - 4} - x)$; c) $\lim_{x \rightarrow 0} \frac{x \sin^2 8x}{\operatorname{arctg}^3 4x}$; d) $\lim_{x \rightarrow 0} \frac{1 - \cos \frac{x}{4}}{\arcsin^2 3x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 - 3\sqrt[3]{x})^{\frac{2}{\sqrt[3]{x}}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{x - 10}{x + 1} \right)^{3x+1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{6x^2 + 2}{6x^2 + 1} \right)^{x^2 + 4x - 1}$; d) $\lim_{x \rightarrow \infty} \left(\frac{x - 1}{7x + 3} \right)^{3x+7}$; e) $\lim_{x \rightarrow 2} (5 - 2x)^{\frac{4}{x-2}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 - x} - \sqrt[3]{1 + x}}{\ln(1 + 2x)}$; b) $\lim_{x \rightarrow 3} \frac{3 - x}{\sin(3\pi x)}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{5^{2x} - 9^x}{\operatorname{arctg} 4x - 3x}$; b) $\lim_{x \rightarrow -3} \frac{\ln(4 + x)}{\operatorname{tg}(2x + 6)}$;

c) $\lim_{x \rightarrow 4} \frac{\log_4 x - 1}{x - 4}$; d) $\lim_{x \rightarrow \infty} (4x - 7)[\ln 2x - \ln(2x - 3)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sqrt[3]{x^4 + 1} - 1$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} -\cos x, & x < 0, \\ x - 1, & 0 < x \leq 2, \\ 1, & x > 2. \end{cases}$ b) $f(x) = \begin{cases} x - 1, & x \leq 0, \\ -\ln x, & 0 < x \leq 1, \\ 1, & x > 1. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 5^{-\frac{1}{x-2}}$; b) $f(x) = \frac{\sin x}{|x|}$.

Limits and Continuity

Variant №5

Exercise 1. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \frac{2x^5 + x^4 + x^3 - 8}{x^5 - x^3 + 3}$; **b)** $\lim_{x \rightarrow \infty} \frac{x^4 + 5x^3 - x^2 + 1}{11 + 3x^2 + x^6}$;

c) $\lim_{x \rightarrow \infty} \frac{x - 2x^2 + x^7}{x^4 - 9x^3 - 1}$; **d)** $\lim_{x \rightarrow \infty} \frac{(3-x)^4 - (2-x)^4}{(1-x)^4 - (1+x)^4}$.

Exercise 2. Evaluate limits: **a)** $\lim_{x \rightarrow 2} \frac{x^2 - 12x + 20}{x^2 - 5x + 6}$; **b)** $\lim_{x \rightarrow 1} \frac{\sqrt{2-x} - 1}{x^2 - 2x + 1}$.

Exercise 3. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \left(\frac{x^3}{6x^2 - 4} - \frac{x^2}{6x + 9} \right)$;

b) $\lim_{x \rightarrow \infty} x (\sqrt[3]{64x^3 - 1} - 4x)$; **c)** $\lim_{x \rightarrow 0} \frac{x (\sin 3x - \sin x)}{\arctg^2 x}$; **d)** $\lim_{x \rightarrow 0} \frac{\arcsin^2 x}{\cos 4x - 1}$.

Exercise 4. Evaluate limits: **a)** $\lim_{x \rightarrow 0} (1 + 3x^2)^{\frac{2}{x^2}}$; **b)** $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+5} \right)^{4x+1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 3x - 1}{x^2 + 3x + 2} \right)^{\frac{x-1}{3}}$; **d)** $\lim_{x \rightarrow \infty} \left(\frac{8x-1}{x+8} \right)^{x+2}$; **e)** $\lim_{x \rightarrow 1} (4-3x)^{\frac{2x+1}{1-x}}$.

Exercise 5. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - \sqrt{1+x}}{\sin 2x \cos 4x}$; **b)** $\lim_{x \rightarrow \pi} \frac{\sin 5x}{\operatorname{tg} 3x}$.

Exercise 6. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{2 \operatorname{tg} x - \sin x}$; **b)** $\lim_{x \rightarrow 2} \frac{5^x - 25}{x - 2}$;

c) $\lim_{x \rightarrow \infty} (11 - 4x)[\ln(x-8) - \ln x]$; **d)** $\lim_{x \rightarrow 0} \frac{x}{\ln(3x+5) - \ln 5}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sin \sqrt[3]{x^2} - x$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} x-1, & x < 1, \\ \ln x, & 1 < x < e^2, \\ 2, & x \geq e^2. \end{cases}$ **b)** $f(x) = \begin{cases} 0, & x \leq -\frac{\pi}{2}, \\ -\operatorname{tg} x, & -\frac{\pi}{2} < x \leq 0, \\ x^2 + 1, & x > 0. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 4^{\frac{1}{4-x}}$; **b)** $f(x) = \frac{1}{3^{\operatorname{tg} x} + 1}$.

Limits and Continuity

Variant №6

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{12x^4 + 1}{4 - x^2 - x^3 + 2x^4};$

b) $\lim_{x \rightarrow \infty} \frac{x^3 + x^2 - 1}{x^2 + 7x^4 + x^6};$

c) $\lim_{x \rightarrow \infty} \frac{4 + 6x^2 + x^5}{x^3 - 5x^2 + x^4 - 1};$

d) $\lim_{x \rightarrow \infty} \frac{(x + 6)^3 - (x + 1)^3}{(2x + 3)^2 + (x + 4)^2}.$

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 2} \frac{12 - 8x + x^2}{x^2 - 7x + 10};$

b) $\lim_{x \rightarrow -1} \frac{\sqrt{2 + x} - 1}{x^2 - 1}.$

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow 1} \left(\frac{4}{(x - 1)(x - 5)} - \frac{11}{(x - 1)(x - 12)} \right);$

b) $\lim_{x \rightarrow \pm\infty} x(\sqrt{5x^2 + 1} - x);$

c) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x \cdot \arcsin 4x}{(1 - \cos x)};$

d) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg}^2 8x}{\sin 3x - \sin 5x}.$

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 - 9x)^{\frac{3}{x}};$

b) $\lim_{x \rightarrow \infty} \left(\frac{3x + 1}{3x - 1} \right)^{2x+3};$

c) $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 3}{2x^2 - 1} \right)^{\frac{x^2}{4}};$

d) $\lim_{x \rightarrow \infty} \left(\frac{x - 2}{5x + 1} \right)^{5x-4};$

e) $\lim_{x \rightarrow 3} (2x - 5)^{\frac{4x}{x-3}}.$

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + 2x} - 1}{e^{3x} - 1};$

b) $\lim_{x \rightarrow \pi} \frac{\cos 3x - \cos x}{\operatorname{tg}^2(2x)}.$

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{e^{2x} - e^{-5x}}{\operatorname{tg} x - x^3};$

b) $\lim_{x \rightarrow 2} \frac{\ln^2(3 - x)}{8 - x^3};$

c) $\lim_{x \rightarrow \infty} x \ln^4 \sqrt{\frac{2x + 5}{2x - 9}};$

d) $\lim_{x \rightarrow \infty} (3x + 1)[\ln x - \ln(x + 2)].$

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = e^{\operatorname{arctg} \sqrt{x}} - 1$ with respect to the infinitesimal $\beta(x) = \operatorname{tg} x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 2^x, & x < 0, \\ x^2 + 1, & 0 < x \leq 1, \\ 2x, & x > 1. \end{cases}$

b) $f(x) = \begin{cases} \ln(-x), & x < 0, \\ \sqrt{x}, & 0 \leq x \leq 4, \\ 3, & x > 4. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 7^{\frac{1}{x-5}};$

b) $f(x) = \frac{\sqrt{15 + x} - 3}{x^2 - 36}.$

Limits and Continuity

Variant №7

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{3 - 5x - x^4}{2x^4 + x^3 - 1}$;

b) $\lim_{x \rightarrow \infty} \frac{x^3 + 4x^2 - 2x - 7}{x^5 + 6x^2 + 11}$;

c) $\lim_{x \rightarrow \infty} \frac{x + 5x^5 + x^7}{x^5 + 6x^4 - 1}$;

d) $\lim_{x \rightarrow \infty} \frac{(x + 2)^3 + (x - 2)^3}{x^4 + 2x^2 - 1}$.

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 2} \frac{20 - 12x + x^2}{x^2 - 5x + 6}$;

b) $\lim_{x \rightarrow 2} \frac{\sqrt{3 - x} - 1}{x^2 - 4x + 4}$.

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{7x^2 + 2} - \frac{2x^2}{14x - 7} \right)$;

b) $\lim_{x \rightarrow \infty} x (\sqrt[3]{x^3 + 4} - x)$;

c) $\lim_{x \rightarrow 0} \frac{x(1 - \cos 2x)}{\arcsin^3 2x}$;

d) $\lim_{x \rightarrow 0} \frac{\sin 3x - \sin 5x}{\operatorname{arctg} 4x}$.

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 + 9\sqrt{\operatorname{tg} x})^{\sqrt{\operatorname{ctg} x}}$;

b) $\lim_{x \rightarrow \infty} \left(\frac{8x - 5}{8x + 3} \right)^{2x+3}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 8}{x^2 - 2x - 7} \right)^x$;

d) $\lim_{x \rightarrow \infty} \left(\frac{2x + 5}{x - 1} \right)^{3x+4}$;

e) $\lim_{x \rightarrow 1} (4 - 3x)^{\frac{5x}{x-1}}$.

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt{1 - 2x} - \sqrt{1 + x}}{\operatorname{tg} x \cos 4x}$;

b) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\ln(x - 1)}$.

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{7^{5x} - 3^{-x}}{\operatorname{tg} 3x - \sin 2x}$;

b) $\lim_{x \rightarrow 2} \frac{e^x - e^2}{\operatorname{arctg}(x^2 - 4)}$;

c) $\lim_{x \rightarrow 0} \frac{1}{4x} [\ln(x + 5) - \ln 5]$;

d) $\lim_{x \rightarrow \infty} x \ln \sqrt[7]{\frac{4x + 5}{4x - 9}}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = 1 - \cos \sqrt[3]{x}$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 0, & x \leq 0, \\ \sin x, & 0 < x < \pi, \\ x - \pi, & x > \pi. \end{cases}$

b) $f(x) = \begin{cases} x^3, & x \leq 0, \\ -\ln x, & 0 < x \leq 1, \\ 2, & x > 1. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 2^{\frac{1}{x+5}}$;

b) $f(x) = \operatorname{arctg} \frac{1}{4 - x}$.

Limits and Continuity

Variant №8

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{1 + 2x + x^3 - x^4}{2x^4 + x + 7};$

b) $\lim_{x \rightarrow \infty} \frac{7x^5 + x^4 - x^2}{x^6 + 4x^5 + 15};$

c) $\lim_{x \rightarrow \infty} \frac{9 + x - 3x^2 + x^6}{x^3 + 5x^2 - 13};$

d) $\lim_{x \rightarrow \infty} \frac{(x+1)^3 - (x+1)^2}{(x-1)^3 - (x+1)^3}.$

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow -2} \frac{2x^3 + 7x^2 + 6x}{2x^2 + 5x + 2};$

b) $\lim_{x \rightarrow 1} \frac{\sqrt{10-x} - 3}{3x^2 + x - 4}.$

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow 6} \left(\frac{1}{x-6} - \frac{108}{x^3 - 256} \right);$

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{4x^2 + 4x + 3} - 2x);$

c) $\lim_{x \rightarrow 0} \frac{x \sin^2 6x}{\arcsin 2x \operatorname{tg}^2 x};$

d) $\lim_{x \rightarrow 0} \frac{\arctg^2 8x}{1 - \cos x}.$

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 + 3x)^{-\frac{4}{x}};$

b) $\lim_{x \rightarrow \infty} \left(\frac{10x-3}{10x-1} \right)^{5x};$

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x + 7}{x^2 + 2x + 2} \right)^{\frac{x^2+1}{5}};$

d) $\lim_{x \rightarrow \infty} \left(\frac{x-2}{3x+5} \right)^{3x+3};$

e) $\lim_{x \rightarrow 3} (3x-8)^{\frac{2x}{3-x}}.$

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+5x} - \sqrt[3]{1+3x}}{\arcsin 2x \cos 4x};$

b) $\lim_{x \rightarrow -3} \frac{\sin \pi x}{x+3}.$

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{\sin x + \operatorname{tg} x};$

b) $\lim_{x \rightarrow -3} \frac{2^{-x} - 8}{x+3};$

c) $\lim_{x \rightarrow 0} \frac{x}{\ln(4x+5) - \ln 5};$

d) $\lim_{x \rightarrow \infty} (3x-5)[\ln x - \ln(x-4)].$

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \operatorname{tg}(\sqrt[3]{x+1} - 1)$ with respect to the infinitesimal $\beta(x) = \sin x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} \frac{1}{x}, & x < -1, \\ 2x+1, & -1 < x \leq 0, \\ \sqrt{x}, & x > 0. \end{cases}$

b) $f(x) = \begin{cases} 2, & x \leq -\pi, \\ \operatorname{ctg} x, & -\pi < x \leq -\frac{\pi}{2}, \\ \sin x, & x > -\frac{\pi}{2}. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 4^{\frac{1}{6-x}};$

b) $f(x) = \frac{x^2 + 6x - 7}{x^3 - 1}.$

Limits and Continuity

Variant №9

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{4 + 9x - x^2 - x^5}{3x^5 + x^4 + 2}$;

b) $\lim_{x \rightarrow \infty} \frac{4x^3 + x^2 + 12}{x^7 + 5x^4 + x^2}$;

c) $\lim_{x \rightarrow \infty} \frac{1 - 2x^2 - x^3 + 8x^4}{x^3 + 4x - 10}$;

d) $\lim_{x \rightarrow \infty} \frac{(1 + 2x)^3 - 8x^3}{(1 + 2x)^2 + 4x^2}$.

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x^2 - 6x + 8}$;

b) $\lim_{x \rightarrow 0} \frac{x^2 + 7x}{\sqrt{3x + 1} - \sqrt{1 - 3x}}$.

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{8x^2 - 9} - \frac{x^2}{8x + 16} \right)$;

b) $\lim_{x \rightarrow \infty} x^2 (\sqrt[3]{x^3 + 3} - x)$;

c) $\lim_{x \rightarrow 0} \frac{x^2 \sin^2 9x}{\arctg^4 3x}$;

d) $\lim_{x \rightarrow 0} \frac{\cos^2 x - \cos^3 x}{\arcsin^2 4x}$.

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 - 3 \sin^4 x)^{\frac{2}{\sin^4 x}}$;

b) $\lim_{x \rightarrow \infty} \left(\frac{x - 6}{x + 2} \right)^{3x - 2}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x + 5}{x^2 + 4x + 7} \right)^{3x + 1}$;

d) $\lim_{x \rightarrow \infty} \left(\frac{4x - 2}{x + 5} \right)^{x + 1}$;

e) $\lim_{x \rightarrow -1} (2 + x)^{\frac{6x}{x + 1}}$.

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt{1 + 6x} - \sqrt{1 + 4x}}{\operatorname{tg} 2x \cos 2x}$;

b) $\lim_{x \rightarrow \pi} \frac{x^2 - \pi^2}{\operatorname{tg} 2x}$.

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{5^{2x} - 2^{3x}}{\sin 3x - \operatorname{tg} 2x}$;

b) $\lim_{x \rightarrow 4} \frac{\ln(x - 3)}{16 - x^2}$;

c) $\lim_{x \rightarrow 7} \frac{\log_7 x - 1}{x - 7}$;

d) $\lim_{x \rightarrow \infty} (4x - 7)[\ln(2x + 5) - \ln 2x]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sqrt[3]{\ln(1 + \sqrt[3]{x})}$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} x^3 + 1, & x < 0, \\ \cos x, & 0 < x \leq \frac{\pi}{2}, \\ x - \frac{\pi}{2}, & x > \frac{\pi}{2}. \end{cases}$

b) $f(x) = \begin{cases} 1, & x \leq 0, \\ \frac{1}{x-3}, & 0 < x < 3, \\ x, & x \geq 3. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 3^{\frac{1}{x+4}}$;

b) $f(x) = x \cdot \frac{|x + 2|}{x + 2} - 3$.

Limits and Continuity

Variant №10

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{3x^3 - 5x^2 + x + 9}{-12x^3 + 4x - 5}$; b) $\lim_{x \rightarrow \infty} \frac{x^4 - 5x^3 + x^2 + 1}{x^5 + 2x^2 - 9}$;

c) $\lim_{x \rightarrow \infty} \frac{x^2 + 4x^3 + x^6}{x^4 + 7x^3 - 18}$; d) $\lim_{x \rightarrow \infty} \frac{(x+1)^3 + (x-1)^3}{x^3 - 3x}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow 3} \frac{x^3 - 6x^2 + 9x}{x^4 - 18x^2 + 81}$; b) $\lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x^2 - 4x - 5}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow 1} \left(\frac{3}{(x-1)(x-10)} - \frac{2}{(x-1)(x-7)} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 + 5x + 6} - x)$; c) $\lim_{x \rightarrow 0} \frac{\arcsin^2 9x}{(1 - \cos 3x)}$; d) $\lim_{x \rightarrow 0} \frac{\sin 6x - \sin 2x}{\arctg 4x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 - 9x)^{\frac{2}{3x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{x+5}{x-7} \right)^{\frac{x+1}{6}}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 6}{x^2 + 2x + 1} \right)^{2x-1}$; d) $\lim_{x \rightarrow \infty} \left(\frac{x+2}{4x-3} \right)^{5x+3}$; e) $\lim_{x \rightarrow -1} (2+x)^{\frac{5x}{x+1}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\arctg 4x}{\sqrt{1+2x} - \sqrt{1+4x}}$; b) $\lim_{x \rightarrow 2} \frac{\ln(7-3x)}{x-2}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{e^x - e^{3x}}{\arcsin 7x - 2x}$; b) $\lim_{x \rightarrow 4} \frac{3^x - 81}{x-4}$;

c) $\lim_{x \rightarrow 2} \frac{x-2}{\log_2 x - 1}$; d) $\lim_{x \rightarrow \infty} (3x+1)[\ln x - \ln(x+2)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = 1 - \cos \sqrt[3]{x}$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

$$\text{a) } f(x) = \begin{cases} -2^x, & x < 0, \\ -x-1, & 0 < x < 2, \\ -3, & x \geq 2. \end{cases} \quad \text{b) } f(x) = \begin{cases} x^2, & x \leq 2, \\ \ln(x-2), & 2 < x < 3, \\ 4, & x \geq 3. \end{cases}$$

Exercise 9. Examine the continuity of functions and graph these functions.

$$\text{a) } f(x) = 5^{\frac{1}{8-x}}; \quad \text{b) } f(x) = \frac{1}{1-3^{x-2}}.$$

Limits and Continuity

Variant №11

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{8x^3 - x^2 + 4x - 1}{5 + x - 2x^3};$

b) $\lim_{x \rightarrow \infty} \frac{11 - 3x^3 + x^5}{x - 4x^4 + x^6};$

c) $\lim_{x \rightarrow \infty} \frac{7 + 3x^3 + 3x^4}{x^3 + 5x^2 - 5};$

d) $\lim_{x \rightarrow \infty} \frac{(3 - x)^3}{(x + 1)^2 - (x + 1)^3}.$

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 2} \frac{x^4 - 8x^2 + 16}{x^3 - 4x^2 + 4x};$

b) $\lim_{x \rightarrow 0} \frac{\sqrt{1 + 5x} - \sqrt{1 - 5x}}{7x + x^2}.$

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{9x^2 - 2} - \frac{x^2}{9x + 27} \right);$

b) $\lim_{x \rightarrow \infty} x (\sqrt[3]{8x^3 + 3} - 2x);$

c) $\lim_{x \rightarrow 0} \frac{\arcsin^3 6x}{x^2 \arctg 3x};$

d) $\lim_{x \rightarrow 0} \frac{(1 - \cos 4x)}{\operatorname{tg} 2x (\sin 6x + \sin 2x)}.$

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 - 3\sqrt[4]{\operatorname{tg} x})^{\sqrt[4]{\operatorname{ctg} x}};$

b) $\lim_{x \rightarrow \infty} \left(\frac{2x + 3}{2x - 1} \right)^{3x-5};$

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 8}{x^2 + 3x} \right)^{5x+1};$

d) $\lim_{x \rightarrow \infty} \left(\frac{x + 2}{6x - 1} \right)^{x+2};$

e) $\lim_{x \rightarrow 2} (5 - 2x)^{\frac{8x}{x-2}}.$

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\ln(1 + 2 \sin 3x)}{\sqrt{4x + 1} - 1};$

b) $\lim_{x \rightarrow 2} \frac{3^x - 9}{\operatorname{tg}(\pi x)}.$

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{9^x - 2^{3x}}{x + \sin(x^2)};$

b) $\lim_{x \rightarrow e} \frac{\ln x - 1}{\arcsin(x - e)};$

c) $\lim_{x \rightarrow 0} \frac{1}{2x} [\ln(6x + 4) - \ln 4];$

d) $\lim_{x \rightarrow \infty} x \ln \sqrt[5]{\frac{2x + 3}{2x - 7}}.$

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = e^{\operatorname{tg} \sqrt{x}} - 1$ with respect to the infinitesimal $\beta(x) = \arctg x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} x^2 - 1, & x < 1, \\ \ln x, & 1 < x < e^3, \\ 3, & x \geq e^3. \end{cases}$

b) $f(x) = \begin{cases} \cos x, & x \leq -\frac{\pi}{2}, \\ \operatorname{tg} x, & -\frac{\pi}{2} < x < \frac{\pi}{4}, \\ 2, & x \geq \frac{\pi}{4}. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 2^{-\frac{1}{1-x}};$

b) $f(x) = \frac{\sqrt{11+x} - 4}{x^2 - 25}.$

Limits and Continuity

Variant №12

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{1 - x^2 + 5x^3 - x^4}{x^4 + 4x - 13}$;

b) $\lim_{x \rightarrow \infty} \frac{x + 7x^2 + x^3}{5 + x^4 + 12x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{2 - x + 7x^5 + x^6}{x^5 + 3x^4 - 1}$;

d) $\lim_{x \rightarrow \infty} \frac{(3 - 4x)^2}{(x - 3)^3 - (x + 3)^3}$.

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 5x + 6}$;

b) $\lim_{x \rightarrow 1} \frac{\sqrt{x+6} - \sqrt{8-x}}{2x^2 + 5x - 7}$.

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow 10} \left(\frac{1}{x-10} - \frac{300}{x^3 - 1000} \right)$;

b) $\lim_{x \rightarrow \pm\infty} x(\sqrt{x^2 - 4} - x)$;

c) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x \cdot \arcsin 3x}{1 - \cos 3x}$;

d) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} 8x}{\sin 2x - \sin 4x}$.

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 - 5x)^{\frac{3}{x}}$;

b) $\lim_{x \rightarrow \infty} \left(\frac{6x - 7}{6x + 4} \right)^{3x+2}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x + 2}{x^2 - 4x - 1} \right)^{5x-1}$;

d) $\lim_{x \rightarrow \infty} \left(\frac{x - 2}{4x + 5} \right)^{2x+1}$;

e) $\lim_{x \rightarrow -1} (3 + 2x)^{\frac{3}{x+1}}$.

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{2x+1}}{\operatorname{arctg} 2x}$;

b) $\lim_{x \rightarrow 0} \frac{5^x - 1}{\sin \pi x}$.

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow -2} \frac{7^{-x} - 49}{x + 2}$;

b) $\lim_{x \rightarrow 0} \frac{e^{4x} - e^{-x}}{3x - \sin x}$;

c) $\lim_{x \rightarrow 0} \frac{x}{\ln(3x+8) - \ln 8}$;

d) $\lim_{x \rightarrow \infty} (1 + 5x)[\ln 4x - \ln(4x + 8)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \arcsin(\sqrt{9 + x^2} - 3)$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 0, & x < 0, \\ \operatorname{tg} x & 0 < x < \frac{\pi}{4}, \\ 1, & x \geq \frac{\pi}{4}. \end{cases}$

b) $f(x) = \begin{cases} \frac{1}{1-x}, & x < 1, \\ \ln x, & 1 \leq x \leq 2, \\ 0, & x > 2. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 5^{\frac{1}{x-4}}$;

b) $f(x) = \operatorname{arctg} \frac{1}{1-x^2}$.

Limits and Continuity

Variant №13

Exercise 1. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \frac{16x^5 - 3x^3 - x + 1}{2x^2 + x^3 + 4x^5}$; **b)** $\lim_{x \rightarrow \infty} \frac{12 + x^2 - 4x^3}{1 + x - x^2 + x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{x + 9x^3 + x^7}{x^6 + x^4 - 16}$; **d)** $\lim_{x \rightarrow \infty} \frac{8x^3 - 2x}{(x+1)^4 - (x-1)^4}$.

Exercise 2. Evaluate limits: **a)** $\lim_{x \rightarrow 1} \frac{x^2 + 6x - 7}{x^3 + 3x^2 - x - 3}$; **b)** $\lim_{x \rightarrow 2} \frac{\sqrt{8-x} - \sqrt{4+x}}{x^2 - x - 2}$.

Exercise 3. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \left(\frac{x^3}{3x^2 + 5} - \frac{2x^2}{6x - 9} \right)$;

b) $\lim_{x \rightarrow \infty} x(\sqrt[3]{2x+1} - \sqrt[3]{2x-1})$; **c)** $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x \cdot (1 - \cos 4x)}{\operatorname{arctg}^3 x}$; **d)** $\lim_{x \rightarrow 0} \frac{\cos x - \cos^3 x}{x \arcsin 4x}$.

Exercise 4. Evaluate limits: **a)** $\lim_{x \rightarrow 0} (1 + 4x)^{-\frac{3}{x}}$; **b)** $\lim_{x \rightarrow \infty} \left(\frac{5x-1}{5x+3} \right)^{10x-4}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 4x + 2}{x^2 + 5x - 3} \right)^{2x+1}$; **d)** $\lim_{x \rightarrow \infty} \left(\frac{2x-1}{x+5} \right)^{6x+1}$; **e)** $\lim_{x \rightarrow -3} (4+x)^{\frac{5}{x+3}}$.

Exercise 5. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{\arcsin 5x}{\sqrt{3+x} - \sqrt{3}}$; **b)** $\lim_{x \rightarrow 3} \frac{\ln(7-2x)}{\sin 2\pi x}$.

Exercise 6. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{3^{5x} - 2^{-7x}}{\operatorname{tg} 3x - \operatorname{tg} x}$; **b)** $\lim_{x \rightarrow -2} \frac{e^{-x} - e^2}{\operatorname{arctg}(x^2 - 4)}$;

c) $\lim_{x \rightarrow 5} \frac{\log_5 x - 1}{x - 5}$; **d)** $\lim_{x \rightarrow \infty} (4x - 9)[\ln(4x + 5) - \ln 4x]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \operatorname{tg}(e^{\sqrt{x}} - 1)$ with respect to the infinitesimal $\beta(x) = \arcsin x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} \sin 2x, & x < 0, \\ x, & 0 < x < 2, \\ 2, & x \geq 2. \end{cases}$ **b)** $f(x) = \begin{cases} x - 1, & x \leq 3, \\ \ln(x - 3), & 3 < x \leq 4, \\ 0, & x > 4. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 9^{-\frac{1}{x+1}}$; **b)** $f(x) = \frac{x^3 - 27}{x^2 - 9}$.

Limits and Continuity

Variant №14

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{12 - 2x^3 + 9x^4}{18x^4 - 5x^3 + x + 1}$; b) $\lim_{x \rightarrow \infty} \frac{x^3 + 3x^2 + 4}{5 + 7x + x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{5 - 2x + x^3 + x^5}{10 - x^2 - x^3}$; d) $\lim_{x \rightarrow \infty} \frac{(x+2)^2 - (x-2)^2}{(x+3)^2}$.

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 1} \frac{x^3 + 4x^2 - x - 4}{x^3 + x^2 - 2x}$; b) $\lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{\sqrt{1+5x} - \sqrt{4x+2}}$.

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow 1} \left(\frac{2}{(x-1)(x-3)} - \frac{7}{(x-1)(x-8)} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 - 11x + 18} - x)$; c) $\lim_{x \rightarrow 0} \frac{\arcsin^3 3x}{\operatorname{tg} x \operatorname{arctg}^2 x}$; d) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\cos 7x - \cos 3x}$.

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 + 5 \sin \sqrt{x})^{\operatorname{cosec} \sqrt{x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{2x-1}{2x+1} \right)^{x+1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x - 1}{x^2 - 2x + 2} \right)^{3x+2}$; d) $\lim_{x \rightarrow \infty} \left(\frac{x-5}{7x+1} \right)^{x+3}$; e) $\lim_{x \rightarrow -3} (4+x)^{\frac{7}{x+3}}$.

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{8+x} - 2}{\arcsin 4x}$; b) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \operatorname{tg} \frac{x}{2}}{\frac{\pi}{2} - x}$.

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{x + \operatorname{tg} x^2}$; b) $\lim_{x \rightarrow 3} \frac{\ln(4-x)}{\arcsin(x-3)}$;

c) $\lim_{x \rightarrow 0} \frac{1}{3x} [\ln(6x+2) - \ln 2]$; d) $\lim_{x \rightarrow \infty} x \ln \sqrt[5]{\frac{2x-9}{2x-7}}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = x \sin \sqrt[3]{x}$ with respect to the infinitesimal $\beta(x) = \operatorname{tg} x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} \frac{1}{x-2}, & x < 0, \\ -\frac{1}{2}, & 0 < x \leq 2, \\ -\frac{x}{4}, & x > 2. \end{cases}$ b) $f(x) = \begin{cases} 1, & x \leq 2, \\ \ln(x-2), & 2 < x \leq 4, \\ 7, & x > 4. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 7^{\frac{1}{x-4}}$; b) $f(x) = 2 + x \cdot \frac{x+1}{|x+1|}$.

Limits and Continuity

Variant №15

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{3 + 4x + x^2 + 3x^3}{x^3 - 2x - 1}$; b) $\lim_{x \rightarrow \infty} \frac{x^3 + 3x^2 - 1}{x + 12x^3 + x^5}$;

c) $\lim_{x \rightarrow \infty} \frac{x + 6x^3 + x^6}{4 - 12x + x^4}$; d) $\lim_{x \rightarrow \infty} \frac{(x + 2)^2 + (x - 2)^2 - (x + 2)^3}{(4 - x)^3}$.

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x^3 + 2x^2 - x - 2}$; b) $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{\sqrt{2x + 3} - 3}$.

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{5x^2 + 3} - \frac{2x^2}{10x + 25} \right)$;

b) $\lim_{x \rightarrow \infty} x(\sqrt[3]{125x^3 + 1} - 5x)$; c) $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{\operatorname{arctg}^2 x}$; d) $\lim_{x \rightarrow 0} \frac{\sin^2 x - \operatorname{tg}^2 x}{x^3 \arcsin 2x}$.

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 - 9 \operatorname{tg}^3 x)^{\operatorname{ctg}^3 x}$; b) $\lim_{x \rightarrow \infty} \left(\frac{5x - 3}{5x + 4} \right)^{x+4}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 3x - 1}{x^2 - 4x + 2} \right)^{4x-1}$; d) $\lim_{x \rightarrow \infty} \left(\frac{6x - 2}{x + 4} \right)^{3x+3}$; e) $\lim_{x \rightarrow 2} (5 - 2x)^{\frac{8}{x-2}}$.

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + x} - 1}{\operatorname{arctg} 2x}$; b) $\lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\pi - 6x}$.

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{e^{4x} - e^{-2x}}{x + \operatorname{arctg} x}$; b) $\lim_{x \rightarrow 3} \frac{\ln^2(4 - x)}{27 - x^3}$;

c) $\lim_{x \rightarrow 0} \frac{x}{\ln(10x + 3) - \ln 3}$; d) $\lim_{x \rightarrow \infty} (7 - 6x)[\ln 2x - \ln(2x + 3)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sqrt[3]{x^2} - \sqrt{x^3}$ with respect to the infinitesimal $\beta(x) = \sin x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 1, & x < 1, \\ \sqrt{x}, & 1 < x \leq 4, \\ x - 2, & x > 4. \end{cases}$ b) $f(x) = \begin{cases} \frac{1}{4-x}, & x < 4, \\ \sqrt{x}, & 4 \leq x \leq 9, \\ 2, & x > 9. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 8^{\frac{1}{4-x}}$; b) $f(x) = \frac{1}{1 - 2^{1-x}}$.

Limits and Continuity

Variant №16

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{6x^3 - 2x + 1}{4 + x + x^2 - 2x^3};$

b) $\lim_{x \rightarrow \infty} \frac{x^3 + x}{x^4 + 2x^3 + x^2};$

c) $\lim_{x \rightarrow \infty} \frac{x^2 + x^3 + 5x^5}{2x^3 + 4x^2 - 1};$

d) $\lim_{x \rightarrow \infty} \frac{(x+1)^3 - (x-1)^3}{(x+1)^2 + (x-1)^2}.$

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x^2 + x - 2};$

b) $\lim_{x \rightarrow -5} \frac{x^2 - x - 30}{\sqrt{3x+17} - \sqrt{2x+12}}.$

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow 1} \left(\frac{1}{(x-1)(x-2)} - \frac{14}{(x-1)(x-15)} \right);$

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 + 10x + 9} - x);$

c) $\lim_{x \rightarrow 0} \frac{x \arcsin^2 2x}{\operatorname{tg}^3 x};$

d) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg}^4 8x}{(1 - \cos^3 x) \sin^3 4x}.$

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 - 7x)^{\frac{2}{x}};$

b) $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1} \right)^{3x};$

c) $\lim_{x \rightarrow \infty} \left(\frac{3x^2 + 4x - 1}{3x^2 + 2x + 7} \right)^{2x+4};$

d) $\lim_{x \rightarrow \infty} \left(\frac{x+1}{3x+4} \right)^{2x+1};$

e) $\lim_{x \rightarrow -1} (2+x)^{\frac{5x}{x+1}}.$

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\ln(x^2 + 1)}{1 - \sqrt{x^2 + 1}};$

b) $\lim_{x \rightarrow 1} \frac{1 + \cos(\pi x)}{\operatorname{tg}^2(\pi x)}.$

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{3^{5x} - 2^{-3x}}{3x - \operatorname{tg} 3x};$

b) $\lim_{x \rightarrow 3} \frac{\ln(4-x)}{\operatorname{tg}(\pi x)};$

c) $\lim_{x \rightarrow 5} \frac{\log_5 x - 1}{x - 5};$

d) $\lim_{x \rightarrow \infty} (3x+1)[\ln x - \ln(x+2)].$

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sin(\sqrt{9+x} - 3)$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} -x, & x \leq 0, \\ \operatorname{tg} \frac{x}{2}, & 0 < x < \frac{\pi}{2}, \\ 1, & x > \frac{\pi}{2}; \end{cases}$

b) $f(x) = \begin{cases} x, & x \leq 2, \\ \ln(x-2), & 2 < x < 3, \\ 1, & x \geq 3. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 4^{\frac{1}{3-x}};$

b) $f(x) = \frac{\sqrt{x+6} - 3}{x^2 - 9}.$

Limits and Continuity

Variant №17

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{8x^7 - 5x^5 + 3}{1 - 2x + 4x^6 + 2x^7}$; b) $\lim_{x \rightarrow \infty} \frac{3x^3 + x^2 - 3}{2x^5 - 4x^3 + 2x + 1}$;

c) $\lim_{x \rightarrow \infty} \frac{x^4 + 2x^3 + x + 7}{15 + x^2 - 3x^3}$; d) $\lim_{x \rightarrow \infty} \frac{2(x+1)^3 - (x-2)^3}{x^2 + 2x - 3}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow -2} \frac{2x^2 + 9x + 10}{x^2 + 7x + 10}$; b) $\lim_{x \rightarrow 3} \frac{\sqrt{x+13} - 2\sqrt{x+1}}{x^2 - 9}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{2x^2 - 1} - \frac{4x^2}{8x + 3} \right)$;

b) $\lim_{x \rightarrow \infty} x (\sqrt[3]{x^3 - 5} - x)$; c) $\lim_{x \rightarrow 0} \frac{x \operatorname{arctg} 4x}{\arcsin^2 x}$; d) $\lim_{x \rightarrow 0} \frac{1 - \cos 8x}{\operatorname{tg} 6x \cdot \sin x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 + \sin x)^{3 \operatorname{cosec} x}$; b) $\lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x+4} \right)^{4x+1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 3x - 2}{2x^2 + 3x - 1} \right)^{x^2}$; d) $\lim_{x \rightarrow \infty} \left(\frac{2x+1}{x-2} \right)^{3x}$; e) $\lim_{x \rightarrow 2} (5 - 2x)^{\frac{4x}{x-2}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - \sqrt{1+2x}}{\ln(1-7x)}$; b) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{tg}(3x)}{\operatorname{tg} x}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{3^{2x} - 5^{3x}}{2x - \operatorname{arctg}(3x)}$; b) $\lim_{x \rightarrow -3} \frac{\ln(3+x)}{\operatorname{tg}(2x+4)}$;

c) $\lim_{x \rightarrow \infty} x \ln \sqrt[3]{\frac{x+1}{x-1}}$; d) $\lim_{x \rightarrow \infty} (2+3x)[\ln(x+4) - \ln x]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \log_3(1 + \sin x^2)$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} \sin x, & x \leq 0, \\ x^2, & 0 < x < 1, \\ 1, & x > 1; \end{cases}$ b) $f(x) = \begin{cases} 1-x, & x \leq 0, \\ \operatorname{tg} x, & 0 < x < \frac{\pi}{2}, \\ 1, & x \geq \frac{\pi}{2}. \end{cases}$

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = 2^{\frac{1}{x+3}}$; b) $f(x) = \operatorname{arctg} \frac{1}{x-5}$.

Limits and Continuity

Variant №18

Exercise 1. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \frac{3x^3 + 5x + 2}{5x + x^2 - 6x^3 - 1}$; **b)** $\lim_{x \rightarrow \infty} \frac{x^3 + 5x^2 - 4}{1 + 2x + 3x^3 + x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{x^6 + 4x^4 + x^2}{x^3 - 2x^2 + 12}$; **d)** $\lim_{x \rightarrow \infty} \frac{(x+1)^3 - (x+2)^3}{(x+4)^3 + (x+5)^3}$.

Exercise 2. Evaluate limits: **a)** $\lim_{x \rightarrow 4} \frac{2x^2 - 7x - 4}{2x^2 - 13x + 20}$; **b)** $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 2}{x^2 - 5x + 4}$.

Exercise 3. Evaluate limits: **a)** $\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$;

b) $\lim_{x \rightarrow \pm\infty} x(\sqrt{x^2 + 1} - x)$; **c)** $\lim_{x \rightarrow 0} \frac{\operatorname{arctg}^2 2x}{x \arcsin x}$; **d)** $\lim_{x \rightarrow 0} \frac{x \operatorname{tg} \frac{x}{2}}{\cos 4x - \cos^2 4x}$.

Exercise 4. Evaluate limits: **a)** $\lim_{x \rightarrow 0} (1 + 5x)^{-\frac{3}{x}}$; **b)** $\lim_{x \rightarrow \infty} \left(\frac{x+3}{x+4} \right)^{\frac{1}{2x}}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{3x^2 - 4x + 7}{3x^2 + 20x - 1} \right)^{-x+1}$; **d)** $\lim_{x \rightarrow \infty} \left(\frac{x+3}{2x+1} \right)^{3x+1}$; **e)** $\lim_{x \rightarrow -1} (3 + 2x)^{\frac{4}{x+1}}$.

Exercise 5. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-6x}}{\operatorname{tg} 2x \cos x}$; **b)** $\lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x}$.

Exercise 6. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{3^{2x} - 5^{3x}}{2x - \operatorname{arctg} 3x}$; **b)** $\lim_{x \rightarrow -2} \frac{\ln(3+x)}{\operatorname{tg}(2x+4)}$;

c) $\lim_{x \rightarrow 0} \frac{1}{9x} [\ln(9x+4) - \ln 4]$; **d)** $\lim_{x \rightarrow 0} \frac{x}{\ln(x+2) - \ln 2}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sqrt[4]{1+2x} - 1$ with respect to the infinitesimal $\beta(x) = \operatorname{tg} x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} x+1, & x \leq 0, \\ 1, & 1 < x < 1, \\ \sqrt{x}, & x > 1; \end{cases}$ **b)** $f(x) = \begin{cases} 3, & x \leq 0, \\ \frac{1}{x}, & 0 < x < 1, \\ 2^x, & x \geq 1. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 3^{\frac{1}{2-x}}$; **b)** $f(x) = \frac{x^2 - 4x + 3}{x^3 - 1}$.

Limits and Continuity

Variant №19

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{1 + 3x - 14x^3}{2x^3 + 5x^2 - 4}$; b) $\lim_{x \rightarrow \infty} \frac{5x^3 - 4x - 12}{x^4 + 3x^2 + 2x - 1}$;

c) $\lim_{x \rightarrow \infty} \frac{x^5 - 3x^3 + x + 4}{12 - 2x^2 + x^4}$; d) $\lim_{x \rightarrow \infty} \frac{(x+1)^3 - (x-1)^3}{(x+1)^2 - (x-1)^2}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow -2} \frac{x^2 - 6x - 16}{x^2 - x - 6}$; b) $\lim_{x \rightarrow 7} \frac{\sqrt{x-2} - \sqrt{5}}{x^2 - 3x - 28}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{4x^2 - 3} - \frac{2x^2}{8x + 1} \right)$;

b) $\lim_{x \rightarrow \infty} x (\sqrt[3]{8x^3 + 5} - 2x)$; c) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg}^4 x}{x \arcsin^3 3x}$; d) $\lim_{x \rightarrow 0} \frac{\operatorname{tg}^2 5x}{\cos 6x - \cos x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 - 3x)^{\frac{6}{x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{2x+1}{2x+3} \right)^{5x+2}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 3x + 6}{x^2 + 5x + 1} \right)^{\frac{x}{2}}$; d) $\lim_{x \rightarrow \infty} \left(\frac{4x+1}{x+3} \right)^{x+2}$; e) $\lim_{x \rightarrow -3} (4+x)^{\frac{5}{x+3}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\sqrt{1+4x} - 1}{e^{2x} - 1}$; b) $\lim_{x \rightarrow \pi} \frac{1 + \cos 3x}{\sin^2 7x}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{e^{7x} - e^{5x}}{\sin 4x - \sin 2x}$; b) $\lim_{x \rightarrow 3} \frac{4^x - 64}{x - 3}$;

c) $\lim_{x \rightarrow 6} \frac{\log_6 x - 1}{x - 6}$; d) $\lim_{x \rightarrow \infty} (4x+1)[\ln 5x - \ln(5x+3)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sqrt[3]{1+3x^2} - 1$ with respect to the infinitesimal $\beta(x) = \arcsin x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} \cos x, & x \leq 0, \\ x+1, & 0 < x < 2, \\ 3, & x > 2. \end{cases}$ b) $f(x) = \begin{cases} -1, & x \leq 0, \\ \ln x, & 0 < x \leq 1, \\ x, & x > 1. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 7^{\frac{1}{x-3}}$; b) $f(x) = \frac{1 - \cos x}{x}$.

Limits and Continuity

Variant №20

Exercise 1. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \frac{7x^3 + 8x^2 + 7}{1 + x + 6x^2 + x^3}$; **b)** $\lim_{x \rightarrow \infty} \frac{3x^3 + 2x^2 - x - 1}{12 + x^4 + 4x^5}$;

c) $\lim_{x \rightarrow \infty} \frac{x^6 + 2x^4 + 14}{x - x^2 - 15x^3 + x^5}$; **d)** $\lim_{x \rightarrow \infty} \frac{x^3 - (x-1)^3}{(x+1)^4 - x^4}$.

Exercise 2. Evaluate limits: **a)** $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 12x + 20}$; **b)** $\lim_{x \rightarrow 0} \frac{\sqrt{1-x^3} - 1}{x^3 + 7x}$.

Exercise 3. Evaluate limits: **a)** $\lim_{x \rightarrow 1} \left(\frac{1}{(x-1)(x-3)} - \frac{6}{(x-1)(x-14)} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 - 3x + 2} - x)$; **c)** $\lim_{x \rightarrow 0} \frac{\sin 4x - \sin 2x}{\operatorname{arctg} 4x}$; **d)** $\lim_{x \rightarrow 0} \frac{x(1 - \cos x)}{\arcsin^3 x}$.

Exercise 4. Evaluate limits: **a)** $\lim_{x \rightarrow 0} (1 + 3 \operatorname{tg} x)^{\operatorname{ctg} x}$; **b)** $\lim_{x \rightarrow \infty} \left(\frac{x-2}{x+2} \right)^{\frac{1}{x}}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{3x^2 + 4x - 1}{3x^2 + 2x + 7} \right)^{2x+5}$; **d)** $\lim_{x \rightarrow \infty} \left(\frac{x-1}{2x+5} \right)^{x+2}$; **e)** $\lim_{x \rightarrow 2} (5 - 2x)^{\frac{3}{x-2}}$.

Exercise 5. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-x}}{\sin x \cos 3x}$; **b)** $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \sin 2x}{(\pi - 4x)^2}$.

Exercise 6. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{7^{2x} - 5^{3x}}{\arcsin x + x^3}$; **b)** $\lim_{x \rightarrow 3} \frac{e^x - e^3}{\operatorname{tg}(x^2 - 9)}$;

c) $\lim_{x \rightarrow \infty} x \ln \sqrt[3]{\frac{x+4}{x-4}}$; **d)** $\lim_{x \rightarrow \infty} (1 - 2x)[\ln(2x + 3) - \ln 2x]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \operatorname{tg} \frac{\pi x^3}{2}$ with respect to the infinitesimal $\beta(x) = e^x - 1$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 2x+1, & x < -1, \\ x^3, & -1 < x < 1, \\ x, & x \geq 1. \end{cases}$ **b)** $f(x) = \begin{cases} 2, & x \leq -\frac{\pi}{2}, \\ \operatorname{ctg} x, & -\frac{\pi}{2} < x < 0, \\ x, & x \geq 0. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 5^{\frac{1}{1-x}}$; **b)** $f(x) = \frac{1}{5^{x-2} - 1}$.

Limits and Continuity

Variant №21

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{x^8 + 2x^3 + 1}{1 - 3x + x^2 - 2x^8}$; b) $\lim_{x \rightarrow \infty} \frac{x^3 + 2x^2 - 3x + 1}{7 + 13x^2 + x^5}$;

c) $\lim_{x \rightarrow \infty} \frac{x^4 + 2x^3 + 5x}{14 - x + x^2}$; d) $\lim_{x \rightarrow \infty} \frac{(2x + 1)^3 - (2x + 3)^3}{(2x + 1)^2 + (2x + 3)^2}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 8x + 12}$; b) $\lim_{x \rightarrow 2} \frac{\sqrt{3x - 5} - \sqrt{4x - 7}}{x^2 - 4}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{5x^2 - 4} - \frac{x^2}{5x + 1} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 + 10x + 9} - x)$; c) $\lim_{x \rightarrow 0} \frac{\arctg^2 x}{x \arcsin 5x}$; d) $\lim_{x \rightarrow 0} \frac{\cos^2 8x - \cos^3 8x}{\sin^2 2x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 - 2x)^{\frac{3}{x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{4x - 3}{4x + 5} \right)^{x+3}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x^2 + x - 1} \right)^{-x^2}$; d) $\lim_{x \rightarrow \infty} \left(\frac{5x - 3}{x + 2} \right)^{4x+3}$; e) $\lim_{x \rightarrow 1} (4 - 3x)^{\frac{3x+1}{1-x}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 - 7x} - \sqrt[3]{1 - 3x}}{\sin 2x \cos 7x}$; b) $\lim_{x \rightarrow 1} \frac{\sin(7\pi x)}{\sin(8\pi x)}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{e^{5x} - e^{3x}}{\arctg x - x^2}$; b) $\lim_{x \rightarrow -4} \frac{2^{-x} - 16}{x + 4}$;

c) $\lim_{x \rightarrow 0} \frac{7}{2x} [\ln(x + 7) - \ln 7]$; d) $\lim_{x \rightarrow \infty} x \ln \sqrt{\frac{2x - 1}{2x + 1}}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = e^{\operatorname{tg} \sqrt{x}} - 1$ with respect to the infinitesimal $\beta(x) = \arcsin x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 0, & x < 1, \\ \ln x, & 1 < x \leq e, \\ 1, & x > e. \end{cases}$ b) $f(x) = \begin{cases} \frac{1}{x-2}, & x < 2, \\ x - 1, & 2 \leq x < 3, \\ 4, & x \geq 3. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 9^{\frac{1}{2-x}}$; b) $f(x) = \frac{\sqrt{x + 14} - 4}{x^2 - 4}$.

Limits and Continuity

Variant №22

Exercise 1. Evaluate limits:

a) $\lim_{x \rightarrow \infty} \frac{12x^3 - 2x^2 - 1}{x + 3x^2 + 4x^3}$; b) $\lim_{x \rightarrow \infty} \frac{x^4 + 2x^3 + x^2 - 1}{x^5 + 4x^2 + 15}$;

c) $\lim_{x \rightarrow \infty} \frac{x^7 + 3x^5 + x^3}{12x^3 + x^4 + x^5}$; d) $\lim_{x \rightarrow \infty} \frac{(x+1)^3 + (x-1)^3}{x^3 + 1}$.

Exercise 2. Evaluate limits:

a) $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 12x + 20}$; b) $\lim_{x \rightarrow -1} \frac{\sqrt{2x+3} - \sqrt{x+2}}{x^3 + x}$.

Exercise 3. Evaluate limits:

a) $\lim_{x \rightarrow 2} \left(\frac{1}{2-x} - \frac{12}{8-x^3} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 - 6x - 16} - x)$; c) $\lim_{x \rightarrow 0} \frac{\arcsin^2 8x}{x \arctg x}$; d) $\lim_{x \rightarrow 0} \frac{\operatorname{tg}^2 6x}{\cos 3x - \cos x}$.

Exercise 4. Evaluate limits:

a) $\lim_{x \rightarrow 0} (1 + \operatorname{tg}^2 x)^{4 \operatorname{ctg}^2 x}$; b) $\lim_{x \rightarrow 0} \left(\frac{2+x}{2-x} \right)^{\frac{4}{x}}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 5x + 7}{2x^2 + 5x + 3} \right)^x$; d) $\lim_{x \rightarrow \infty} \left(\frac{x+4}{3x+5} \right)^{x+3}$; e) $\lim_{x \rightarrow 3} (2x-5)^{\frac{2x}{x-3}}$.

Exercise 5. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+6x} - \sqrt[3]{1+4x}}{\arctg 2x \cos 2x}$; b) $\lim_{x \rightarrow \pi} \frac{x^2 - \pi^2}{\sin x}$.

Exercise 6. Evaluate limits:

a) $\lim_{x \rightarrow 0} \frac{3^{2x} - 2^x}{x - \operatorname{tg} 4x}$; b) $\lim_{x \rightarrow 3} \frac{\ln(x-2)}{9-x^2}$;

c) $\lim_{x \rightarrow 0} \frac{x}{\ln(2x+3) - \ln 3}$; d) $\lim_{x \rightarrow \infty} (2x-6)[\ln x - \ln(x+5)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sqrt[3]{(e^x - 1)^2}$ with respect to the infinitesimal $\beta(x) = \ln(1+x)$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 2^x, & x < 0, \\ x+1, & 0 < x < 2, \\ 3, & x \geq 2. \end{cases}$ b) $f(x) = \begin{cases} 1, & x \leq 1, \\ \ln(x-1), & 1 < x \leq 2, \\ 2x-2, & x > 2. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 6^{\frac{1}{x-1}}$; b) $f(x) = \operatorname{arccctg} \frac{1}{x-3}$.

Limits and Continuity

Variant №23

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{5x^2 - 6x + 2}{13 + x - 10x^2}$; b) $\lim_{x \rightarrow \infty} \frac{x^4 + 2x^2 + 3}{1 - 3x + x^5 + 12x^6}$;

c) $\lim_{x \rightarrow \infty} \frac{2 - x + 2x^4 + x^5}{x^4 + 3x^2 - 12}$; d) $\lim_{x \rightarrow \infty} \frac{(x + 3)^3 + (x + 4)^3}{(x + 3)^4 - (x + 4)^4}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow -2} \frac{2x^2 + 5x + 2}{2x^3 + 7x^2 + 6x}$; b) $\lim_{x \rightarrow 4} \frac{x^2 - 16}{\sqrt{x - 3} - 1}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{3x^2 - 2} - \frac{x^2}{3x + 2} \right)$;

b) $\lim_{x \rightarrow \infty} (\sqrt[3]{x^2} - \sqrt{x(x-2)})$; c) $\lim_{x \rightarrow 0} \frac{\operatorname{tg}^3 2x}{x \arcsin^2 x}$; d) $\lim_{x \rightarrow 0} \frac{\arctg^2 8x}{1 - \cos 4x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 + 5x)^{\frac{3}{x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{x-2}{x+1} \right)^{2x-1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{5x^2 + 3x - 1}{5x^2 + 3x + 3} \right)^{x^2}$; d) $\lim_{x \rightarrow \infty} \left(\frac{6x-2}{x+5} \right)^{x-1}$; e) $\lim_{x \rightarrow -1} (2+x)^{\frac{8x}{x+1}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1+8x}}{\ln(1 - \sin x)}$; b) $\lim_{x \rightarrow \pi} \frac{x^2 - \pi^2}{\sin x}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{3^{4x} - 2^{2x}}{3 \arcsin x - \operatorname{tg} x}$; b) $\lim_{x \rightarrow 7} \frac{2^x - 128}{x - 7}$;

c) $\lim_{x \rightarrow e} \frac{\ln x - 1}{x - e}$; d) $\lim_{x \rightarrow \infty} (1 + 4x)[\ln(x+5) - \ln x]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sin(\sqrt[3]{x^2 + 1} - 1)$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} x, & x < 0, \\ \operatorname{tg} x, & 0 < x \leq \frac{\pi}{4}, \\ 1, & x > \frac{\pi}{4}. \end{cases}$ b) $f(x) = \begin{cases} \frac{1}{1-x}, & x < 1, \\ x, & 1 \leq x \leq 2, \\ 3, & x > 2. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 3^{\frac{1}{x-4}}$; b) $f(x) = \frac{x^2 - 3x + 2}{x^3 - 8}$.

Limits and Continuity

Variant №24

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{1 + 12x + x^3 - x^4}{5x^4 + 2x^3 + 1}$; b) $\lim_{x \rightarrow \infty} \frac{x^3 + 3x^2 - 14x}{1 + 2x^2 + x^3 + x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{x^5 + 3x^2 + x}{15 - x + x^2}$; d) $\lim_{x \rightarrow \infty} \frac{(x + 10)^2 + (3x + 1)^2}{(x + 6)^3 - (x + 1)^3}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow 3} \frac{x^4 - 18x^2 + 81}{x^3 - 6x^2 + 9x}$; b) $\lim_{x \rightarrow 3} \frac{\sqrt{2x + 3} - 3}{x^2 - 3x}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow 1} \left(\frac{1}{(x-1)(x-4)} - \frac{4}{(x-1)(x-13)} \right)$;

b) $\lim_{x \rightarrow \pm\infty} x(\sqrt{x^2 + 9} - x)$; c) $\lim_{x \rightarrow 0} \frac{x \operatorname{arctg} 7x}{\arcsin^2 x}$; d) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 8x}{1 - \cos 4x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 + 2\sqrt{x})^{\frac{8}{\sqrt{x}}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{5x}{5x - 2} \right)^{1-3x}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 7x - 1}{2x^2 + 3x - 7} \right)^{-x^2}$; d) $\lim_{x \rightarrow \infty} \left(\frac{x+1}{4x-3} \right)^{3x-2}$; e) $\lim_{x \rightarrow 1} (4 - 3x)^{\frac{6x}{x-1}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sqrt[3]{1-2x} - 1}$; b) $\lim_{x \rightarrow 4} \frac{2^x - 16}{\sin(\pi x)}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{e^{6x} - e^{-3x}}{2 \operatorname{arctg} x - x}$; b) $\lim_{x \rightarrow e} \frac{\ln x - 1}{\operatorname{tg}(\pi x - \pi e)}$;

c) $\lim_{x \rightarrow 0} \frac{1}{3x} [\ln(6x + 4) - \ln 4]$; d) $\lim_{x \rightarrow \infty} x \ln \sqrt{\frac{3x-1}{3x+1}}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \ln(1 + \sqrt[3]{x \sin x})$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} -x, & x < 0, \\ \sin x, & 0 < x < \pi, \\ x - \pi, & x \geq \pi. \end{cases}$ b) $f(x) = \begin{cases} 1, & x \leq -\frac{\pi}{2}, \\ \operatorname{tg} x, & -\frac{\pi}{2} < x \leq 0, \\ x + 1, & x > 0. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 8 \frac{1}{x+5}$; b) $f(x) = \frac{|\sin x|}{x}$.

Limits and Continuity

Variant №25

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{9x^4 + 2x^3 + 5}{8x + x^2 - 3x^4}$; b) $\lim_{x \rightarrow \infty} \frac{4x^4 + x^2 - 6x + 1}{13x + 7x^2 - x^3}$;

c) $\lim_{x \rightarrow \infty} \frac{3x + 4x^2 + x^5}{x^4 - 3x^3 + 14}$; d) $\lim_{x \rightarrow \infty} \frac{(3-x)^4 - (2-x)^4}{(1-x)^3 - (1+x)^3}$.

Exercise 2. Evaluate limits a) $\lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{x^2 - 5x + 4}$; b) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{\sqrt{3x+5} - \sqrt{2x+8}}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{6x^2 + 1} - \frac{2x^2}{12x + 9} \right)$;

b) $\lim_{x \rightarrow \infty} (\sqrt[3]{(x+1)^2} - \sqrt[3]{(x-1)^2})$; c) $\lim_{x \rightarrow 0} \frac{\arcsin^2 x}{x \operatorname{tg} 3x}$; d) $\lim_{x \rightarrow 0} \frac{x(\sin 6x - \sin 2x)}{1 - \cos 2x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 + 3x)^{\frac{4}{x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{3x+2}{3x-1} \right)^{4x-3}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 21x - 7}{2x^2 + 18x + 9} \right)^{2x+1}$; d) $\lim_{x \rightarrow \infty} \left(\frac{7x-2}{x+5} \right)^{2x+3}$; e) $\lim_{x \rightarrow 3} (2x-5)^{\frac{2x}{x-3}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\ln(1-3x)}{\sqrt{8x+4}-2}$; b) $\lim_{x \rightarrow 2} \frac{\ln(9-2x^2)}{\sin(2\pi x)}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{8^x - 3^{-5x}}{\operatorname{tg} x - 3x}$; b) $\lim_{x \rightarrow -3} \frac{e^{-x} - e^3}{\operatorname{arctg}(x^2 - 9)}$;

c) $\lim_{x \rightarrow \infty} x \ln \sqrt{\frac{2x+1}{2x-1}}$; d) $\lim_{x \rightarrow \infty} (5x+4)[\ln x - \ln(x-3)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = 1 - \cos \sqrt[5]{x}$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} -x, & x < 0, \\ x^3, & 0 < x < 2, \\ 6-x, & x \geq 2. \end{cases}$ b) $f(x) = \begin{cases} \ln(-x), & x < 0, \\ x, & 0 \leq x < 1, \\ 2, & x \geq 1. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 2^{\frac{1}{7-x}}$; b) $f(x) = \frac{1}{2^{4-x} - 1}$.

Limits and Continuity

Variant №26

- Exercise 1.** Evaluate limits:
- a) $\lim_{x \rightarrow \infty} \frac{x^4 + 4x^3 + x^2 + 2}{13 + 7x + 5x^4}$; b) $\lim_{x \rightarrow \infty} \frac{12 + x^2 - 9x^3}{x^6 + 4x^4 - 17}$;
- c) $\lim_{x \rightarrow \infty} \frac{x^5 + 4x^2 + x - 1}{3 - 12x^3 + x^4}$; d) $\lim_{x \rightarrow \infty} \frac{(6-x)^2 - (6+x)^2}{(6-x)^2 - (1-x)^2}$.
- Exercise 2.** Evaluate limits:
- a) $\lim_{x \rightarrow 2} \frac{x^3 - 4x^2 + 4x}{x^4 - 8x^2 + 16}$; b) $\lim_{x \rightarrow -3} \frac{\sqrt{x+7} - 2}{x^2 + 5x + 6}$.
- Exercise 3.** Evaluate limits:
- a) $\lim_{x \rightarrow 3} \left(\frac{1}{3-x} - \frac{27}{27-x^3} \right)$;
- b) $\lim_{x \rightarrow \pm\infty} (\sqrt{4x^2 + 3x + 1} - 2x)$; c) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 6x}{\sin x \cdot \operatorname{tg} 2x}$; d) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg}^2 x}{x (\sin 4x - \sin 2x)}$.
- Exercise 4.** Evaluate limits:
- a) $\lim_{x \rightarrow 0} (1 + 4 \operatorname{tg} x)^{\operatorname{ctg} x}$; b) $\lim_{x \rightarrow \infty} \left(\frac{2x-5}{2x+1} \right)^{x+1}$;
- c) $\lim_{x \rightarrow \infty} \left(\frac{3x^2 - 5x}{3x^2 - 5x + 7} \right)^{x+1}$; d) $\lim_{x \rightarrow \infty} \left(\frac{x+3}{2x-1} \right)^{4x+3}$; e) $\lim_{x \rightarrow 1} (4x-3)^{\frac{6x}{1-x}}$.
- Exercise 5.** Evaluate limits:
- a) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{3x+1}}{\cos(\pi(x+1)/2)}$; b) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\ln(\operatorname{tg} x)}{\cos 2x}$.
- Exercise 6.** Evaluate limits:
- a) $\lim_{x \rightarrow 0} \frac{e^{5x} - e^{-4x}}{\operatorname{arctg} 2x - x}$; b) $\lim_{x \rightarrow -3} \frac{6^{-x} - 216}{x+3}$;
- c) $\lim_{x \rightarrow 0} \frac{x}{\ln(2-3x) - \ln 2}$; d) $\lim_{x \rightarrow \infty} (1-6x)[\ln(3x-5) - \ln 3x]$.
- Exercise 7.** Determine the order of the infinitesimal $\alpha(x) = e^{\sin^3 x} - 1$ with respect to the infinitesimal $\beta(x) = \arcsin x$ as $x \rightarrow 0$.
- Exercise 8.** Examine the continuity of functions and graph these functions.
- a) $f(x) = \begin{cases} -1, & x < 1, \\ x-2, & 1 < x < 4, \\ \sqrt{x}, & x \geq 4. \end{cases}$ b) $f(x) = \begin{cases} -x, & x \leq 0, \\ \operatorname{ctg} x, & 0 < x \leq \frac{\pi}{4}, \\ 2, & x > \frac{\pi}{4}. \end{cases}$
- Exercise 9.** Examine the continuity of functions and graph these functions.
- a) $f(x) = e^{-\frac{1}{x-5}}$; b) $f(x) = \frac{\sqrt{5+x} - 3}{x^2 - 16}$.

Limits and Continuity

Variant №27

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{7 + x + x^2 - 8x^3}{2x^3 + 4x + 3}$; b) $\lim_{x \rightarrow \infty} \frac{x^3 + 3x^2 - 2x^2}{5 + x + 3x^2 + 11x^4}$;

c) $\lim_{x \rightarrow \infty} \frac{2 + x^3 + x^6}{x^4 + 6x^2 - 12}$; d) $\lim_{x \rightarrow \infty} \frac{(2x - 3)^3 - (x + 5)^3}{(3x - 1)^3 + (2x + 3)^3}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10}$; b) $\lim_{x \rightarrow 0} \frac{x^2 + x}{\sqrt{7x + 1} - \sqrt{1 - 7x}}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow \infty} \left(\frac{2x^3}{4x^2 - 1} - \frac{3x^2}{6x + 5} \right)$;

b) $\lim_{x \rightarrow \infty} x(\sqrt[3]{8x^3 - 2} - 2x)$; c) $\lim_{x \rightarrow 0} \frac{x \operatorname{tg}^3 2x}{\arcsin^4 x}$; d) $\lim_{x \rightarrow 0} \frac{x \operatorname{arctg} 4x}{\cos^4 x - \cos^2 x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 - 8x)^{\frac{1}{2x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{x + 2}{x - 1} \right)^{4x - 3}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 6x + 5}{x^2 - 5x + 5} \right)^{3x + 2}$; d) $\lim_{x \rightarrow \infty} \left(\frac{6x - 1}{x + 3} \right)^{4x - 1}$; e) $\lim_{x \rightarrow \frac{\pi}{2}} (1 + \cos 5x)^{\operatorname{ctg} x}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\arcsin 3x}{\sqrt{2 + x} - \sqrt{2}}$; b) $\lim_{x \rightarrow 1} \frac{3^x - 3}{\operatorname{tg}(\pi x)}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{6^{4x} - 2^x}{\arctg x + x^4}$; b) $\lim_{x \rightarrow e^0} \frac{\ln x - 1}{\sin(\pi x - \pi e)}$;

c) $\lim_{x \rightarrow 0} \frac{1}{7x} [\ln(7x + 3) - \ln 3]$; d) $\lim_{x \rightarrow 2} \frac{\log_2 x - 1}{x - 2}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \arctg(\sqrt{1 + x^5} - 1)$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} 1 - x, & x < 0, \\ \cos x, & 0 \leq x < \frac{\pi}{2}, \\ x - \frac{\pi}{2}, & x > \frac{\pi}{2}. \end{cases}$ b) $f(x) = \begin{cases} 2, & x \leq 1, \\ x, & 1 < x \leq 2, \\ \frac{1}{2 - x}, & x > 2. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 4^{\frac{1}{x-5}}$; b) $f(x) = \arctg \frac{1}{x^2 - 4}$.

Limits and Continuity

Variant №28

Exercise 1. Evaluate limits: **a)** $\lim_{x \rightarrow \infty} \frac{8x^3 + x + x^2 - 1}{x + 4x^2 - x^3}$; **b)** $\lim_{x \rightarrow \infty} \frac{4 + 3x^2 - x^3}{x^5 + 9x^4 + 12}$;

c) $\lim_{x \rightarrow \infty} \frac{x^6 + 5x^2 + 4}{x + 4x^2 - 12x^5}$; **d)** $\lim_{x \rightarrow \infty} \frac{(x+1)^4 - (x-1)^4}{(x+1)^3 + (x-1)^3}$.

Exercise 2. Evaluate limits: **a)** $\lim_{x \rightarrow 1} \frac{x^3 + 3x^2 - x - 3}{x^2 + 6x - 7}$; **b)** $\lim_{x \rightarrow 1} \frac{3x^2 - 4x + 1}{\sqrt{2x+3} - \sqrt{3x+2}}$.

Exercise 3. Evaluate limits: **a)** $\lim_{x \rightarrow 1} \left(\frac{2}{(x-1)(x-5)} - \frac{5}{(x-1)(x-11)} \right)$;

b) $\lim_{x \rightarrow \pm\infty} (\sqrt{x^2 + 10x + 21} - x)$; **c)** $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{\sin 5x - \sin x}$; **d)** $\lim_{x \rightarrow 0} \frac{\arcsin^3 x}{(1 - \cos x) \sin 2x}$.

Exercise 4. Evaluate limits: **a)** $\lim_{x \rightarrow 0} (1 + 2 \sin^3 x)^{\operatorname{cosec}^3 x}$; **b)** $\lim_{x \rightarrow \infty} \left(\frac{x+5}{x-3} \right)^{3x-1}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{7x^2 + 18x - 15}{7x^2 + 11x + 15} \right)^{x+2}$; **d)** $\lim_{x \rightarrow \infty} \left(\frac{x-3}{2x+5} \right)^{x-1}$; **e)** $\lim_{x \rightarrow 3} (3x-8)^{\frac{5x}{3-x}}$.

Exercise 5. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{3 \operatorname{arctg} x}$; **b)** $\lim_{x \rightarrow -2} \frac{\operatorname{tg}(\pi x)}{x+2}$.

Exercise 6. Evaluate limits: **a)** $\lim_{x \rightarrow 0} \frac{e^{5x} - e^x}{\sin(3x) - x}$; **b)** $\lim_{x \rightarrow e} \frac{\ln x - 1}{\arcsin(x - e)}$;

c) $\lim_{x \rightarrow 0} \frac{x}{\ln(5x+7) - \ln 7}$; **d)** $\lim_{x \rightarrow \infty} (2x+4)[\ln 3x - \ln(3x+6)]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \sin(e^{\sqrt[3]{x}} - 1)$ with respect to the infinitesimal $\beta(x) = \operatorname{arctg} x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

$$\mathbf{a)} \ f(x) = \begin{cases} 2^{-x}, & x < 0, \\ \cos x, & 0 < x \leq \pi, \\ -1 & x > \pi. \end{cases} \quad \mathbf{b)} \ f(x) = \begin{cases} \ln(1-x), & x < 1, \\ (x-1)^2, & 1 \leq x \leq 2, \\ 2, & x > 2. \end{cases}$$

Exercise 9. Examine the continuity of functions and graph these functions.

$$\mathbf{a)} \ f(x) = 6^{\frac{1}{x-1}}; \quad \mathbf{b)} \ f(x) = \frac{x^3 - 8}{x^2 - 4}.$$

Limits and Continuity

Variant №29

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{8x^6 + 3x^5 + 10}{4 + x + 5x^3 - 2x^6}$; b) $\lim_{x \rightarrow \infty} \frac{4 + 3x - x^2}{x^3 + 6x^2 + 13x - 1}$;

c) $\lim_{x \rightarrow \infty} \frac{x^4 + 5x^2 + x}{1 - 3x^2 + 2x^3}$; d) $\lim_{x \rightarrow \infty} \frac{(x+2)^4 - (x-2)^4}{(x+5)^2 + (x-5)^2}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow 1} \frac{x^3 + x^2 - 2x}{x^3 + 4x^2 - x - 4}$; b) $\lim_{x \rightarrow 5} \frac{2x^2 - 6x + 20}{\sqrt{2x+1} - \sqrt{x+6}}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{2x^2 + 3} - \frac{5x^2}{10x - 4} \right)$;

b) $\lim_{x \rightarrow \infty} x (\sqrt[3]{27x^3 + 2} - 3x)$; c) $\lim_{x \rightarrow 0} \frac{x \arcsin^2 5x}{\arctg^3 x}$; d) $\lim_{x \rightarrow 0} \frac{\cos x - \cos^2 x}{\sin^2 6x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 + 8x)^{-\frac{1}{4x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{2x-3}{2x+5} \right)^{x-2}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 2x + 3}{2x^2 + 2x + 1} \right)^{3x^2 - 7}$; d) $\lim_{x \rightarrow \infty} \left(\frac{3x+2}{x+5} \right)^{5x+3}$; e) $\lim_{x \rightarrow -1} (2+x)^{\frac{5x}{x+1}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{\sin(\pi(x+2))}$; b) $\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\pi - x}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{e^x - e^{-2x}}{\sin 4x - \operatorname{tg} x}$; b) $\lim_{x \rightarrow 2} \frac{\ln^2(x-1)}{8 - x^3}$;

c) $\lim_{x \rightarrow \infty} x \ln \sqrt[3]{\frac{3x+5}{3x-1}}$; d) $\lim_{x \rightarrow \infty} (1 - 5x)[\ln(x+4) - \ln x]$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \ln(1 + \sqrt[3]{x^5})$ with respect to the infinitesimal $\beta(x) = \sin x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

a) $f(x) = \begin{cases} x-1, & x < 1, \\ 2 \ln x, & 1 < x \leq e, \\ 2, & x > e. \end{cases}$ b) $f(x) = \begin{cases} 0, & x \leq -\frac{\pi}{2}, \\ \operatorname{tg} x, & -\frac{\pi}{2} < x \leq 0, \\ x+1, & x > 0. \end{cases}$

Exercise 9. Examine the continuity of functions and graph these functions.

a) $f(x) = 2^{\frac{1}{7-x}}$; b) $f(x) = \frac{\sqrt{1 - \cos 2x}}{x}$.

Limits and Continuity

Variant №30

Exercise 1. Evaluate limits: a) $\lim_{x \rightarrow \infty} \frac{8x^7 + 3x^5 - 1}{5 - x + 2x^2 - 8x^7}$; b) $\lim_{x \rightarrow \infty} \frac{3x^3 + 2x^2 - 4}{x^4 + 5x^2 - 2x + 1}$;

c) $\lim_{x \rightarrow \infty} \frac{4x + x^3 + x^5}{x^3 + 3x^2 - 17}$; d) $\lim_{x \rightarrow \infty} \frac{(3-x)^2 + (3+x)^2}{(3-x)^2 - (3+x)^2}$.

Exercise 2. Evaluate limits: a) $\lim_{x \rightarrow 1} \frac{x^3 + 2x^2 - x - 2}{2x^2 - x - 1}$; b) $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{\sqrt{2-x} - \sqrt{x+6}}$.

Exercise 3. Evaluate limits: a) $\lim_{x \rightarrow 4} \left(\frac{1}{4-x} - \frac{48}{64-x^3} \right)$;

b) $\lim_{x \rightarrow \pm\infty} x (\sqrt{16x^2 + 1} - 4x)$; c) $\lim_{x \rightarrow 0} \frac{x (\sin 7x - \sin x)}{\arctg^2 x}$; d) $\lim_{x \rightarrow 0} \frac{\cos^2 4x - 1}{x \arcsin 5x}$.

Exercise 4. Evaluate limits: a) $\lim_{x \rightarrow 0} (1 - 7\sqrt{\operatorname{tg} x})^{\sqrt{\operatorname{ctg} x}}$; b) $\lim_{x \rightarrow \infty} \left(\frac{x+3}{x+5} \right)^{x+4}$;

c) $\lim_{x \rightarrow \infty} \left(\frac{4x^2 + 4x - 1}{4x^2 + 2x + 3} \right)^{1-2x}$; d) $\lim_{x \rightarrow \infty} \left(\frac{x+1}{8x-3} \right)^{2x-1}$; e) $\lim_{x \rightarrow -1} (2+x)^{\frac{3x}{x+1}}$.

Exercise 5. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{x \sin x}$; b) $\lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2 \cos x}{\pi - 3x}$.

Exercise 6. Evaluate limits: a) $\lim_{x \rightarrow 0} \frac{4^{2x} - 3^{-x}}{2 \sin x - \arctg x}$; b) $\lim_{x \rightarrow 2} \frac{e^x - e^2}{\sin(4-x^2)}$;

c) $\lim_{x \rightarrow 0} \frac{1}{2x} [\ln(8x+2) - \ln 2]$; d) $\lim_{x \rightarrow 0} \frac{x}{\ln(x+4) - \ln 4}$.

Exercise 7. Determine the order of the infinitesimal $\alpha(x) = \log_2 \sqrt[5]{1+x^3 \sin x}$ with respect to the infinitesimal $\beta(x) = x$ as $x \rightarrow 0$.

Exercise 8. Examine the continuity of functions and graph these functions.

$$\text{a) } f(x) = \begin{cases} x^3, & x < 0, \\ \operatorname{tg} x, & 0 < x \leq \frac{\pi}{4}, \\ 1, & x > \frac{\pi}{4}. \end{cases} \quad \text{b) } f(x) = \begin{cases} \frac{1}{x}, & x < 0, \\ x+1, & 0 \leq x < 2, \\ 4, & x \geq 2. \end{cases}$$

Exercise 9. Examine the continuity of functions and graph these functions.

$$\text{a) } f(x) = 3^{\frac{1}{x+2}}; \quad \text{b) } f(x) = \frac{1}{1+2^{\operatorname{tg} x}}.$$

Limits and Continuity

LITERATURE

1. Вища математика. Диференціальне та інтегральне числення функцій однієї змінної: збірник задач до розрахункової роботи та приклади розв'язування типових задач [Електронний ресурс] : навчальний посібник для здобувачів ступеня бакалавра / КПІ ім. Ігоря Сікорського ; уклад.: Качаєнко О. Б., Коваль О. О., Поліщук О. Б., Стогній В. І. – Київ: КПІ ім. Ігоря Сікорського, 2022. – 117 с.
2. Вступ до математичного аналізу. Диференціальне числення функції однієї змінної. Збірник задач [Електронний ресурс]: навчальний посібник для студентів інженерних спеціальностей / КПІ ім. Ігоря Сікорського ; уклад.: М. Є. Дудкін, О. Ю. Дюженкова, І. В. Степахно.– Київ : КПІ ім. Ігоря Сікорського, 2021. – 65 с.
3. Вступ до математичного аналізу. Диференціальне числення функцій однієї змінної. Навчально-методичний посібник [Електронний ресурс] / КПІ ім. Ігоря Сікорського ; уклад.: Т. В. Авдєєва, О. В. Борисенко, О. Ю. Дюженкова, В. В. Листопадова. – Київ : КПІ ім. Ігоря Сікорського, 2021. – 84 с.
4. Методичні вказівки та варіанти типово-розрахункових робіт з вищої математики для студентів технічних спеціальностей. Вступ до математичного аналізу. Диференціальне числення функції однієї змінної [Електронний ресурс] / НТУУ «КПІ»; уклад.: Г. В. Журавська, І. М. Копась, Г. М. Кулик, Н. В. Рева, Н. В. Степаненко. –Київ : НТУУ «КПІ», 2016. – 90 с.
5. Дубовик В. П. Вища математика: навч. посіб. для студ. вищ. навч. закл. / В.П. Дубовик., І. І. Юрик. - 4-те вид. - К. : Ігнатекс-Україна., 2013. – 648 с.
6. Вища математика. Збірник задач: навч. посіб. / В. П. Дубовик, І. І. Юрик, І. П. Вовкодав, В. І. Дев'ятко, Р. К. Клименко, В. В. Крочук, М. А. Мартиненко ; за ред. В. П. Дубовика, І. І. Юрика. – К. : А.С.К., 2011. – 480 с.
7. Demidovich B. (Editor), Yankovsky G. (Translator), Problems in Mathematical Analysis, Mir Publishers, American First edition, January 1 1989, p. 496. – ISBN-10: 5030009434.

Електронне мережне навчальне видання

ВИЩА МАТЕМАТИКА

ГРАНИЦІ. НЕПЕРЕРВНІСТЬ

Практикум

(Англійською мовою)

Укладачі: Массалітіна Є.В., Новикова Г.К., Пилипенко В.А.

Практикум до розділу «Границі. Неперервність» з курсу «Вища математика» для студентів технічних спеціальностей містить 30 варіантів, кожен варіант складається з 9 завдань (26 задач). Самостійне виконання цих завдань забезпечує свідоме оволодіння навчальним матеріалом, який передбачено робочою програмою з вищої математики.

Практикум може бути рекомендований в якості розрахункової роботи за темою «Границі. Неперервність» для студентів першого курсу технічних спеціальностей.

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і розповсюджувачів видавничої продукції ДК № 5354 від 25.05.2017 р.