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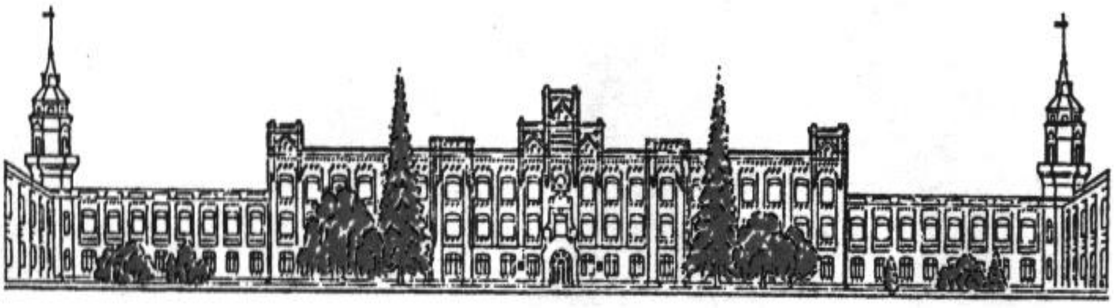

HIGHER MATHEMATICS

QUADRATIC CURVES
and
QUADRIC SURFACES

Practice exercises collection

Recommended by the Methodological Council
of the Igor Sikorsky Kyiv Polytechnic Institute
as a study aid for bachelor's degree applicants
on the technical specialties

Compilers: V.A. Pylypenko, E.V. Massalitina

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Reviewer *Dykhovychnyi O.O.*, PhD, Associate Professor,
Department of Mathematical Analysis and Probability Theory,
Igor Sikorsky Kyiv Polytechnic Institute

Responsible Editor *Dudkin M.E.*, Doctor of Physical and Mathematical Sciences, Professor

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The practice book offers additional individual exercises for university students studying Quadratic Curves and Quadric Surfaces in the course of Higher Mathematics of Igor Sikorsky KPI. The book contains 30 different variants and each variant consists of 5 exercises (23 tasks). Students master the material being studied and consolidate the acquired knowledge by solving such individual tasks.

The practice book can be recommended as an individual work on Quadratic Curves and Quadric Surfaces for first-year students of technical specialties.

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Quadratic Curves and Quadric Surfaces

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INTRODUCTION

The Quadratic Curves and Quadric Surfaces section is included in the course of Higher Mathematics for engineering students of Igor Sikorsky KPI. An important factor in the successful assimilation of the educational material by the students is solving practical tasks on their own.

The practice book offers a systematized set of exercises that students of technical specialties should be able to solve when studying Quadratic Curves and Quadric Surfaces. The book contains 30 different variants and each variant consists of 5 exercises (23 tasks).

This practice book helps students to develop practical skills in solving basic exercises: to reduce equations of quadratic curves to a canonical form, to determine the type of these curves and find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses and directrixes; to determine the type of quadric surfaces such as ellipsoids, elliptic paraboloids, hyperbolic paraboloids, hyperboloids of one sheet and two sheets, cylindrical surfaces, conical surfaces, to find equations of surfaces of revolution and graph them.

GENERAL RECOMMENDATIONS

The practice book is designed to control and improve the knowledge of university students in the study of Quadratic Curves and Quadric Surfaces in the course of Higher Mathematics. The main goal is to develop and consolidate the skills of independent work of students in the study of educational material.

In order to successfully complete the exercises, students need to thoroughly study the lecture material and analyze the examples solved in practical classes. Only after that students can start solving their individual tasks.

Students have to adhere to the following requirements:

1. The number of the variant of the individual exercises corresponds to the ordinal number of the student in the list of the study group;
2. Individual work is written in a separate notebook, which should contain:
   - the title page;
   - the results table;
   - solved exercises (the solution of each exercise starts from a new page).
3. Before solving each exercise, the condition and all specific data for the corresponding variant are completely rewritten.
4. The solution of each task must contain detailed explanations and necessary formulas.
5. Completed work must be handed over to the teacher for verification within the prescribed time limit.

Students who do not submit their completed individual work on time will not be allowed to take the exam.

Quadratic Curves and Quadric Surfaces

Variant №1

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 2x - 4y - 31 = 0;$

b) $9x^2 + 4y^2 + 36x - 24y + 36 = 0;$

c) $36x^2 - 16y^2 + 216x - 32y - 268 = 0;$

d) $y^2 + 8x + 8y + 8 = 0;$

e) $4x^2 + 25y^2 + 24x - 50y + 161 = 0;$

f) $y = 3 + \frac{4}{3}\sqrt{x^2 - 4x + 13}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2 + z^2 + 2x - 2z + 1 = 0;$

b) $x^2 + 4y^2 - 4x = 0;$

c) $y^2 - 9z^2 - 18z = 0;$

d) $x^2 + 6x - 7z + 9 = 0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + 6x + y^2 + z^2 + 8 = 0;$

b) $9x^2 + 16y^2 + z^2 = 144;$

c) $4x^2 - y^2 + 49z^2 = 196;$

d) $-x^2 + y^2 + 25z^2 = -25;$

e) $16x^2 + 4y^2 = 64z;$

f) $z^2 - x^2 = y;$

g) $-x^2 + y^2 + 9z^2 = 0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2 + y^2 = 81, \\ z = 0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \frac{y^2}{4} + \frac{z^2}{9} = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} x = \frac{z}{2}, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

d) the parabola $\begin{cases} x^2 = 5y, \\ z = 0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{y^2}{36} - z^2 = 1, \\ x = 0 \end{cases}$ is revolved about the y -axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0, -3, 0)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{36} + \frac{z^2}{25} = 1, \\ y = 1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №2

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+6x-2y-15=0$;

b) $x^2+4y^2+2x+32y+49=0$;

c) $16x^2-9y^2-96x-54y+207=0$;

d) $x^2-4x+6y-14=0$;

e) $9x^2-y^2+54x-6y+72=0$;

f) $x=-1-\frac{3}{5}\sqrt{-y^2+8y+9}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2+z^2+4y+4z+4=0$;

b) $x^2+9z^2-18x=0$;

c) $x^2-4y^2+4x=0$;

d) $3y^2-2z-6=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2-8y+z^2+15=0$;

b) $x^2+16y^2+4z^2=64$;

c) $25x^2+4y^2-z^2=100$;

d) $4x^2-y^2+z^2=-16$;

e) $y^2+36z^2=36x$;

f) $x^2-y^2=z$;

g) $4x^2-y^2+9z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2+z^2=4, \\ x=0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \frac{x^2}{16}+\frac{z^2}{4}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} y=\frac{x}{3}, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

d) the parabola $\begin{cases} y^2=6z, \\ x=0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2-\frac{z^2}{25}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,-3)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{36}+\frac{y^2}{16}=1, \\ z=2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №3

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+6x+6y+2=0$;

b) $25x^2+4y^2-200x+16y+316=0$;

c) $x^2-4y^2+8y-31=0$;

d) $y^2-x-8y+15=0$;

e) $9x^2+16y^2-54x+128y+337=0$;

f) $y=-2-\frac{4}{5}\sqrt{-x^2-8x+9}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+y^2-6x+6y+9=0$;

b) $y^2+9z^2+6y=0$;

c) $x^2-4z^2-8z=0$;

d) $x^2-4x+5y+4=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2+10z+24=0$;

b) $36x^2+y^2+4z^2=144$;

c) $-x^2+4y^2+9z^2=36$;

d) $4x^2+4y^2-z^2=-4$;

e) $4x^2+16z^2=16y$;

f) $y^2-z^2=x$;

g) $25x^2+4y^2-z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=9, \\ y=0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{4}+\frac{y^2}{25}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} z=\frac{y}{4}, \\ x=0 \end{cases}$ is revolved about the y - axis; and the z - axis;

d) the parabola $\begin{cases} z^2=2x, \\ y=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{16}-y^2=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(-3,0,0)$ and its directrix is the ellipse $\begin{cases} \frac{y^2}{36}+\frac{z^2}{9}=1, \\ x=1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №4

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 6x + 8y + 16 = 0$; b) $9x^2 + 16y^2 - 18x - 128y + 121 = 0$;

c) $25x^2 - 9y^2 + 150x + 54y + 369 = 0$; d) $x^2 + 8x - 4y + 8 = 0$;

e) $9x^2 + y^2 - 18x - 2y + 46 = 0$; f) $x = 2 + \frac{3}{4}\sqrt{y^2 + 2y + 17}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2 + z^2 - 2x - 2z + 1 = 0$; b) $x^2 + 4y^2 + 8y = 0$;

c) $y^2 - 9z^2 - 6y = 0$; d) $2z^2 + x - 2 = 0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 - 12x + y^2 + z^2 + 35 = 0$; b) $25x^2 + 4y^2 + z^2 = 100$;

c) $16x^2 - y^2 + 4z^2 = 64$; d) $-x^2 + y^2 + 9z^2 = -9$;

e) $4x^2 + y^2 = 4z$; f) $z^2 - x^2 = y$;

g) $-x^2 + y^2 + 25z^2 = 0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2 + y^2 = 100, \\ z = 0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \frac{y^2}{4} + \frac{z^2}{36} = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} x = \frac{z}{5}, \\ y = 0 \end{cases}$ is revolved about the z - axis and the x - axis;

d) the parabola $\begin{cases} x^2 = 3y, \\ z = 0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{y^2}{9} - z^2 = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0, -2, 0)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{25} + \frac{z^2}{9} = 1, \\ y = 2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №5

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 6x - 6y - 7 = 0;$ b) $25x^2 + 9y^2 + 200x - 36y + 211 = 0;$

c) $4x^2 - 25y^2 + 24x - 200y - 464 = 0;$ d) $y^2 + 2x + 2y - 3 = 0;$

e) $16x^2 + 36y^2 + 32x - 72y + 628 = 0;$ f) $x = 4 - \frac{4}{3}\sqrt{y^2 - 4y - 5}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2 + z^2 + 4y - 4z + 4 = 0;$ b) $4x^2 + z^2 - 4z = 0;$

c) $9x^2 - y^2 + 18x = 0;$ d) $z^2 - 3y - 6z + 9 = 0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + y^2 + 14y + z^2 + 48 = 0;$ b) $x^2 + 4y^2 + 9z^2 = 36;$

c) $9x^2 + 25y^2 - z^2 = 225;$ d) $4x^2 - y^2 + z^2 = -4;$

e) $y^2 + 16z^2 = 16x;$ f) $x^2 - y^2 = z;$

g) $4x^2 - y^2 + 25z^2 = 0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2 + z^2 = 121, \\ x = 0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \frac{x^2}{9} + \frac{z^2}{16} = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} y = \frac{x}{6}, \\ z = 0 \end{cases}$ is revolved about the x - axis and the y - axis;

d) the parabola $\begin{cases} y^2 = 4z, \\ x = 0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2 - \frac{z^2}{4} = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,-2)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{25} + \frac{y^2}{16} = 1, \\ z = 1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №6

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 + 6x - 8y + 21 = 0$; b) $x^2 + 4y^2 + 4x + 8y - 28 = 0$;

c) $16x^2 - 25y^2 - 96x - 100y + 444 = 0$; d) $x^2 - 8x + y + 14 = 0$;

e) $4x^2 - 49y^2 + 24x - 196y - 160 = 0$; f) $y = 1 - \frac{5}{4}\sqrt{-x^2 - 4x + 12}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2 + y^2 + 6x + 6y + 9 = 0$; b) $9y^2 + z^2 + 18y = 0$;

c) $x^2 - 4z^2 - 4x = 0$; d) $2y^2 - 3x + 9 = 0$;

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + y^2 + z^2 - 6z + 5 = 0$; b) $9x^2 + y^2 + 16z^2 = 144$;

c) $-x^2 + 49y^2 + 4z^2 = 196$; d) $25x^2 + y^2 - z^2 = -25$;

e) $16x^2 + 4z^2 = 64y$; f) $y^2 - z^2 = x$;

g) $9x^2 + y^2 - z^2 = 0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2 + z^2 = 1, \\ y = 0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{25} + \frac{y^2}{9} = 1, \\ z = 0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} y = \frac{z}{2}, \\ x = 0 \end{cases}$ is revolved about the z - axis and the y - axis;

d) the parabola $\begin{cases} x^2 = 5z, \\ y = 0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2 - \frac{y^2}{36} = 1, \\ z = 0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(3, 0, 0)$ and its directrix is the ellipse $\begin{cases} \frac{y^2}{25} + \frac{z^2}{36} = 1, \\ x = -1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №7

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+6x+4y-23=0$; b) $49x^2+9y^2-98x+18y-383=0$;

c) $x^2-4y^2-6x+32y-71=0$; d) $y^2-8x-2y-15=0$;

e) $25x^2+4y^2-50x+24y+61=0$; f) $x=-1+\dfrac{5}{3}\sqrt{-y^2-2y+8}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+z^2-2x+2z+1=0$; b) $9x^2+y^2+6y=0$;

c) $y^2-4z^2+8z=0$; d) $x^2+4x+7z+4=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+8x+y^2+z^2+12=0$; b) $4x^2+16y^2+z^2=64$;

c) $4x^2-y^2+25z^2=100$; d) $-x^2+4y^2+z^2=-16$;

e) $x^2+9y^2=9z$; f) $z^2-x^2=y$;

g) $-x^2+4y^2+9z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+y^2=64, \\ z=0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \dfrac{y^2}{25}+\dfrac{z^2}{9}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} z=\dfrac{x}{3}, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

d) the parabola $\begin{cases} y^2=6x, \\ z=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} y^2-\dfrac{z^2}{25}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,3,0)$ and its directrix is the ellipse $\begin{cases} \dfrac{x^2}{16}+\dfrac{z^2}{36}=1, \\ y=-2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №8

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 2x + 6y - 6 = 0;$

b) $16x^2 + 25y^2 - 64x - 50y - 311 = 0;$

c) $25x^2 - 4y^2 + 150x + 16y + 309 = 0;$

d) $x^2 + 2x - 6y - 5 = 0;$

e) $4x^2 + y^2 - 24x - 4y + 76 = 0;$

f) $y = -1 + \frac{3}{4}\sqrt{x^2 - 8x}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2 + z^2 - 4y - 4z + 4 = 0;$

b) $4x^2 + z^2 - 8x = 0;$

c) $9x^2 - y^2 - 6y = 0;$

d) $3y^2 + 2z - 4 = 0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + y^2 - 10y + z^2 + 21 = 0;$

b) $x^2 + 36y^2 + 4z^2 = 144;$

c) $9x^2 + 4y^2 - z^2 = 36;$

d) $25x^2 - y^2 + 25z^2 = -25;$

e) $16y^2 + 4z^2 = 16x;$

f) $x^2 - y^2 = z;$

g) $4x^2 - y^2 + 9z^2 = 0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2 + z^2 = 9, \\ x = 0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \frac{x^2}{9} + \frac{z^2}{36} = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} x = \frac{y}{4}, \\ z = 0 \end{cases}$ is revolved about the y - axis and the x - axis;

d) the parabola $\begin{cases} z^2 = 2y, \\ x = 0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{16} - z^2 = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,2)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{9} + \frac{y^2}{25} = 1, \\ z = -2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №9

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2-6x-4y-12=0;$ b) $36x^2+16y^2+72x-32y-524=0;$

c) $16x^2-25y^2+32x-150y-609=0;$ d) $y^2+x+2y-3=0;$

e) $x^2+9y^2+2x-54y+188=0;$ f) $y=1-\frac{4}{3}\sqrt{x^2-2x+10}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+y^2+6x-6y+9=0;$ b) $y^2+4z^2+4y=0;$

c) $x^2-9z^2+18z=0;$ d) $x^2-6x-5y+9=0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2+12z+32=0;$ b) $25x^2+y^2+4z^2=100;$

c) $-x^2+4y^2+16z^2=64;$ d) $9x^2+y^2-z^2=-9;$

e) $4x^2+z^2=4y;$ f) $y^2-z^2=x;$

g) $25x^2+y^2-z^2=0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=16, \\ y=0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{16}+\frac{y^2}{25}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} y=\frac{z}{5}, \\ x=0 \end{cases}$ is revolved about the z - axis and the y - axis;

d) the parabola $\begin{cases} x^2=3z, \\ y=0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2-\frac{y^2}{9}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(2,0,0)$ and its directrix is the ellipse $\begin{cases} \frac{y^2}{9}+\frac{z^2}{25}=1, \\ x=-2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №10

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 + 2x - 6y + 6 = 0$;
b) $16x^2 + 49y^2 + 128x + 98y - 479 = 0$;
c) $x^2 - 4y^2 - 4x - 32y - 24 = 0$;
d) $x^2 - 2x + 4y - 3 = 0$;
e) $4x^2 - y^2 + 16x - 8y = 0$;
f) $x = -4 + \frac{3}{5}\sqrt{-y^2 + 2y + 24}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2 + z^2 + 2x + 2z + 1 = 0$;
b) $x^2 + 9y^2 + 18y = 0$;
c) $4y^2 - z^2 - 8y = 0$;
d) $2z^2 - x + 3 = 0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 - 14x + y^2 + z^2 + 45 = 0$;
b) $4x^2 + y^2 + 9z^2 = 36$;
c) $25x^2 - y^2 + 9z^2 = 225$;
d) $4x^2 + y^2 - z^2 = -4$;
e) $49x^2 + z^2 = 49y$;
f) $z^2 - x^2 = y$;
g) $4x^2 + 16y^2 - z^2 = 0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2 + y^2 = 25, \\ z = 0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \frac{y^2}{36} + \frac{z^2}{25} = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} z = \frac{x}{6}, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

d) the parabola $\begin{cases} y^2 = 4x, \\ z = 0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} y^2 - \frac{z^2}{4} = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,2,0)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{16} + \frac{z^2}{25} = 1, \\ y = -1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №11

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+4x+8y-16=0$; b) $4x^2+y^2-32x+6y+9=0$;

c) $4x^2-9y^2-8x+54y-113=0$; d) $y^2-2x-2y-7=0$;

e) $9x^2+49y^2-36x+196y+232=0$; f) $y=-3+\dfrac{4}{5}\sqrt{-x^2-8x+9}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2+z^2-4y+4z+4=0$; b) $x^2+9z^2-6x=0$;

c) $4x^2-y^2-4y=0$; d) $z^2+3y+2z+1=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+6y+z^2+8=0$; b) $x^2+9y^2+16z^2=144$;

c) $4x^2+49y^2-z^2=196$; d) $25x^2-y^2+z^2=-25$;

e) $4y^2+16z^2=64x$; f) $x^2-y^2=z$;

g) $9x^2-y^2+z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2+z^2=81, \\ x=0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \dfrac{x^2}{9}+\dfrac{z^2}{4}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} y=\dfrac{x}{2}, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

d) the parabola $\begin{cases} y^2=5z, \\ x=0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2-\dfrac{z^2}{36}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,-3)$ and its directrix is the ellipse $\begin{cases} \dfrac{x^2}{36}+\dfrac{y^2}{25}=1, \\ z=1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №12

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2-4x+4y-17=0$; b) $25x^2+36y^2-200x-72y-464=0$;

c) $9x^2-4y^2+36x+32y+8=0$; d) $x^2+8x-y+13=0$;

e) $25x^2+16y^2-100x-32y+516=0$; f) $x=1-\frac{3}{4}\sqrt{y^2+4y+20}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+y^2-6x-6y+9=0$; b) $y^2+4z^2-8z=0$;

c) $x^2-9z^2+6x=0$; d) $2y^2+3x+6=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2-8z+15=0$; b) $4x^2+y^2+16z^2=64$;

c) $-x^2+25y^2+4z^2=100$; d) $x^2+4y^2-z^2=-16$;

e) $x^2+36z^2=36y$; f) $y^2-z^2=x$;

g) $9x^2+4y^2-z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=4, \\ y=0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{4}+\frac{y^2}{16}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} z=\frac{y}{3}, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

d) the parabola $\begin{cases} z^2=6x, \\ y=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{25}-y^2=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(-3,0,0)$ and its directrix is the ellipse $\begin{cases} \frac{y^2}{36}+\frac{z^2}{16}=1, \\ x=2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №13

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 4x - 8y + 4 = 0;$

b) $49x^2 + 25y^2 + 392x - 150y - 216 = 0;$

c) $16x^2 - 9y^2 + 64x - 36y - 116 = 0;$

d) $y^2 + 8x + 4y - 4 = 0;$

e) $x^2 + 4y^2 + 4x - 16y + 56 = 0;$

f) $x = 4 + \frac{4}{3}\sqrt{y^2 - 6y}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2 + z^2 + 2y + 2z + 1 = 0;$

b) $4x^2 + y^2 + 4y = 0;$

c) $9y^2 - z^2 + 6z = 0;$

d) $2x^2 - z + 1 = 0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + 10x + y^2 + z^2 + 24 = 0;$

b) $4x^2 + 36y^2 + z^2 = 144;$

c) $4x^2 - y^2 + 9z^2 = 36;$

d) $-x^2 + 4y^2 + 4z^2 = -4;$

e) $9x^2 + 4y^2 = 36z;$

f) $z^2 - x^2 = y;$

g) $-x^2 + 25y^2 + 4z^2 = 0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2 + y^2 = 9, \\ z = 0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \frac{y^2}{4} + \frac{z^2}{25} = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} x = \frac{z}{4}, \\ y = 0 \end{cases}$ is revolved about the z - axis and the x - axis;

d) the parabola $\begin{cases} x^2 = 2y, \\ z = 0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{y^2}{16} - z^2 = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,-3,0)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{36} + \frac{z^2}{9} = 1, \\ y = 1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №14

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+4x-4y-1=0$;

b) $25x^2+64y^2+50x+256y-1319=0$;

c) $4x^2-y^2-16x-6y+23=0$;

d) $x^2-8x+6y-2=0$;

e) $4x^2-25y^2+16x-50y-9=0$;

f) $y=2+\frac{5}{4}\sqrt{-x^2-2x+15}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+y^2-4x+4y+4=0$;

b) $9x^2+z^2-18x=0$;

c) $4x^2-y^2+8x=0$;

d) $y^2-6y-5z+9=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2-12y+z^2+35=0$;

b) $x^2+25y^2+4z^2=100$;

c) $16x^2+4y^2-z^2=64$;

d) $9x^2-y^2+z^2=-9$;

e) $y^2+16z^2=16x$;

f) $x^2-y^2=z$;

g) $25x^2-y^2+z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2+z^2=100, \\ x=0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \frac{x^2}{36}+\frac{z^2}{4}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} y=\frac{x}{5}, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

d) the parabola $\begin{cases} y^2=3z, \\ x=0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2-\frac{z^2}{9}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,-2)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{25}+\frac{y^2}{9}=1, \\ z=2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №15

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+4x+6y+9=0$;

b) $49x^2+36y^2-294x+72y-1287=0$;

c) $25x^2-9y^2-100x+36y-161=0$;

d) $y^2-x+6y+7=0$;

e) $36x^2+16y^2-72x-64y+100=0$;

f) $x=1-\dfrac{5}{3}\sqrt{-y^2-8y-7}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+z^2-6x-6z+9=0$;

b) $9y^2+z^2-6z=0$;

c) $4x^2-z^2-4z=0$;

d) $3x^2+2y+2=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2+14z+48=0$;

b) $9x^2+y^2+4z^2=36$;

c) $-x^2+9y^2+25z^2=225$;

d) $x^2+4y^2-z^2=-4$;

e) $x^2+49z^2=49y$;

f) $y^2-z^2=x$;

g) $25x^2+4y^2-z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=121, \\ y=0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \dfrac{x^2}{16}+\dfrac{y^2}{9}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} z=\dfrac{y}{6}, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

d) the parabola $\begin{cases} z^2=4x, \\ y=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} \dfrac{x^2}{4}-y^2=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(-2,0,0)$ and its directrix is the ellipse $\begin{cases} \dfrac{y^2}{25}+\dfrac{z^2}{16}=1, \\ x=1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №16

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2-2x-4y-20=0$;

b) $4x^2+9y^2+16x-54y+61=0$;

c) $16x^2-36y^2+96x-72y+684=0$;

d) $x^2-2x-4y-15=0$;

e) $4x^2+y^2+24x-2y+53=0$;

f) $y=3-\dfrac{3}{4}\sqrt{x^2-4x-12}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2+z^2-2y+2z+1=0$;

b) $4x^2+y^2+8x=0$;

c) $9y^2-z^2-18y=0$;

d) $z^2+3x-4z+4=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2-6x+y^2+z^2+5=0$;

b) $16x^2+9y^2+z^2=144$;

c) $49x^2-y^2+4z^2=196$;

d) $-x^2+25y^2+z^2=-25$;

e) $16y^2+4z^2=64x$;

f) $z^2-x^2=y$;

g) $-x^2+9y^2+z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+y^2=1, \\ z=0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \dfrac{y^2}{25}+\dfrac{z^2}{9}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} z=\dfrac{x}{2}, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

d) the parabola $\begin{cases} y^2=5x, \\ z=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} y^2-\dfrac{z^2}{36}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,3,0)$ and its directrix is the ellipse $\begin{cases} \dfrac{x^2}{25}+\dfrac{z^2}{36}=1, \\ y=-1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №17

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 + 6x - 2y - 26 = 0$;

b) $4x^2 + y^2 + 8x + 8y + 4 = 0$;

c) $9x^2 - 16y^2 - 54x - 96y - 207 = 0$;

d) $y^2 + 2x - 6y + 5 = 0$;

e) $9x^2 + 49y^2 + 18x + 98y + 499 = 0$;

f) $x = -1 - \frac{3}{4}\sqrt{y^2 - 8y}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2 + y^2 - 4x - 4y + 4 = 0$;

b) $x^2 + 4z^2 - 4x = 0$;

c) $x^2 - 9y^2 - 18y = 0$;

d) $2z^2 - 3y - 9 = 0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + y^2 + 8y + z^2 + 12 = 0$;

b) $x^2 + 4y^2 + 16z^2 = 64$;

c) $4x^2 + 25y^2 - z^2 = 100$;

d) $x^2 - y^2 + 4z^2 = -16$;

e) $16y^2 + z^2 = 16x$;

f) $x^2 - y^2 = z$;

g) $9x^2 - y^2 + 4z^2 = 0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2 + z^2 = 64, \\ x = 0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{9} + \frac{z^2}{25} = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} x = \frac{y}{3}, \\ z = 0 \end{cases}$ is revolved about the y - axis and the x - axis;

d) the parabola $\begin{cases} z^2 = 6y, \\ x = 0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{25} - z^2 = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,3)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{16} + \frac{y^2}{36} = 1, \\ z = -2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №18

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+6x+6y-7=0$; b) $4x^2+25y^2-32x+100y+64=0$;

c) $4x^2-y^2-24x+2y+71=0$; d) $x^2+2x+y-3=0$;

e) $49x^2-16y^2-294x-128y+185=0$; f) $y=-2-\dfrac{5}{3}\sqrt{-x^2-8x-7}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+z^2+6x-6z+9=0$; b) $y^2+9z^2-18z=0$;

c) $x^2-4z^2+4x=0$; d) $y^2-7x+6y+9=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2-10z+21=0$; b) $4x^2+y^2+36z^2=144$;

c) $-x^2+9y^2+4z^2=36$; d) $25x^2+25y^2-z^2=-25$;

e) $9x^2+4z^2=36y$; f) $y^2-z^2=x$;

g) $4x^2+16y^2-z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=49, \\ y=0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \dfrac{x^2}{36}+\dfrac{y^2}{9}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} y=\dfrac{z}{4}, \\ x=0 \end{cases}$ is revolved about the z - axis and the y - axis;

d) the parabola $\begin{cases} x^2=2z, \\ y=0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2-\dfrac{y^2}{16}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(2,0,0)$ and its directrix is the ellipse $\begin{cases} \dfrac{y^2}{16}+\dfrac{z^2}{25}=1, \\ x=-1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №19

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2-6x+8y+21=0;$

b) $16x^2+9y^2-32x-72y+16=0;$

c) $9x^2-25y^2+54x+150y-369=0;$

d) $y^2-8x+4y-28=0;$

e) $x^2+4y^2-6x-24y+45=0;$

f) $x=2+\frac{5}{4}\sqrt{-y^2-2y+15}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2+z^2-2y-2z+1=0;$

b) $x^2+9y^2+6x=0;$

c) $y^2-4z^2-8z=0;$

d) $2x^2+z-2=0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+12x+y^2+z^2+32=0;$

b) $4x^2+25y^2+z^2=100;$

c) $4x^2-y^2+16z^2=64;$

d) $-x^2+9y^2+z^2=-9;$

e) $x^2+36y^2=36z;$

f) $z^2-x^2=y;$

g) $-x^2+25y^2+z^2=0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+y^2=16, \\ z=0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \frac{y^2}{16}+\frac{z^2}{25}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} z=\frac{x}{5}, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

d) the parabola $\begin{cases} y^2=3x, \\ z=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} y^2-\frac{z^2}{9}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,2,0)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{9}+\frac{z^2}{25}=1, \\ y=-2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №20

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 6x - 6y + 9 = 0$;

b) $9x^2 + 25y^2 + 72x - 100y + 19 = 0$;

c) $25x^2 - 4y^2 + 150x - 32y + 261 = 0$;

d) $x^2 - 4x - 6y - 2 = 0$;

e) $36x^2 + 16y^2 + 72x - 32y + 628 = 0$;

f) $y = 2 + \frac{4}{3}\sqrt{x^2 - 8x + 7}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2 + y^2 + 4x - 4y + 4 = 0$;

b) $x^2 + 4z^2 + 8z = 0$;

c) $x^2 - 9y^2 - 6x = 0$;

d) $y^2 - 4y + 5z + 4 = 0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + y^2 - 14y + z^2 + 45 = 0$;

b) $x^2 + 9y^2 + 4z^2 = 36$;

c) $25x^2 + 9y^2 - z^2 = 225$;

d) $x^2 - y^2 + 4z^2 = -4$;

e) $16y^2 + z^2 = 16x$;

f) $x^2 - y^2 = z$;

g) $25x^2 - y^2 + 4z^2 = 0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2 + z^2 = 121, \\ x = 0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{25} + \frac{z^2}{36} = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} x = \frac{y}{6}, \\ z = 0 \end{cases}$ is revolved about the y - axis and the x - axis;

d) the parabola $\begin{cases} z^2 = 4y, \\ x = 0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{4} - z^2 = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,2)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{16} + \frac{y^2}{25} = 1, \\ z = -1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №21

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+6x-8y-11=0$;
b) $4x^2+y^2+16x+2y-19=0$;
c) $25x^2-16y^2-150x-64y-239=0$;
d) $y^2+x-4y=0$;
e) $x^2+9y^2+6x+36y+81=0$;
f) $y=1-\frac{3}{4}\sqrt{x^2+4x+20}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+z^2+6x+6z+9=0$;
b) $4y^2+z^2-4z=0$;
c) $9x^2-z^2+18x=0$;
d) $3x^2-2y+6=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2+6z+8=0$;
b) $16x^2+y^2+9z^2=144$;
c) $-x^2+4y^2+49z^2=196$;
d) $x^2+25y^2-z^2=-25$;
e) $4x^2+16z^2=64y$;
f) $y^2-z^2=x$;
g) $x^2+9y^2-z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=81, \\ y=0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \frac{x^2}{4}+\frac{y^2}{9}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} z=\frac{y}{2}, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

d) the parabola $\begin{cases} z^2=5x, \\ y=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{36}-y^2=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(-3,0,0)$ and its directrix is the ellipse $\begin{cases} \frac{y^2}{36}+\frac{z^2}{25}=1, \\ x=1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №22

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 + 6x + 4y - 12 = 0$; b) $9x^2 + 49y^2 - 18x + 98y - 383 = 0$;

c) $4x^2 - y^2 - 24x + 8y + 36 = 0$; d) $x^2 + 4x + 4y = 0$;

e) $x^2 - 9y^2 - 2x - 54y - 80 = 0$; f) $x = -1 + \frac{4}{5}\sqrt{-y^2 - 2y + 24}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2 + z^2 + 2y - 2z + 1 = 0$; b) $9x^2 + y^2 + 18x = 0$;

c) $y^2 - 4z^2 - 4y = 0$; d) $z^2 - 3x - 6z + 9 = 0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 - 8x + y^2 + z^2 + 15 = 0$; b) $16x^2 + 4y^2 + z^2 = 64$;

c) $25x^2 - y^2 + 4z^2 = 100$; d) $-x^2 + y^2 + 4z^2 = -16$;

e) $36x^2 + y^2 = 36z$; f) $z^2 - x^2 = y$;

g) $-x^2 + 9y^2 + 4z^2 = 0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2 + y^2 = 4, \\ z = 0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \frac{y^2}{4} + \frac{z^2}{16} = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} x = \frac{z}{3}, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

d) the parabola $\begin{cases} x^2 = 6y, \\ z = 0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{y^2}{25} - z^2 = 1, \\ x = 0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0, -3, 0)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{36} + \frac{z^2}{16} = 1, \\ y = 2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №23

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 2x + 6y + 1 = 0$;

b) $25x^2 + 16y^2 - 100x - 32y - 284 = 0$;

c) $4x^2 - 25y^2 + 24x + 100y - 164 = 0$;

d) $y^2 - 2x + 2y - 1 = 0$;

e) $4x^2 + y^2 - 24x - 4y + 40 = 0$;

f) $y = -1 + \frac{3}{5}\sqrt{-x^2 + 8x + 9}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2 + y^2 + 4x + 4y + 4 = 0$;

b) $9x^2 + z^2 + 6z = 0$;

c) $x^2 - 4y^2 + 8y = 0$;

d) $2z^2 + 3y + 6 = 0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + y^2 + 10y + z^2 + 24 = 0$;

b) $x^2 + 4y^2 + 36z^2 = 144$;

c) $4x^2 + 9y^2 - z^2 = 36$;

d) $4x^2 - y^2 + 4z^2 = -4$;

e) $4y^2 + z^2 = 16x$;

f) $x^2 - y^2 = z$;

g) $25x^2 - y^2 + 4z^2 = 0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2 + z^2 = 49, \\ x = 0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{25} + \frac{z^2}{4} = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} y = \frac{x}{4}, \\ z = 0 \end{cases}$ is revolved about the x - axis and the y - axis;

d) the parabola $\begin{cases} y^2 = 2z, \\ x = 0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2 - \frac{z^2}{16} = 1, \\ y = 0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0, 0, -3)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{36} + \frac{y^2}{9} = 1, \\ z = 1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №24

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2-6x-4y-3=0$;

b) $16x^2+36y^2+32x-72y-524=0$;

c) $25x^2-16y^2+50x-96y+281=0$;

d) $x^2-8x-y+15=0$;

e) $25x^2+9y^2+50x-54y+331=0$;

f) $x=1-\frac{4}{3}\sqrt{y^2-2y+10}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+z^2-6x+6z+9=0$;

b) $4y^2+z^2-8y=0$;

c) $9x^2-z^2-6z=0$;

d) $y^2+7x-4y+4=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2-12z+35=0$;

b) $4x^2+y^2+25z^2=100$;

c) $-x^2+16y^2+4z^2=64$;

d) $x^2+9y^2-z^2=-9$;

e) $x^2+81z^2=81y$;

f) $y^2-z^2=x$;

g) $x^2+25y^2-z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=121, \\ y=0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \frac{x^2}{4}+\frac{y^2}{36}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} z=\frac{y}{5}, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

d) the parabola $\begin{cases} z^2=3x, \\ y=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{9}-y^2=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(-2,0,0)$ and its directrix is the ellipse $\begin{cases} \frac{y^2}{25}+\frac{z^2}{9}=1, \\ x=2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №25

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+2x-6y-26=0;$

b) $49x^2+16y^2+392x+32y+16=0;$

c) $4x^2-y^2-16x-8y-36=0;$

d) $y^2+8x-2y-7=0;$

e) $16x^2+25y^2+64x+50y+489=0;$

f) $x=-4+\frac{3}{4}\sqrt{y^2-2y-15}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+y^2-2x-2y+1=0;$

b) $x^2+4y^2+4x=0;$

c) $y^2-9z^2+18z=0;$

d) $x^2-6x-3z+9=0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+14x+y^2+z^2+48=0;$

b) $4x^2+9y^2+z^2=36;$

c) $9x^2-y^2+25z^2=225;$

d) $-x^2+y^2+4z^2=-4;$

e) $64x^2+y^2=64z;$

f) $z^2-x^2=y;$

g) $-x^2+25y^2+4z^2=0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+y^2=25, \\ z=0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \frac{y^2}{16}+\frac{z^2}{9}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} x=\frac{z}{6}, \\ y=0 \end{cases}$ is revolved about the z - axis and the x - axis;

d) the parabola $\begin{cases} x^2=4y, \\ z=0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{y^2}{4}-z^2=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,-2,0)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{25}+\frac{z^2}{16}=1, \\ y=1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №26

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+4x+8y+11=0$; b) $x^2+4y^2-8x+24y-12=0$;

c) $9x^2-4y^2-18x+24y+9=0$; d) $x^2+8x+6y+10=0$;

e) $25x^2+16y^2-100x+64y+164=0$; f) $y=-3+\dfrac{5}{3}\sqrt{-x^2-8x-7}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+z^2+4x-4z+4=0$; b) $x^2+9z^2+18z=0$;

c) $4x^2-y^2-8x=0$; d) $2y^2-3z+9=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2-6y+z^2+5=0$; b) $x^2+16y^2+9z^2=144$;

c) $49x^2+4y^2-z^2=196$; d) $x^2-y^2+25z^2=-25$;

e) $16y^2+4z^2=64x$; f) $x^2-y^2=z$;

g) $x^2-y^2+9z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2+z^2=1, \\ x=0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \dfrac{x^2}{9}+\dfrac{z^2}{25}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} x=\dfrac{y}{2}, \\ z=0 \end{cases}$ is revolved about the y - axis and the x - axis;

d) the parabola $\begin{cases} z^2=5y, \\ x=0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \dfrac{x^2}{36}-z^2=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,3)$ and its directrix is the ellipse $\begin{cases} \dfrac{x^2}{25}+\dfrac{y^2}{36}=1, \\ z=-1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №27

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2 + y^2 - 4x + 4y - 8 = 0;$

b) $36x^2 + 25y^2 - 288x - 50y - 299 = 0;$

c) $4x^2 - 9y^2 + 16x + 72y - 164 = 0;$

d) $y^2 - x + 6y + 5 = 0;$

e) $x^2 + 4y^2 - 4x - 32y + 68 = 0;$

f) $x = 1 - \frac{5}{4}\sqrt{-y^2 + 4y + 12}.$

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2 + z^2 + 6y + 6z + 9 = 0;$

b) $y^2 + 9z^2 - 6y = 0;$

c) $4x^2 - z^2 + 4z = 0;$

d) $x^2 + 4x + 7y + 4 = 0.$

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2 + y^2 + z^2 + 8z + 12 = 0;$

b) $16x^2 + y^2 + 4z^2 = 64;$

c) $-x^2 + 4y^2 + 25z^2 = 100;$

d) $4x^2 + y^2 - z^2 = -16;$

e) $49x^2 + z^2 = 49y;$

f) $y^2 - z^2 = x;$

g) $4x^2 + 9y^2 - z^2 = 0.$

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2 + z^2 = 16, \\ y = 0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \frac{x^2}{25} + \frac{y^2}{9} = 1, \\ z = 0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} y = \frac{z}{3}, \\ x = 0 \end{cases}$ is revolved about the z - axis and the y - axis;

d) the parabola $\begin{cases} x^2 = 6z, \\ y = 0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2 - \frac{y^2}{25} = 1, \\ z = 0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(3,0,0)$ and its directrix is the ellipse $\begin{cases} \frac{y^2}{16} + \frac{z^2}{36} = 1, \\ x = -2. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №28

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2-4x-8y-5=0$;

b) $25x^2+49y^2+200x-294y-384=0$;

c) $9x^2-16y^2+36x-64y+116=0$;

d) $x^2-2x-4y-7=0$;

e) $9x^2+4y^2+36x-16y+88=0$;

f) $y=3-\dfrac{4}{3}\sqrt{x^2-8x+7}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+y^2+2x-2y+1=0$;

b) $x^2+4y^2-8y=0$;

c) $y^2-9z^2+6y=0$;

d) $3z^2+2x+4=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2-10x+y^2+z^2+21=0$;

b) $36x^2+4y^2+z^2=144$;

c) $9x^2-y^2+4z^2=36$;

d) $-x^2+25y^2+25z^2=-25$;

e) $4x^2+9y^2=36z$;

f) $z^2-x^2=y$;

g) $-x^2+4y^2+16z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+y^2=49, \\ z=0 \end{cases}$ is revolved about the y - axis;

b) the ellipse $\begin{cases} \dfrac{y^2}{36}+\dfrac{z^2}{9}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis;

c) the line $\begin{cases} z=\dfrac{x}{4}, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

d) the parabola $\begin{cases} y^2=2x, \\ z=0 \end{cases}$ is revolved about the x - axis;

e) the hyperbola $\begin{cases} y^2-\dfrac{z^2}{16}=1, \\ x=0 \end{cases}$ is revolved about the y - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,3,0)$ and its directrix is the ellipse $\begin{cases} \dfrac{x^2}{4}+\dfrac{z^2}{25}=1, \\ y=-1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №29

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+4x-4y-28=0$; b) $64x^2+25y^2+128x+100y-1436=0$;

c) $x^2-4y^2-4x-24y-48=0$; d) $y^2+2x-6y+1=0$;

e) $4x^2+9y^2+16x+54y+133=0$; f) $y=2+\frac{3}{4}\sqrt{x^2+2x+17}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $x^2+z^2+4x+4z+4=0$; b) $4x^2+z^2+4z=0$;

c) $9x^2-y^2+6y=0$; d) $z^2-5y+6z+9=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+12y+z^2+32=0$; b) $x^2+4y^2+25z^2=100$;

c) $4x^2+16y^2-z^2=64$; d) $x^2-y^2+9z^2=-9$;

e) $36y^2+z^2=36x$; f) $x^2-y^2=z$;

g) $x^2-y^2+25z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} y^2+z^2=64, \\ x=0 \end{cases}$ is revolved about the z - axis;

b) the ellipse $\begin{cases} \frac{x^2}{25}+\frac{z^2}{16}=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis;

c) the line $\begin{cases} x=\frac{y}{5}, \\ z=0 \end{cases}$ is revolved about the y - axis and the x - axis;

d) the parabola $\begin{cases} z^2=3y, \\ x=0 \end{cases}$ is revolved about the y - axis;

e) the hyperbola $\begin{cases} \frac{x^2}{9}-z^2=1, \\ y=0 \end{cases}$ is revolved about the x - axis and the z - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(0,0,3)$ and its directrix is the ellipse $\begin{cases} \frac{x^2}{9}+\frac{y^2}{36}=1, \\ z=-1. \end{cases}$

Quadratic Curves and Quadric Surfaces

Variant №30

Exercise 1. Reduce each equation of the quadratic curve to a canonical form using parallel transport of the coordinate system, determine the type of the curve and graph it.

For the curves b), c), d) find their vertices and co-vertices, semi-major and semi-minor axes, eccentricities, focuses, directrixes and in the case of hyperbola find also asymptotes.

a) $x^2+y^2+4x+6y-3=0$;

b) $36x^2+49y^2-216x+98y+1391=0$;

c) $9x^2-25y^2-36x+100y+161=0$;

d) $x^2+4x+y+7=0$;

e) $16x^2-36y^2-32x+144y-128=0$;

f) $x=1-\dfrac{4}{5}\sqrt{-y^2-8y+9}$.

Exercise 2. Determine the type of each given cylindrical surface and gfaph it:

a) $y^2+z^2-6y+6z+9=0$;

b) $9y^2+z^2-18y=0$;

c) $4x^2-z^2+8x=0$;

d) $2y^2-x+2=0$.

Exercise 3. Determine the type of each given quadric surface and gfaph it:

a) $x^2+y^2+z^2-14z+45=0$;

b) $9x^2+4y^2+z^2=36$;

c) $-x^2+25y^2+9z^2=225$;

d) $-x^2+4y^2+z^2=-4$;

e) $x^2+49y^2=49z$;

f) $y^2-z^2=x$;

g) $-x^2+4y^2+16z^2=0$.

Exercise 4. Find the equations of the following surfaces of revolution and graph them:

a) the circle $\begin{cases} x^2+z^2=81, \\ y=0 \end{cases}$ is revolved about the x - axis;

b) the ellipse $\begin{cases} \dfrac{x^2}{36}+\dfrac{y^2}{25}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis;

c) the line $\begin{cases} y=\dfrac{z}{6}, \\ x=0 \end{cases}$ is revolved about the z - axis and the y - axis;

d) the parabola $\begin{cases} x^2=4z, \\ y=0 \end{cases}$ is revolved about the z - axis;

e) the hyperbola $\begin{cases} x^2-\dfrac{y^2}{4}=1, \\ z=0 \end{cases}$ is revolved about the x - axis and the y - axis.

Exercise 5. Find the equation and grap the conical surface that has an apex at the point $M_0(3,0,0)$ and its directrix is the ellipse $\begin{cases} \dfrac{y^2}{9}+\dfrac{z^2}{36}=1, \\ x=-1. \end{cases}$

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Електронне мережне навчальне видання

ВИЩА МАТЕМАТИКА

КРИВІ ТА ПОВЕРХНІ ДРУГОГО ПОРЯДКУ

Практикум

(Англійською мовою)

Укладачі: Массалітіна Є.В., Пилипенко В.А.

Практикум до розділу «Криві та поверхні другого порядку» з курсу «Вища математика» для студентів технічних спеціальностей містить 30 варіантів, кожен варіант складається з 5 завдань (23 задач). Самостійне виконання цих завдань забезпечує свідоме оволодіння навчальним матеріалом, який передбачено робочою програмою з вищої математики.

Практикум може бути рекомендований в якості розрахункової роботи за темою «Криві та поверхні другого порядку» для студентів першого курсу технічних спеціальностей.

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Національний технічний університет України
«Київський політехнічний інститут імені Ігоря Сікорського»
проспект Перемоги, 37, м. Київ, 03056
https://kpi.ua
Свідоцтво про внесення до Державного реєстру видавців, виготовлювачів
і розповсюджувачів видавничої продукції ДК № 5354 від 25.05.2017 р.